Living on the edge of noise-driver order

Rachel Kuske, UBC

In Collaboration with

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Stochastic dynamics (+ uncertainty, stats) at Snowbird SIAM (Applications of) DS Conference

1995:	MS = .5	Plenary = 0	Themes: 0
1997:	MS = 6.5	Plenary = 1	Themes: 1
1999:	MS = 1.5	Plenary = 0	Themes: 0
2001:	MS = 4	Plenary = .5	Themes: 0 (.5)
2003:	MS = 3.5	Plenary = 1.5	Themes: 3
2005:	MS = 8 +CP	Plenary = 2	Themes: 1.5
2007:	MS = 10.5	Plenary = 1	Themes: 1
2009:	MS = 12	Plenary = 2	Themes: 1

Stochastic Frameworks, Paradigms, Building Blocks

Probability densities/moments

Large deviations: Tails of the density

Energy landscape/Potential well dynamics:

Markov chains

S(O/P/D)DE’s vs. discrete models

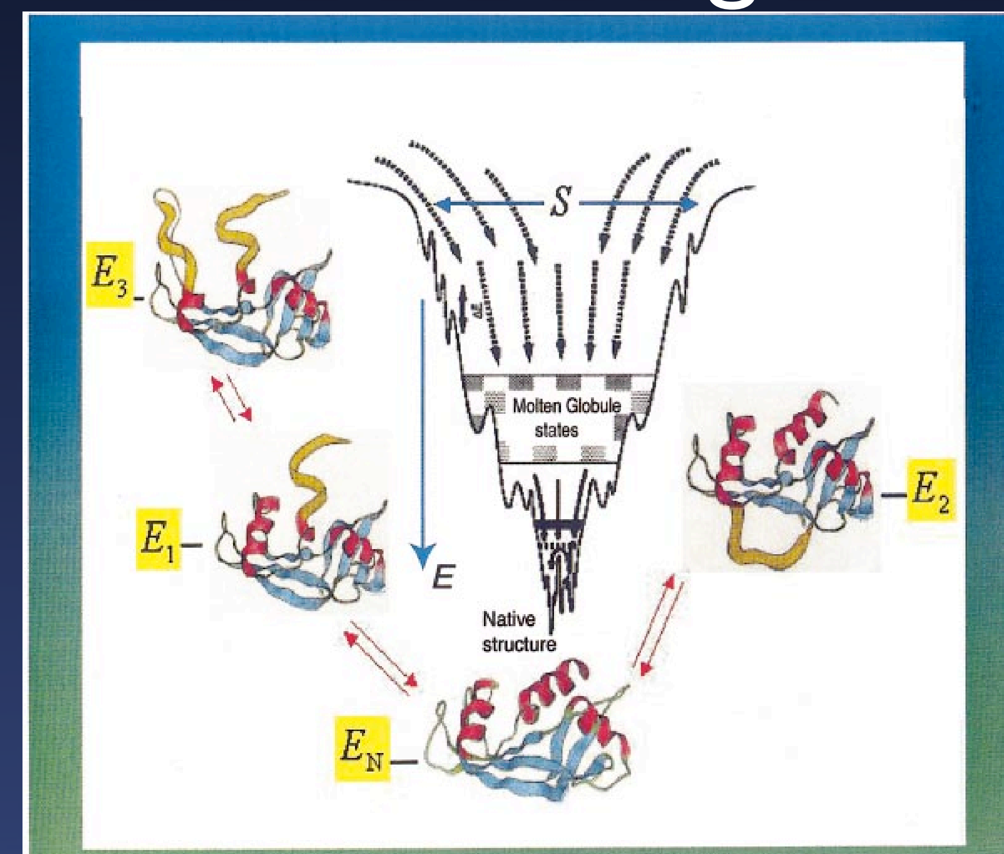
Stochastic bifurcations

Mathematical Technology Transfer

Language/Familiarity in certain fields: Physics, Computational Chemistry, Interacting Particles

Potential well dynamics:
Chemical kinetics

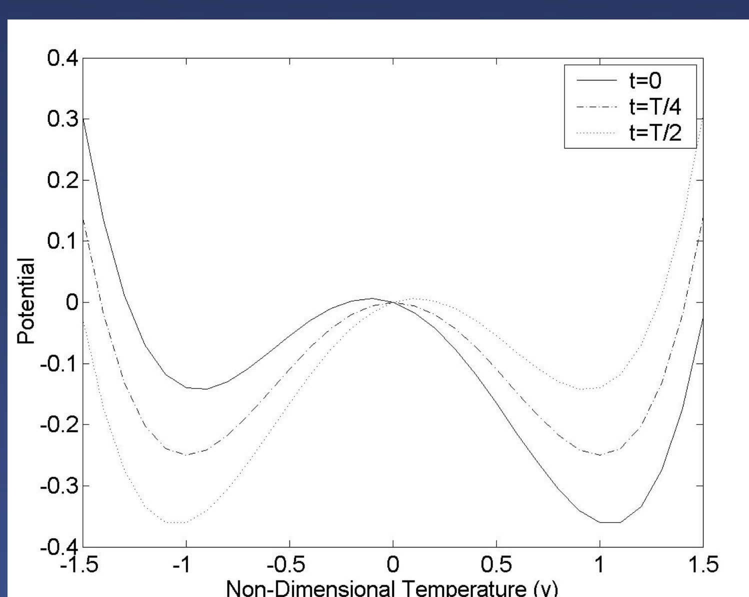
Protein folding:

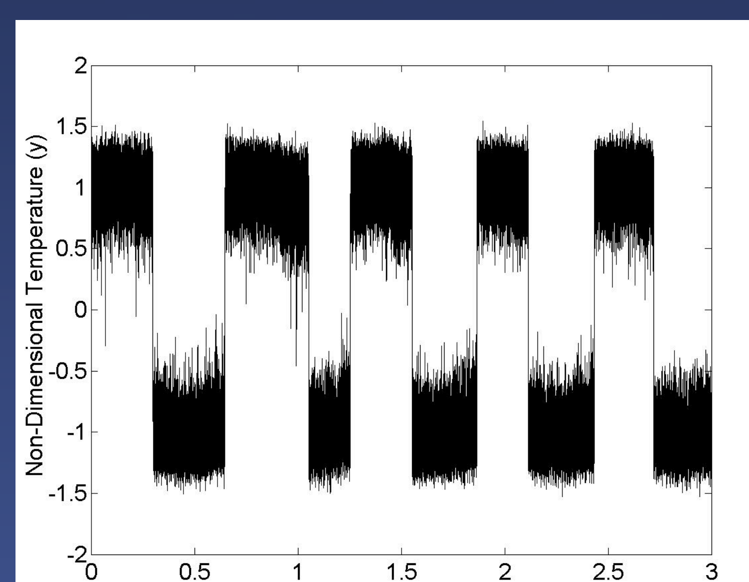


Plotkin, et al, 2002

Global energy balance climate models

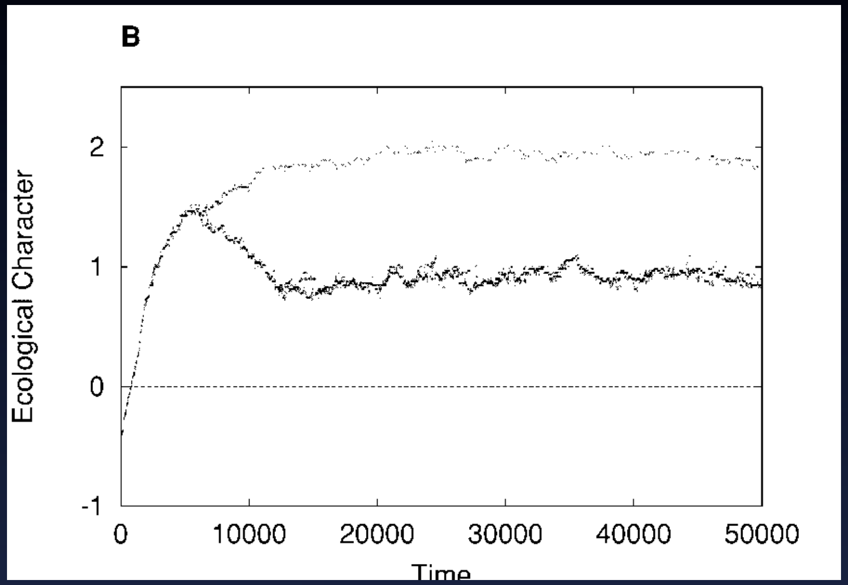
Imkeller, et al,





Mathematical Technology Transfer

Evolutionary branching: state/
time dependent potential



Doebeli, Dieckmann, 2000

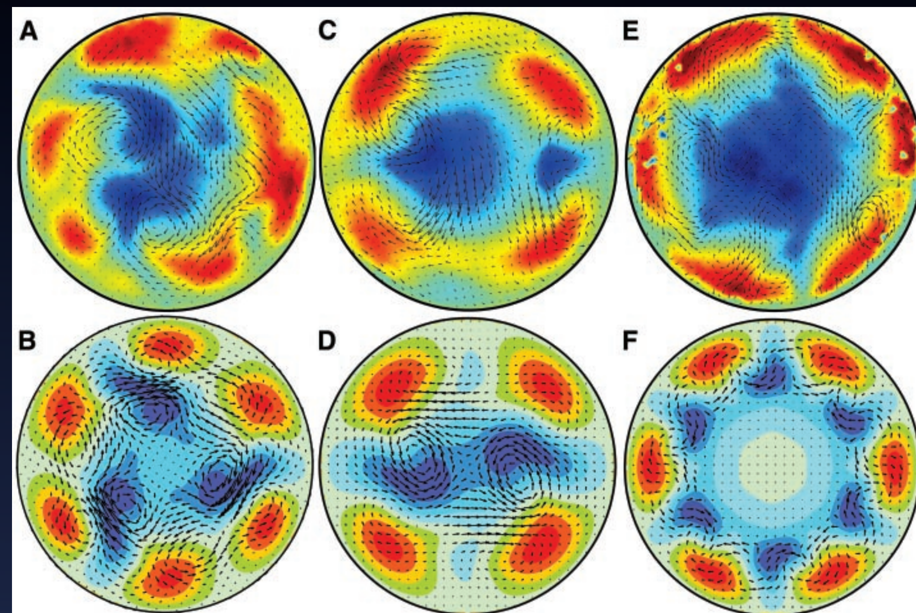
Large deviations: Tails of the density
Value at risk (finance)
Optics: Noise induced perturbations in solitons

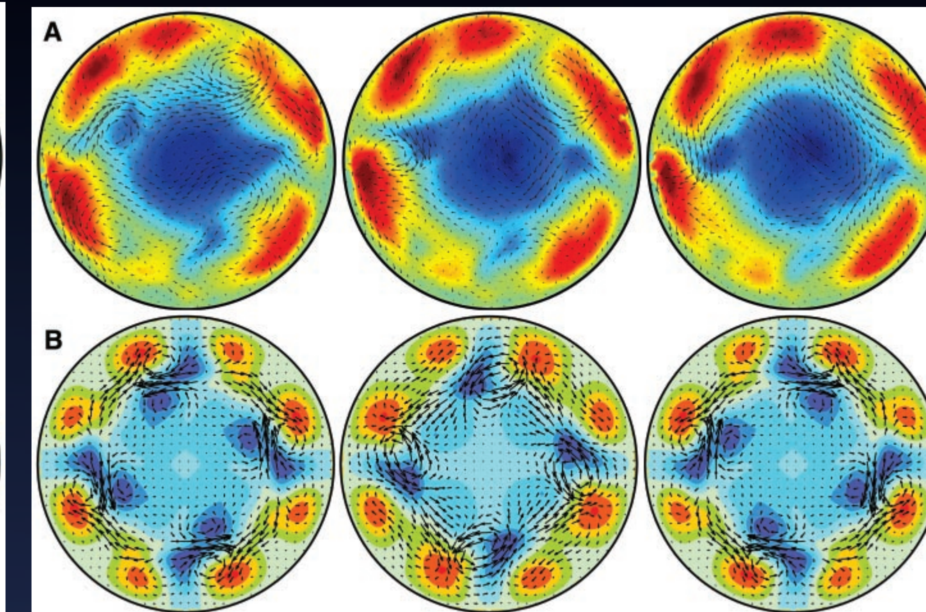
Biondini, Kath

Markov chains: Queueing models
Stochastic parametrization in atmospheric models
Crommelin, 2008

Combined cts/discrete models:
Stochastic bifurcations:

“Stabilized” transients





Laminar-turbulent transitions characterized by series of unstable transient patterns

Hof, et al, Science, 2004

Lower dimensional patterns in brain activity: feedback between different modes

Schiff, 2007

Many examples: Patterns driven by interactions between scales

Noise driven order: Stabilized transients

On the edge: Sources for sensitivity

Living: some examples of MTT to new areas

CAN'T IGNORE:

TRANSIENTS
"SMALL" PERTURBATIONS

Coherence Resonance (CR) as a Building Block ?

Different types, different contexts, different names (autonomous stochastic resonance) MTT

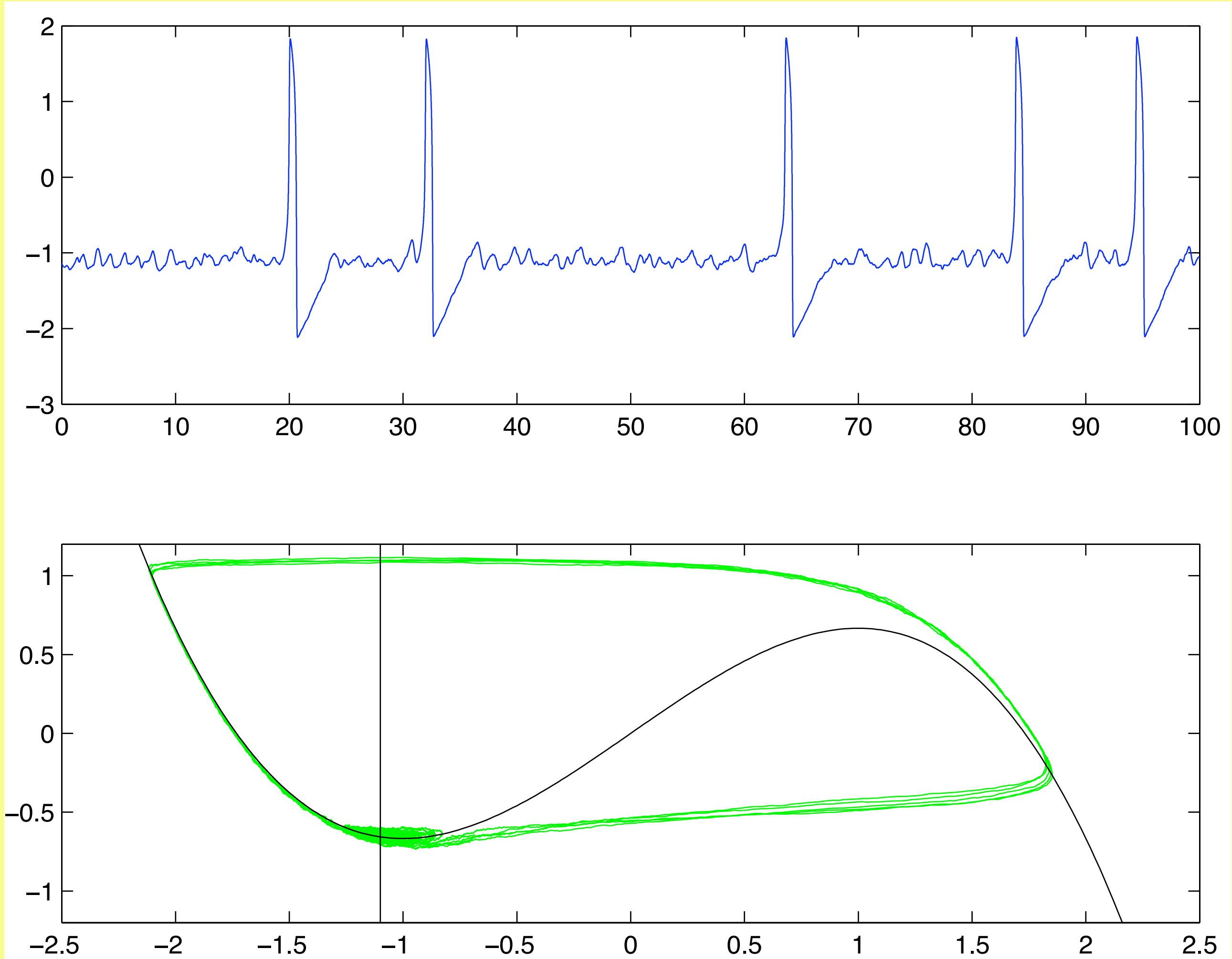
- Coherence resonance I: excursions onto unstable large amplitude oscillations

Pikovsky, Kurths, 1997

FitzHugh Nagumo + noise

Frequency increases with noise

$$\epsilon x_t = -y + (x - x^3/3)$$
$$y_t = x + a + \text{noise}$$



- Gang, Ditzinger, Ning, Haken, 1993, w/o external forcing.
- Shardlow, 2004; De Ville, vd Eijnden, Muratov, et al, 2005,07, SISR for different noise

Gene regulatory circuit

Noise-driven excursions into the competent state

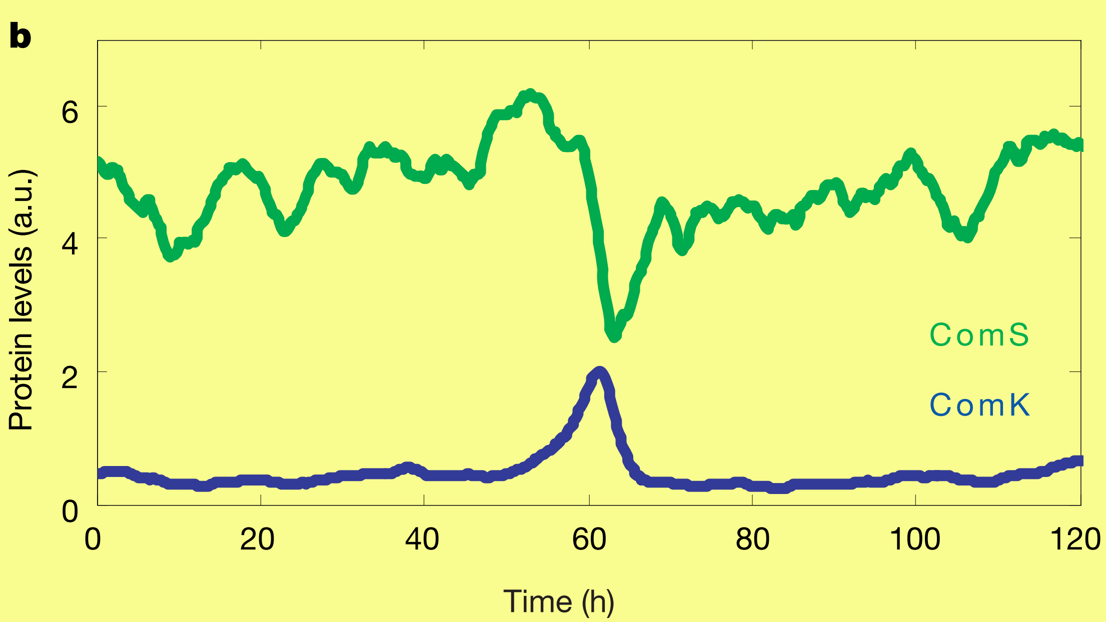
Similar coherence resonance mechanism

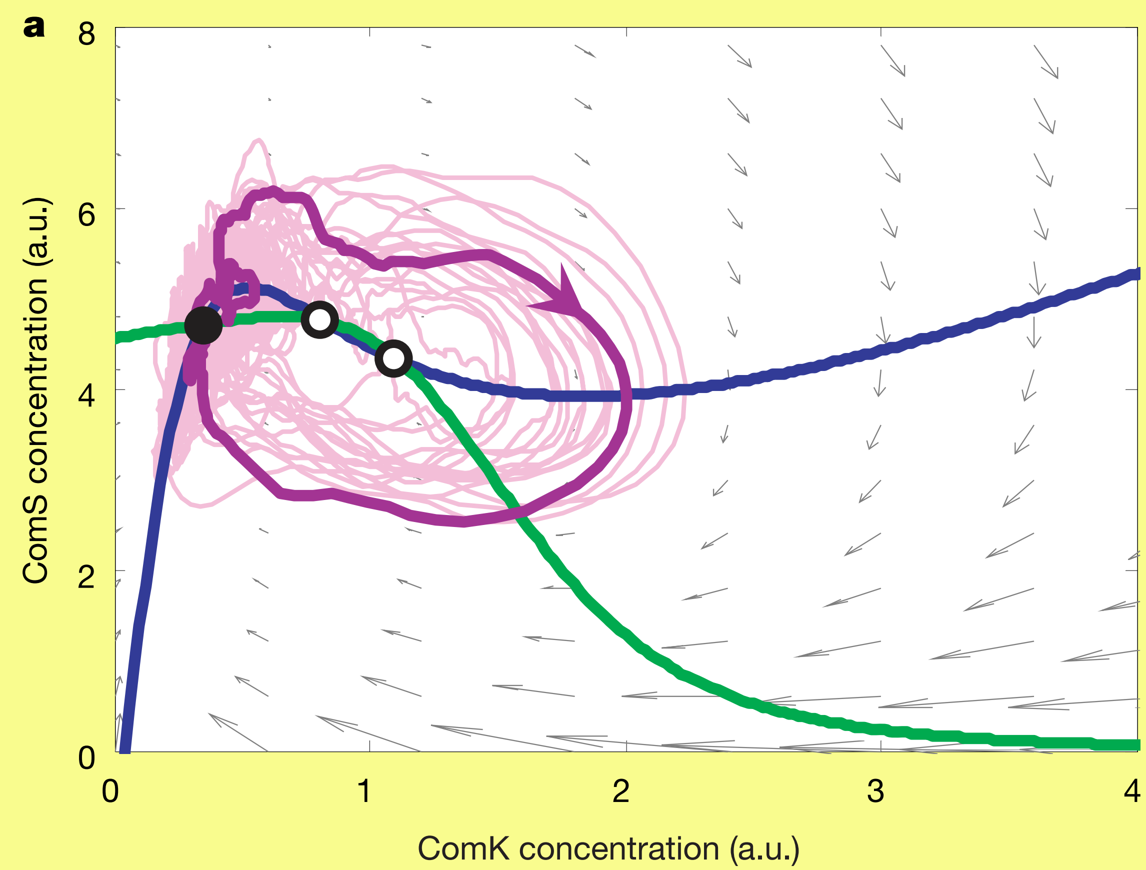
"Master" transcription factor + negative feedback in subcomplex

Suel, et al 2006, Nature

$$\frac{dK}{dt} = a_k + \frac{b_k K^n}{k_0^n + K^n} - \frac{K}{1+K+S}$$

$$\frac{dS}{dt} = \frac{b_s}{1+(K/k_1)^p} - \frac{S}{1+K+S} + \xi(t)$$

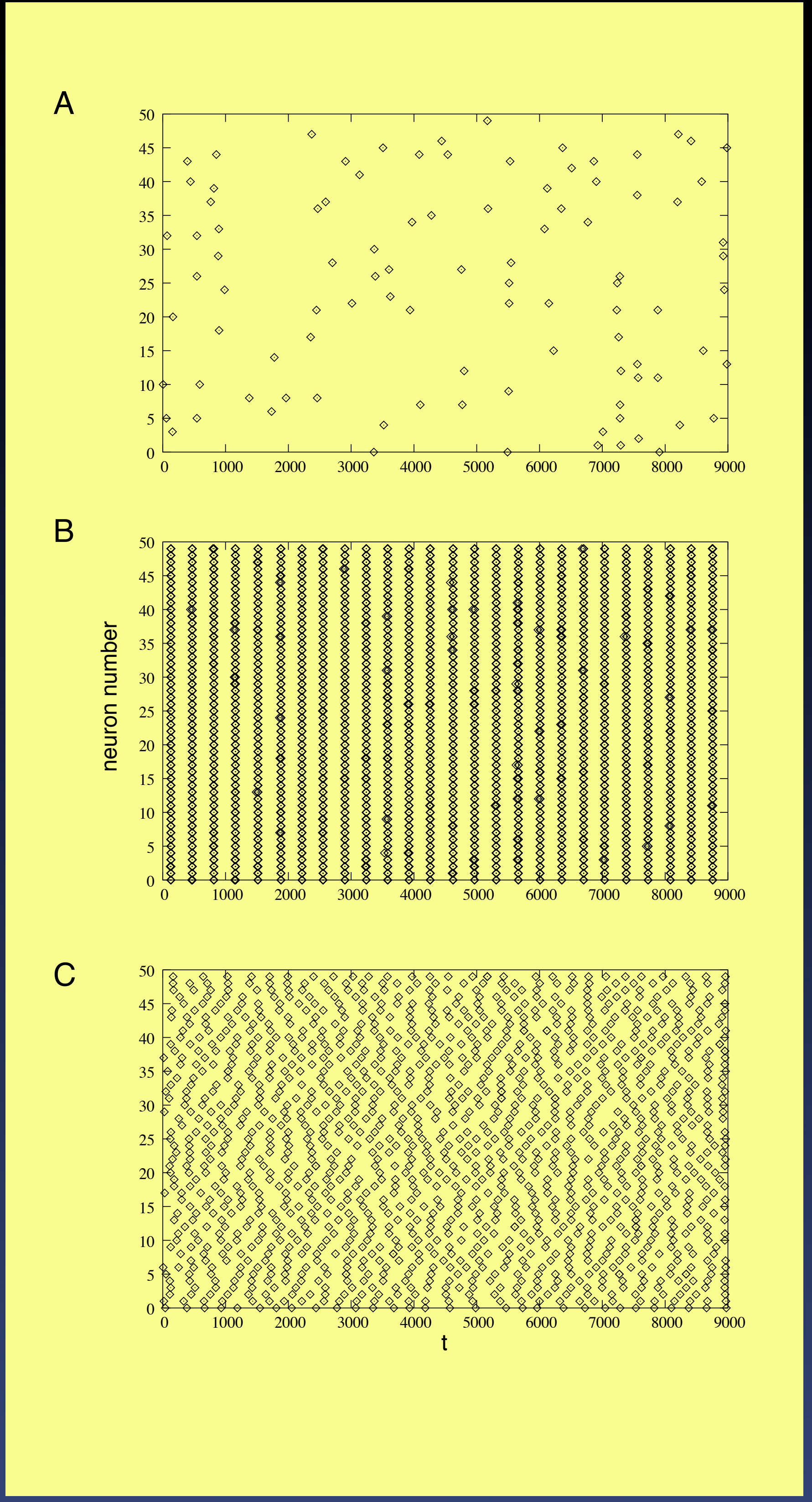


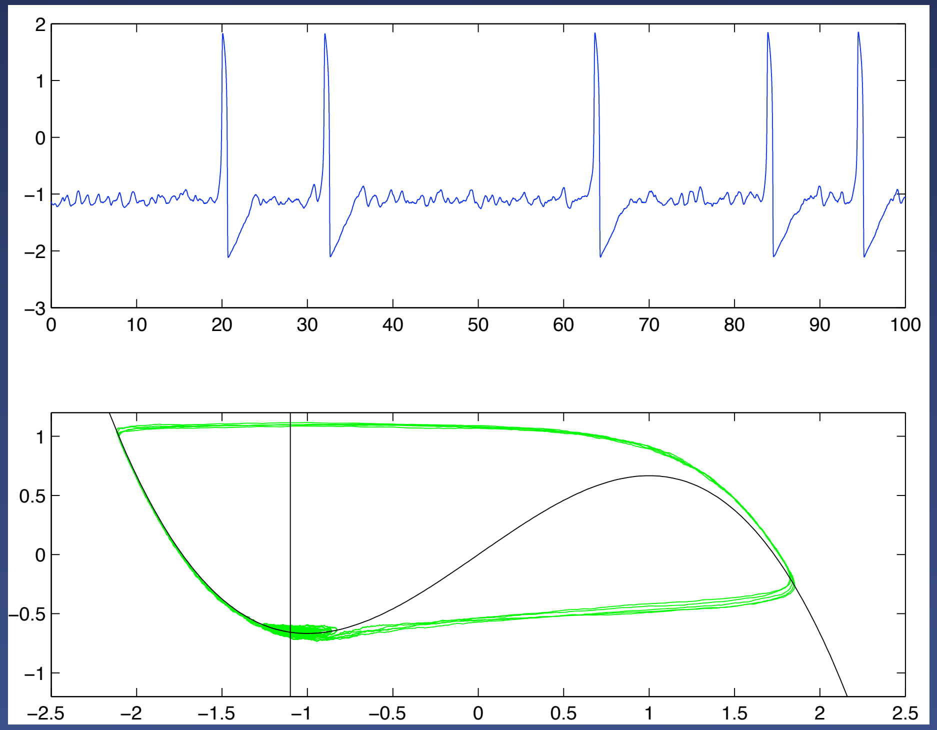


Noise induced synchrony in networks

Optimal noise levels for synchronized transitions to active states

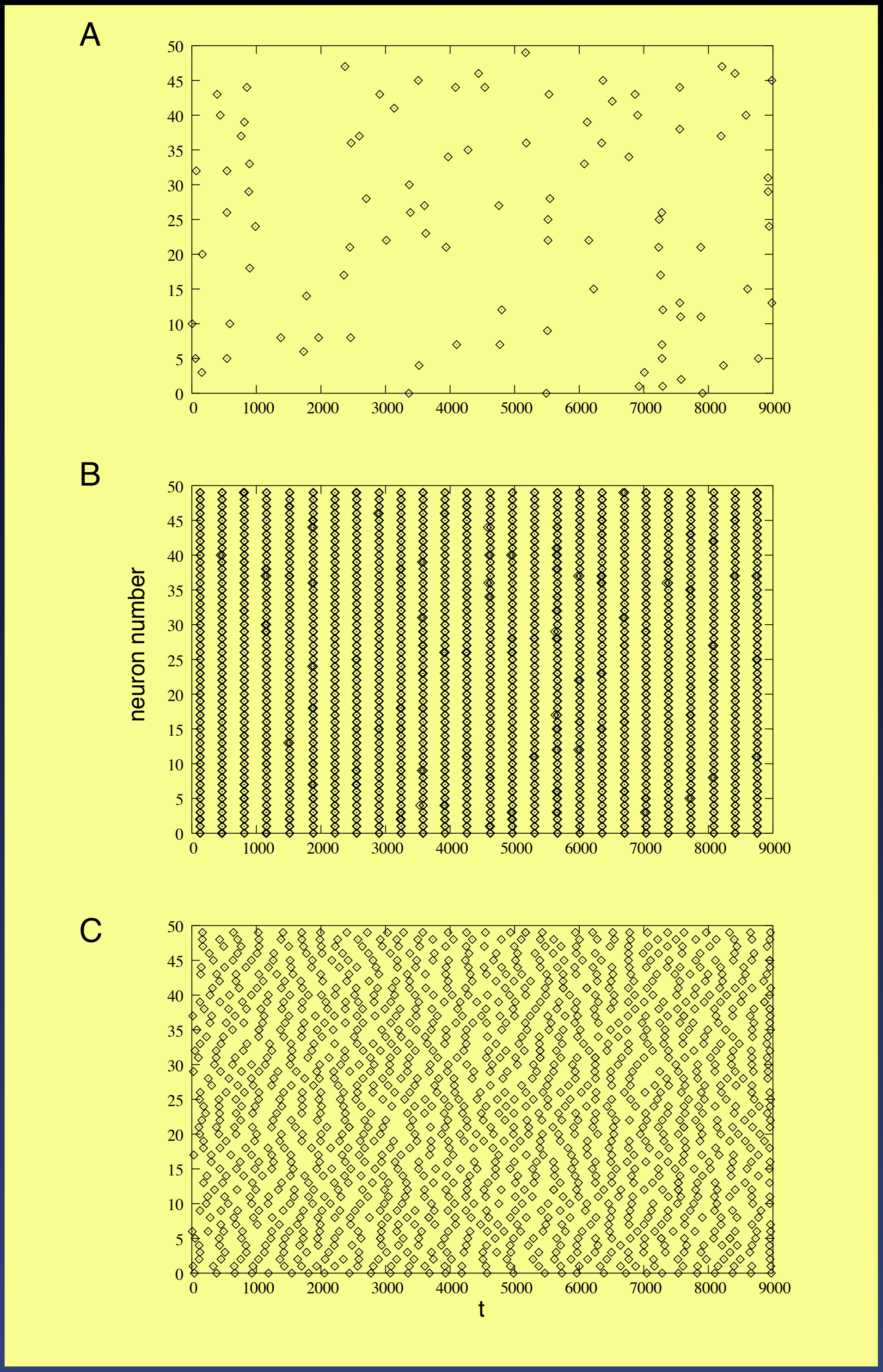
Phenomenon is a sequence of transients driven by noise





Coherence resonance I

Optimal noise levels for synchronized transitions to active states: Simple models



Excitatory/Inhibitory network

Borgers, Epstein, Kopell, 2005

LIF model + CR(II) (multiple scales)

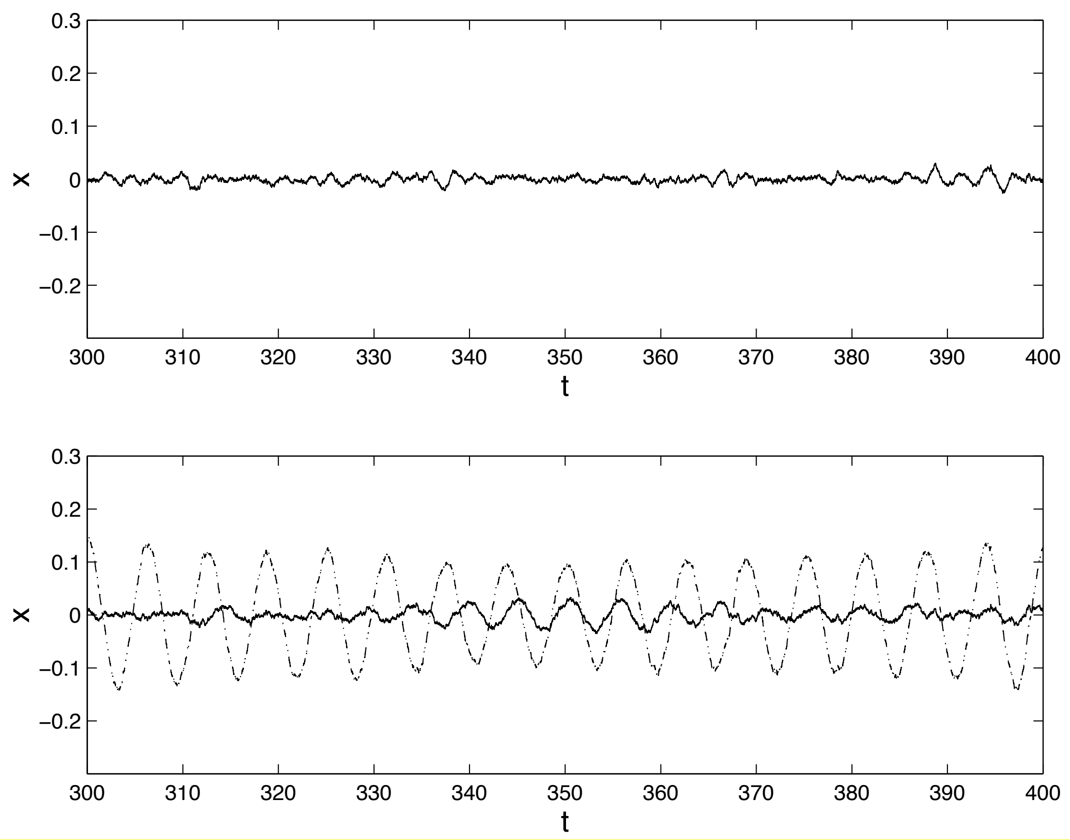
w/ S. Reinker and Y. X. Li, (2006)

- Fast refractory period/inhibition:
- Slow “organizing”: noise sensitive
- Analysis: cts and discrete probability models, insight into network structure

Coherence resonance I

- Coherence resonance II: amplification of modulated oscillations (e.g. sinusoidal)

Near a Hopf bifurcation



,

$$\frac{dx}{dt} = f(x(t), x(t-\tau)) + \delta\eta$$

For $\tau < \tau_c$ (subthreshold),

oscillations are damped:

As $\tau \longrightarrow \tau_c$

“amplification” of noise

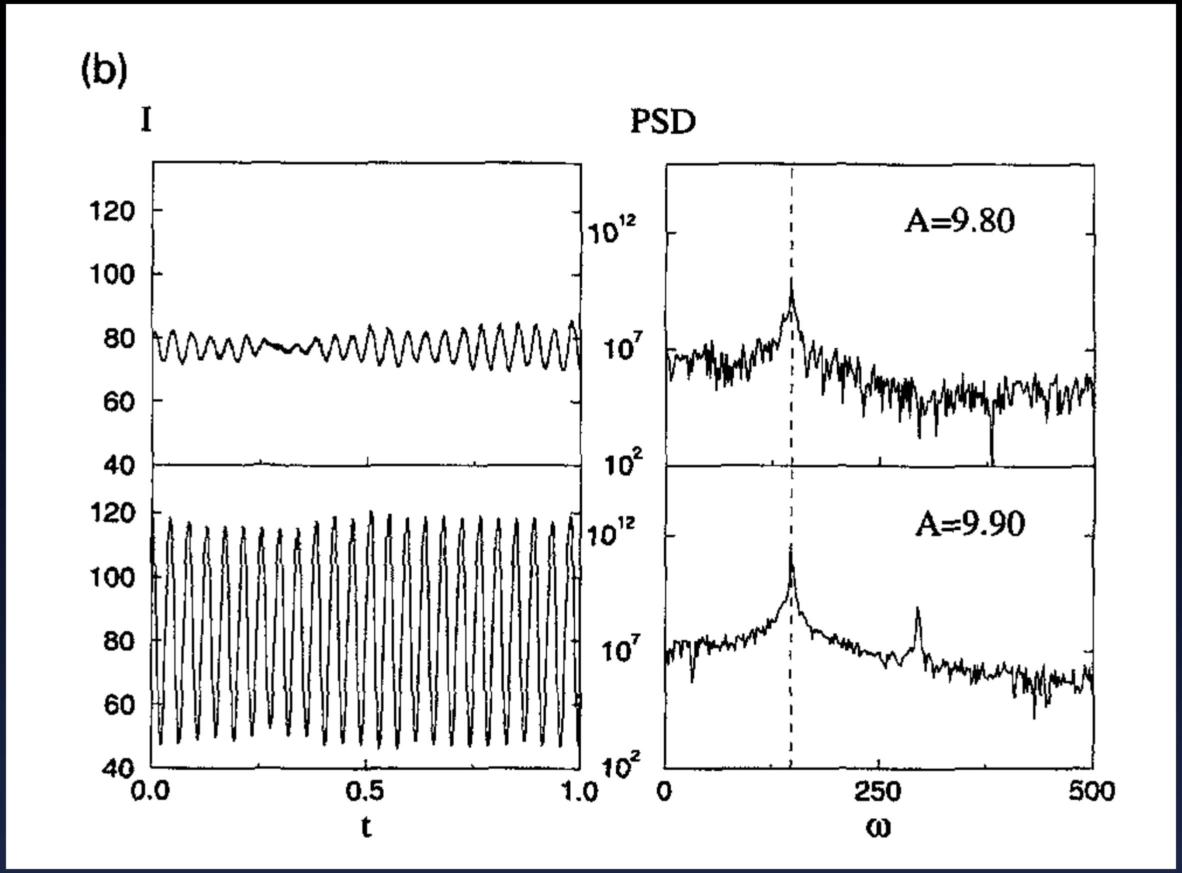
Still **subthreshold** : $\tau < \tau_c$

Noise level is **fixed**

Earlier examples of CR:

Stochastic Ikeda model

Delay from the ring laser



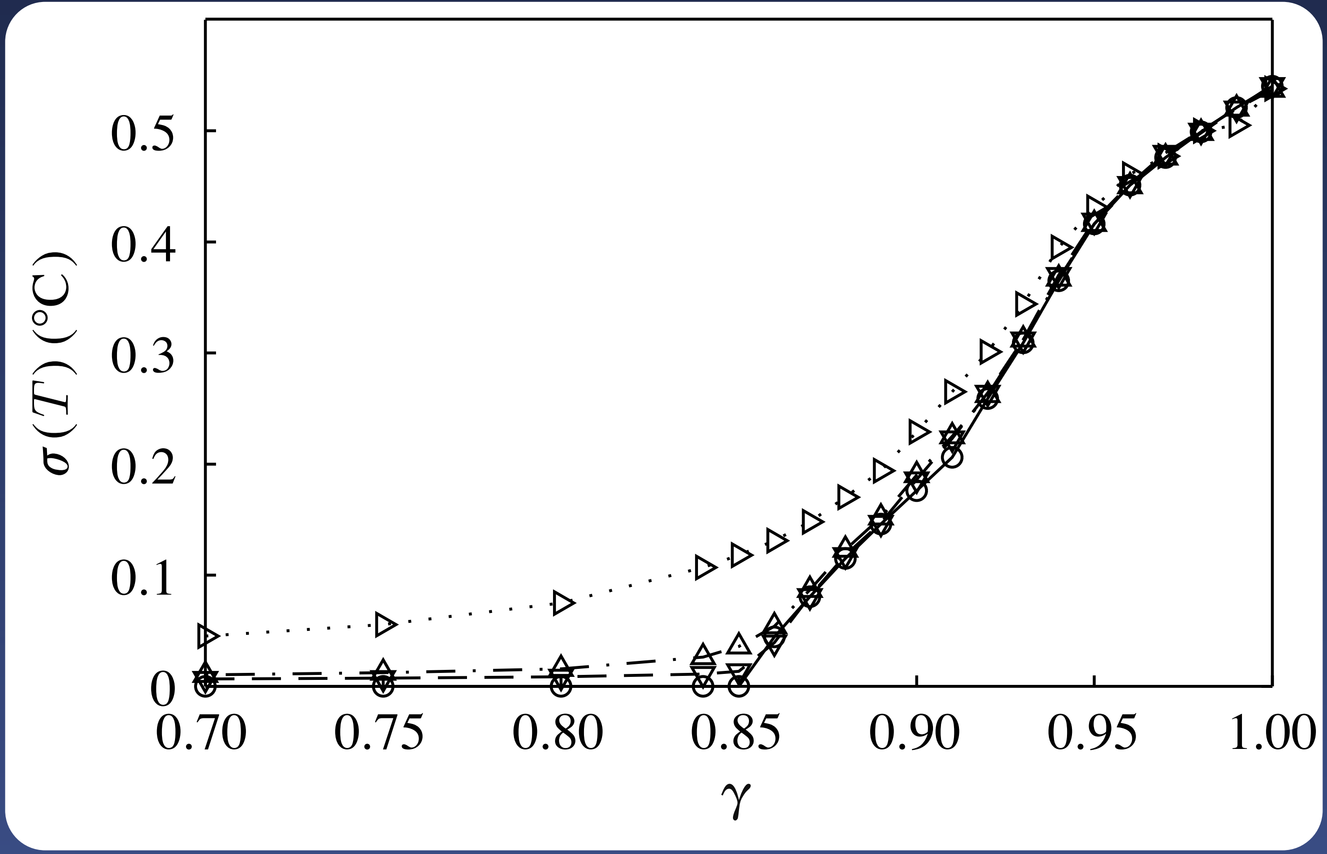
Garcia-Ojalvo, Roy, 1996

Stochastic bifurcations/

Lyapunov exponents

L. Arnold, Namachchivaya, Baxendale, 80's, 90's

SST Oscillations: model of North Atlantic temperature anomaly



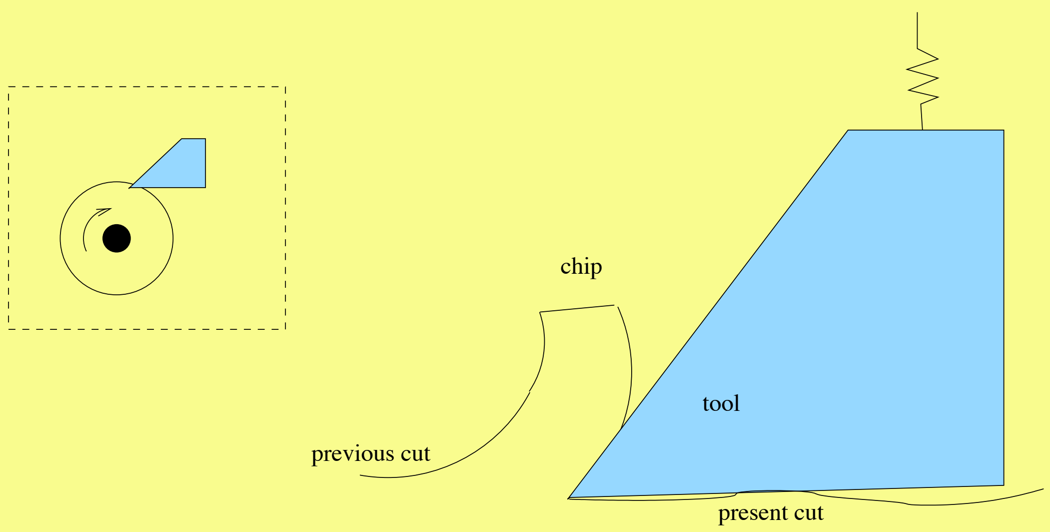
Dijkstra, et al, 2008

CRII in other applications:

Competition of noise sources in delay dynamics: controlling CR with noise

Inherent regular fluctuations in infected populations

Competition of noise sources: CR in delay dynamics



Chatter in machine tool dynamics

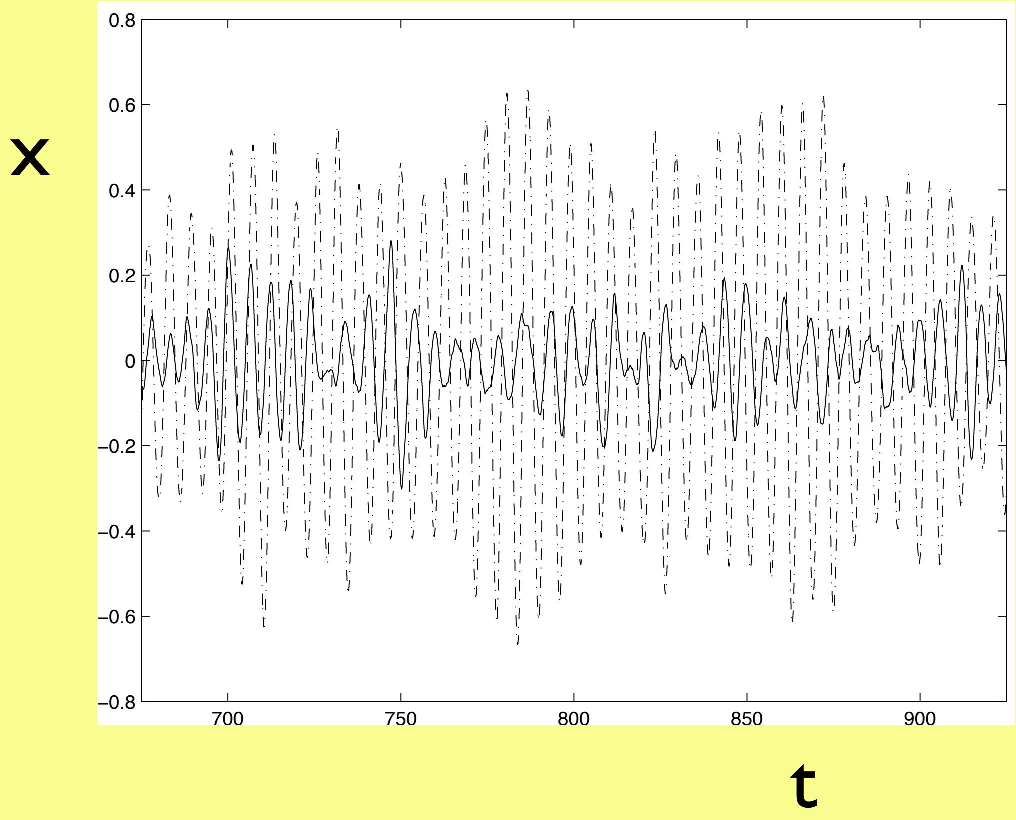
$$
dx = y\,dt \qquad dy = \left(-2\kappa y - x + \sum_{1}^{3} c_j \left[x(t-\tau) - x(t)\right]^j\right)dt + \eta\left(1 + \sum_{1}^{3} c_j \left[x(t-\tau) - x(t)\right]^j\right)
$$

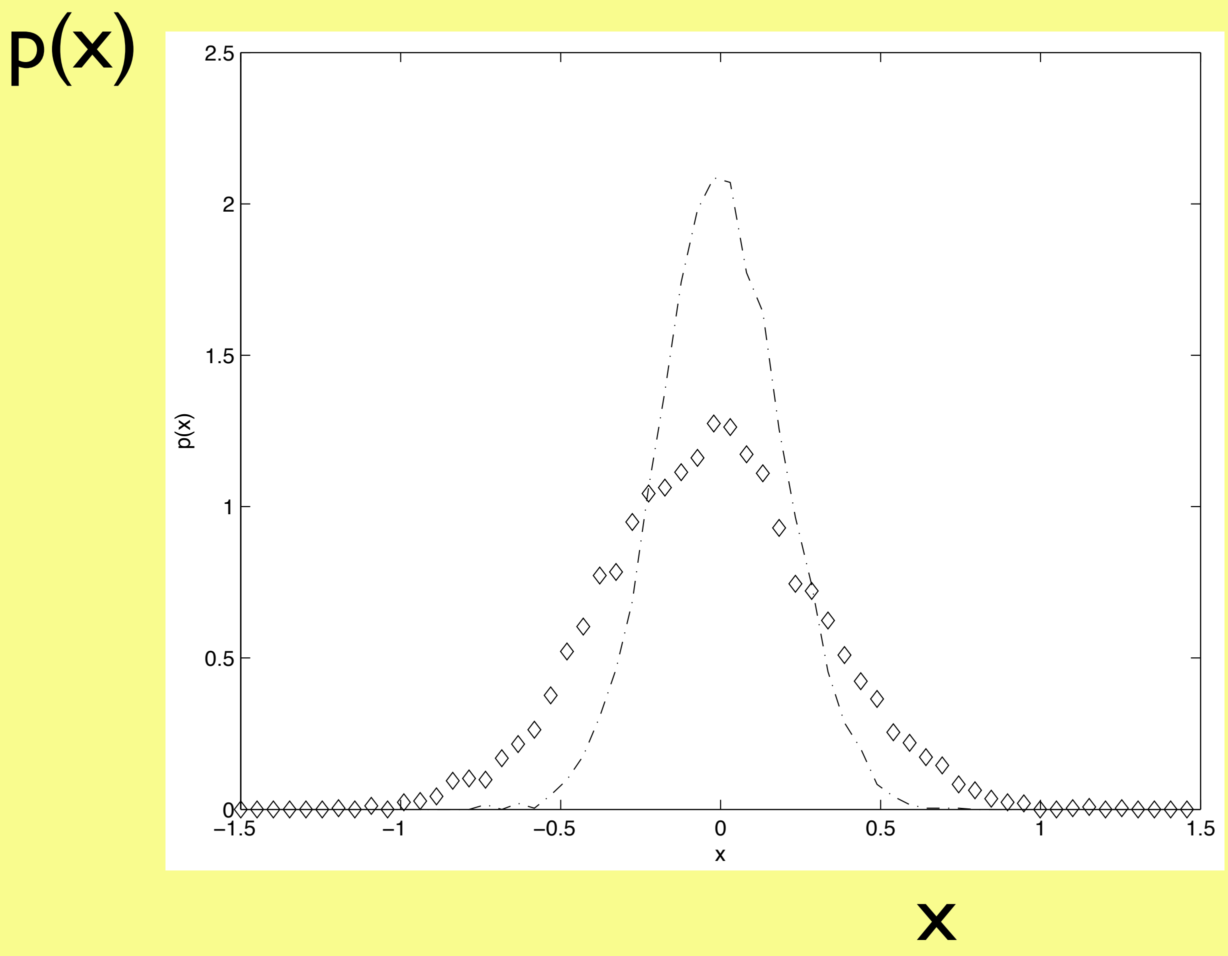
Noise sources:

Material parameters (additive + multiplicative noise)

Speed of rotation (delay)

Variation in material parameters: noise source amplifyies oscillations through CR





Probability density for machine tool position (x=0 is smooth cut)

$$
\begin{aligned}
dx &= y\,dt \\
dy &= (-2\kappa y - x + \sum_1^3 c_j\,[x(t-\tau)-x(t)]^j)dt + \delta dw + \sum_1^3 c_j\,[x(t-\tau)-x(t)]^j\,\delta dw.
\end{aligned}
$$

Variable delay

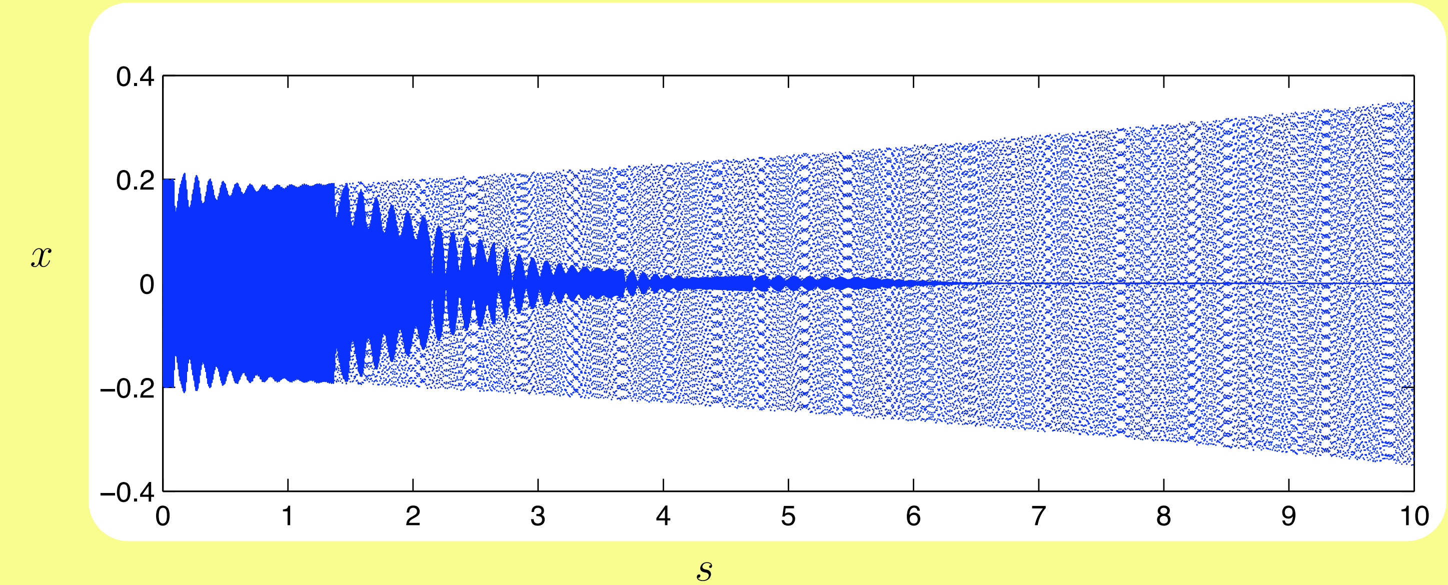
$$\tau = \frac{2\pi\alpha}{\Omega},$$

$$\Omega = \overline{\Omega}(1 + q\zeta(t))$$

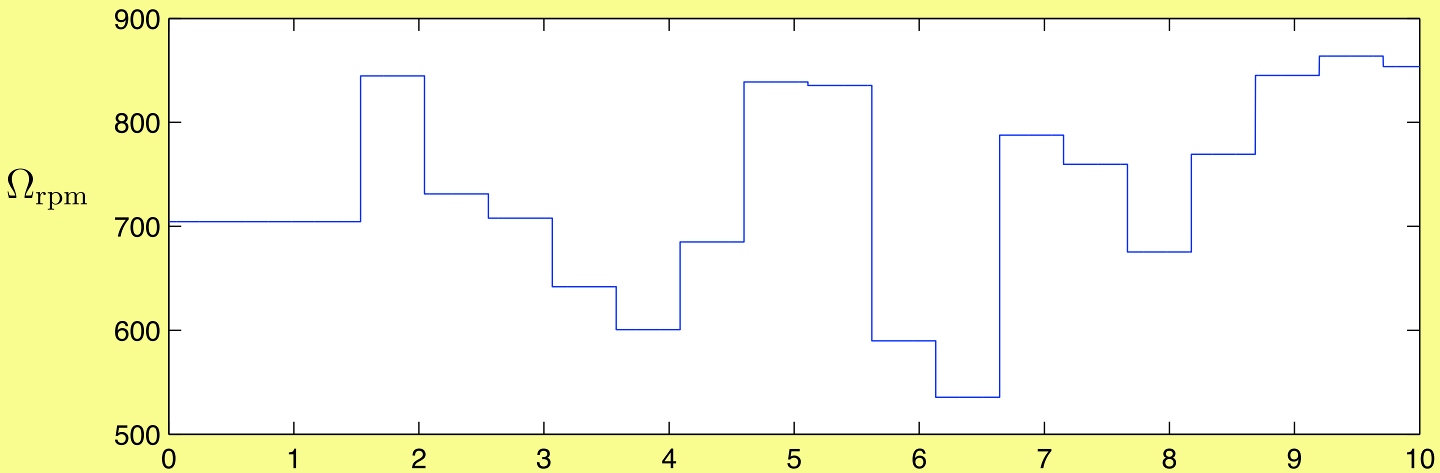
takes uniform values at regular time intervals (L)

MRSSV, (Yilmaz, et al, 2003)

For parameters in region where x=0 is unstable:



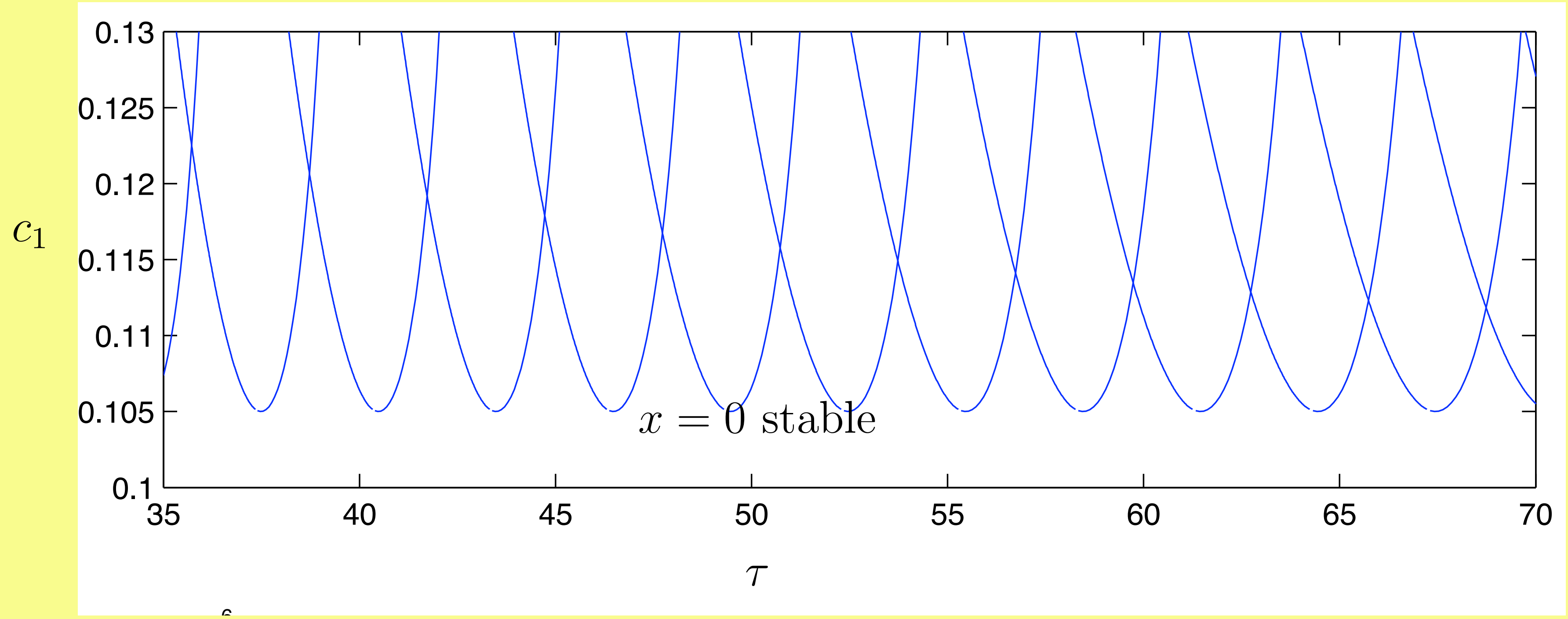
Random changes in delays: suppressing oscillations



- Noise “stabilized” transients
- Importance of multiple time scales
- Reduced systems for different scales
- Yields insight into the phenomenon: coherence resonance
- Noise beneficial/detrimental

Different noise sources: amplifying and suppressing oscillations

Stability region: series of wedges with larger range of stability



Time series: Stochastically modulated oscillations with dominant frequency, amplification factor = distance from critical

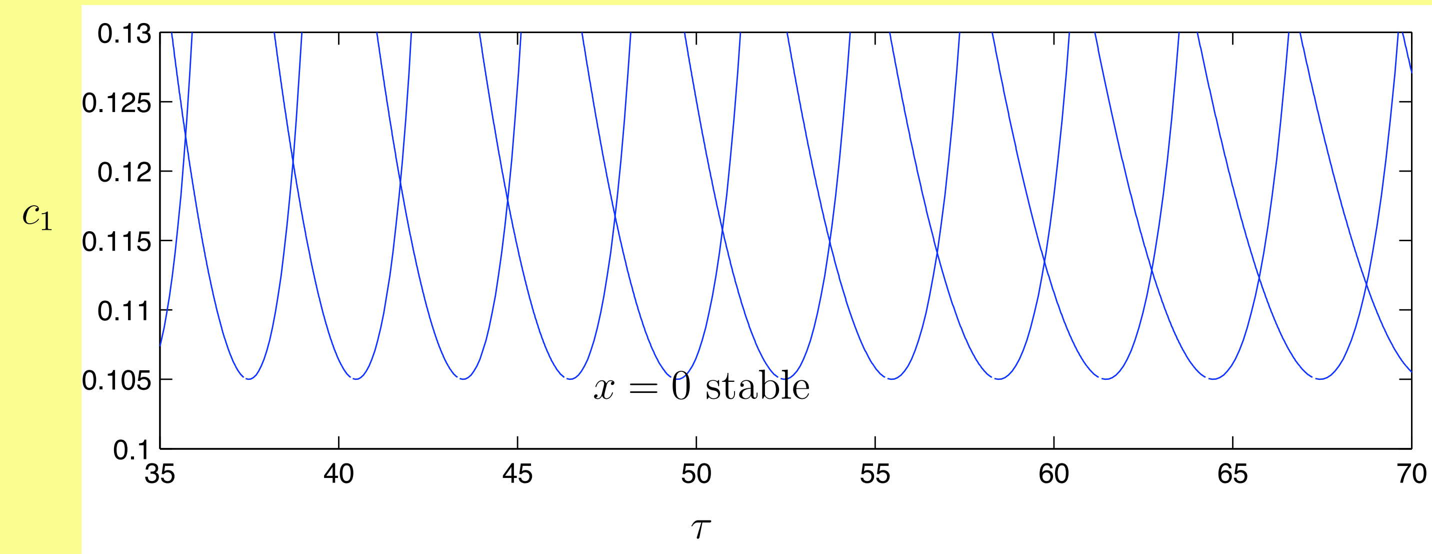
Variation in material parameters:

Fixed $\tau = \frac{2\pi\alpha}{\Omega}$,

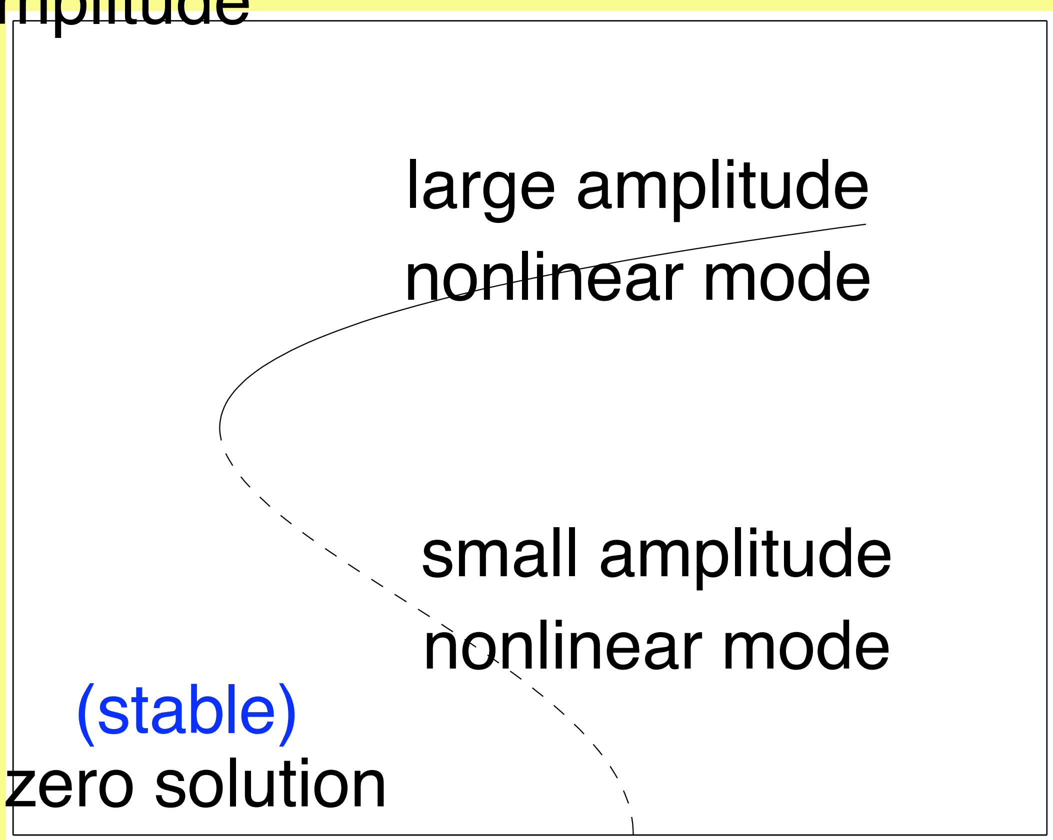
$$dx = y\,dt$$

$$dy = \left(-2\kappa y - x + \sum_{1}^{3} c_j \left[x(t-\tau) - x(t)\right]^j\right)dt + \delta dw + \sum_{1}^{3} c_j \left[x(t-\tau) - x(t)\right]^j \delta dw.$$

Coherence resonance: increased amplitude

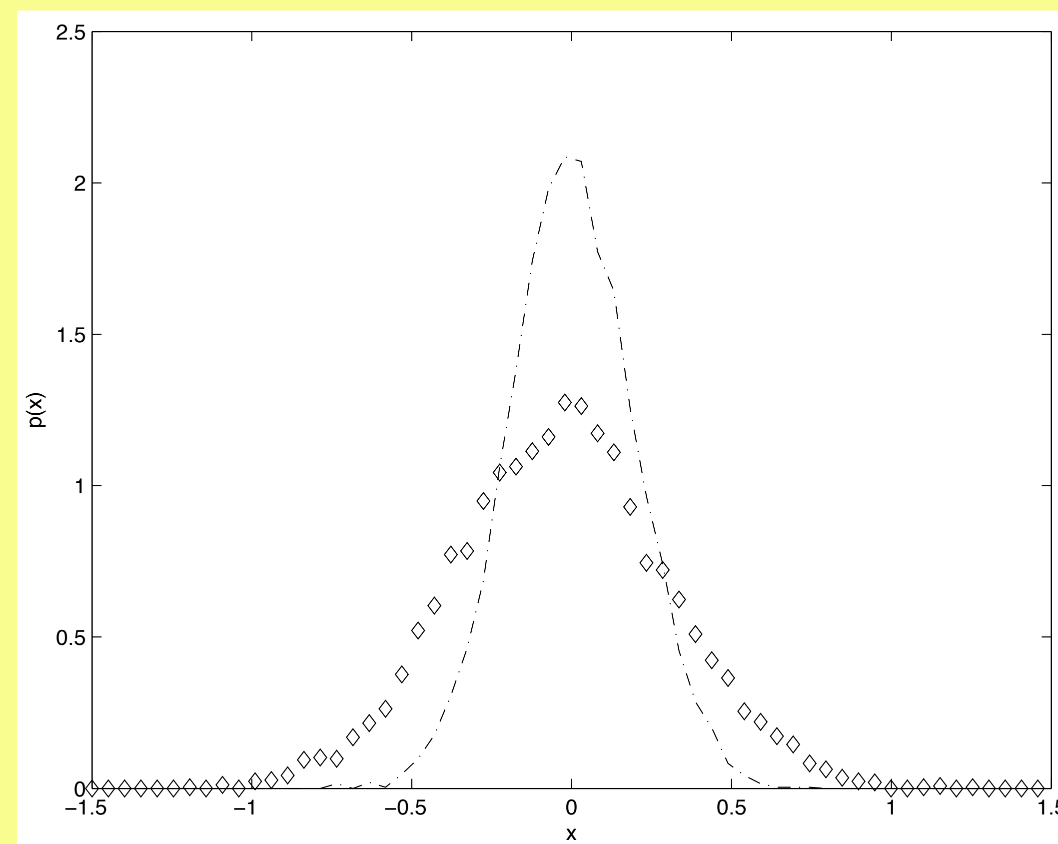


maximum amplitude



control parameter (c_1)

p(x)



x

Buckwar, K. L'Esperance, Soo, 2005

Contrast to approach using the density $p(x,t)$

Fokker-Planck equation the probability densities $p(x,t)$:

$$dx = a(x,t)dt + \sigma dw \quad \text{for SDE's (no delay)}$$

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(ap(x,t)\right) + \frac{\sigma^2}{2}\frac{\partial^2 p(x,t)}{\partial x^2}$$

Fokker-Planck equation for SDDE's

$$dx = a(x(t), x(t-\tau))dt + \sigma dw$$

$$\frac{\partial p(x,t)}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 p(x,t)}{\partial x^2} \underbrace{- \int \frac{\partial}{\partial x}\left(a(x(t), z)p(x,t,z,t-\tau)\right)dz}_{\text{memory effect } (z=x(t-\tau))}$$

Linear operator/generator + initial condition (function)

Linear, Stationary case:

Kuchler, Mensch (1992)

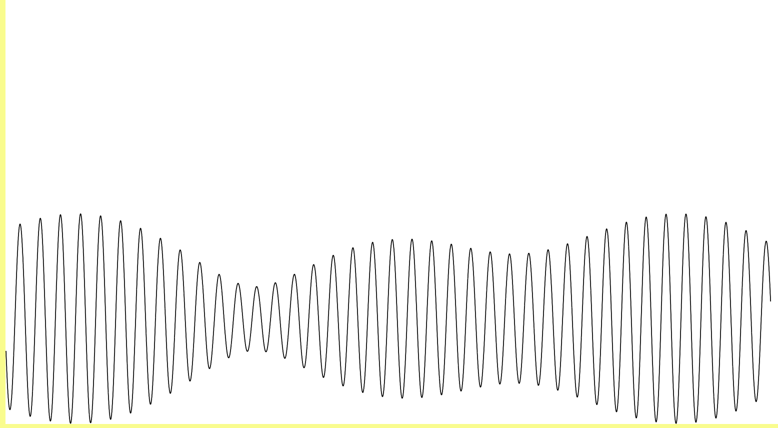
Another view: envelope equations

$$x_{tt} = -\omega^2 x + \beta x_t - a x^3 - b x^2 x_t, \quad \beta \ll 1$$

(Van der Pol/Duffing)

Modulation equations for amplitudes of oscillations:

$A(T)\cos\omega t + B(T)\sin\omega t$



T is a “slow” time : $T = \epsilon^2 t$, $\epsilon \ll 1$.

ϵ typically related to the proximity to a transition point ($\beta = \epsilon^2$) and/or a specific frequency ω

Rigorous results

Stochastic van der Pol-Duffing:

(Arnold, Namachchivaya(1996), Baxendale(2002), Khasminskii (1963))

$$\begin{aligned} dx &= y\,dt \\ dy &= \left[-\omega^2 x + \beta y - a x^3 - b x^2 y\right] dt + \text{noise} \end{aligned}$$

Additive noise: δdw or Multiplicative noise: $\delta x dw$

$\beta = \epsilon^2 \ll 1$: (small damping) “Nontrivial” results: $\delta = \epsilon$, $\epsilon \longrightarrow 0$

Projection/rotation, averaged, certain restrictions on the nonlinearity

Equations for the averaged process = amplitude/envelope of oscillations

Effective noise for the average process: Generator for the averaged process

= generator for original process under projection/averaging

Lyapunov exponents: $\lambda = \lim_{t\to\infty} t^{-1} \log \| x_{\mathrm{L}} \| \longrightarrow \lambda_{\mathrm{det}} + c_\lambda \delta^2$

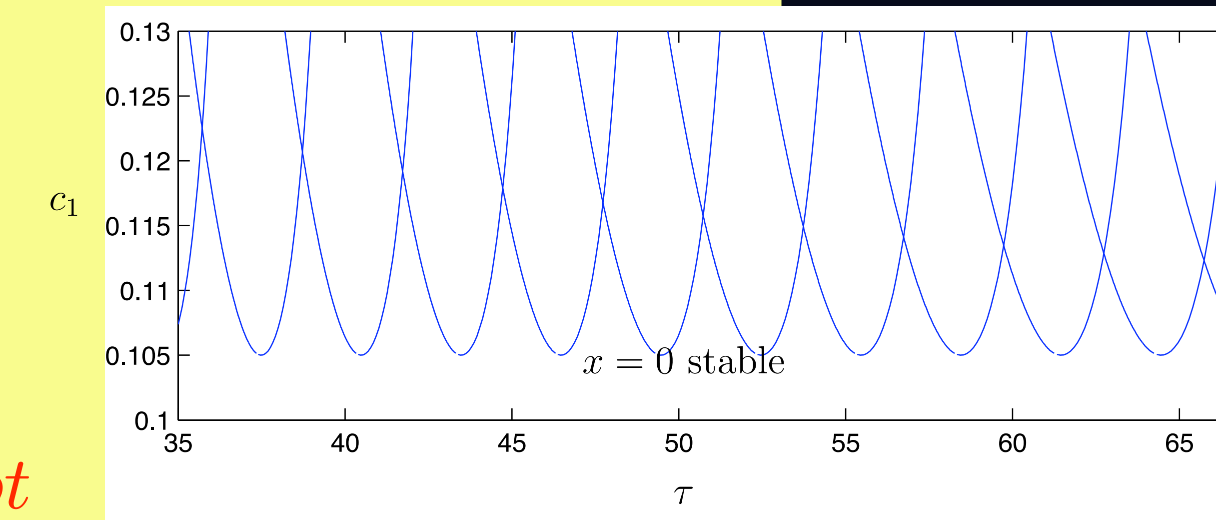
Amplitude equations for SPDE’s: Blomker, et al, 00’s

SDDE: (additive noise)

$$dx = y\,dt, dx = f(x(t), x(t-\tau); c_1)\,dt + \delta dw$$

$\tau \sim \tau_c + \epsilon^2 \tau_2$ or $c_1 = c_{1c} + \epsilon^2 c_{12}$

T is a slow time $T = \epsilon^2 t$,



Look for solution of the form

$$\hat{x}(t) = A(T)\cos bt + B(T)\sin bt$$

Information from the deterministic problem: on t time scale

Derive equations for noisy amplitudes on T time scale

$$\begin{pmatrix} dA \\ dB \end{pmatrix} = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} dT + \Sigma \begin{pmatrix} d\beta_A(T) \\ d\beta_B(T) \end{pmatrix}$$

$\Sigma =$ constant matrix for added noise

Klosek, K. 2005

Find ψ_A, ψ_B and Σ by equating two equations for dx:

Ito's formula: change of variables from x to A, B

Substitution in the SDDE (involves $A(T)$, $A(T-\epsilon^2\tau)$, etc.)

Projection onto primary modes $\cos bt, \sin bt$:

Drift terms: Standard multi-scale analysis for envelopes over long time

Stochastic terms: Treat as Fourier series-type expansion

$$\delta w(t) = \delta \sum f_j \cos(jbt) w_{j1}(t) + g_j \sin(jbt) w_{j2}(t)$$

$$\delta w(t) = \frac{\delta}{\epsilon} \sum_{j=1}^{n} f_j \cos(jbt) w_{j1}(T) + g_j \sin(jbt) w_{j2}(T)$$

Project as a multi-scale process: $T = \epsilon^2 t$ independent of t

Slow decay on T scale: $j = 1$

Fast decay on t scale: $j \neq 1$

Envelope equations for oscillations:

$$\begin{pmatrix} dA \\ dB \end{pmatrix} = \begin{pmatrix} \psi_A(A,B) \\ \psi_B(A,B) \end{pmatrix} dT + \frac{\delta}{\epsilon} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} d\xi_1(T) \\ d\xi_2(T) \end{pmatrix},$$

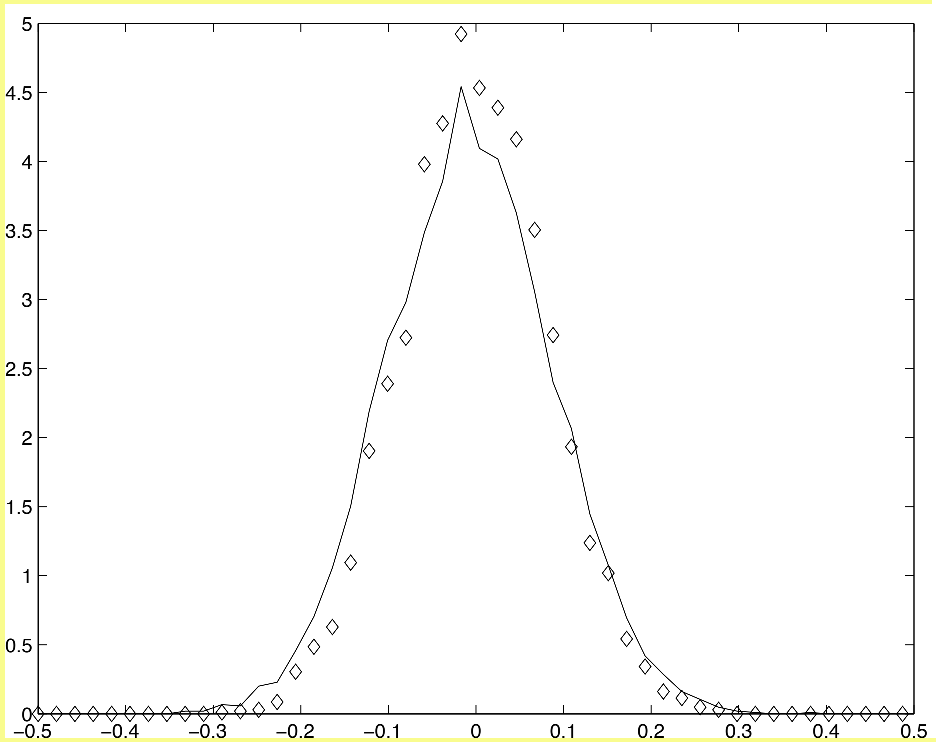
$$\begin{aligned} \psi(A,B) &= c_1^c \left[g_1(\omega,\kappa) \frac{A(T-\epsilon^2\tau) - A(T)}{\epsilon^2} + g_2(\omega,\kappa) \frac{B(T-\epsilon^2\tau) - B(T)}{\epsilon^2} \right] \\ &+ c_{12}[h_1(\omega,\kappa)A(T) + h_2(\omega,\kappa)B(T)] \end{aligned}$$

K, 2006

Time scale of coherence resonance: T

Time scale of oscillations: t

$p(x)$



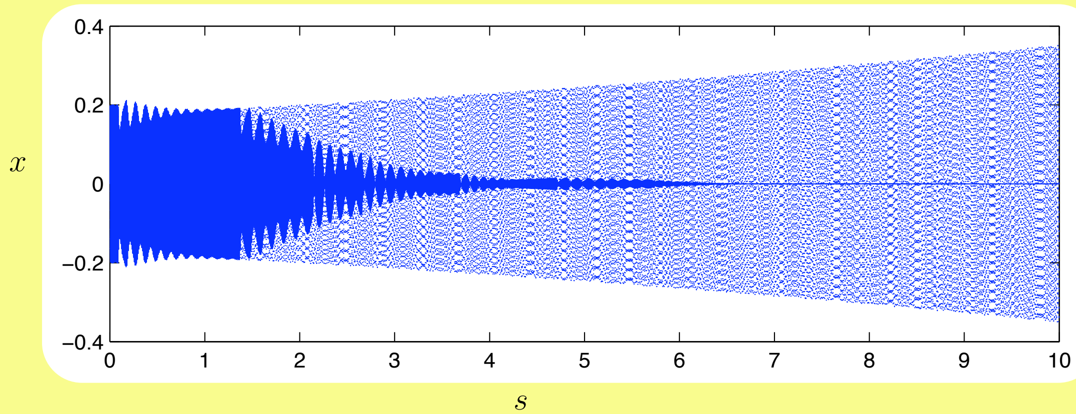
x

$$
\begin{aligned}
dx &= y\,dt \\
dy &= \left(-2\kappa y - x + \sum_{1}^{3} c_j\left[x(t-\tau) - x(t)\right]^j\right)dt + \delta dw + \sum_{1}^{3} c_j\left[x(t-\tau) - x(t)\right]^j \delta dw.
\end{aligned}
$$

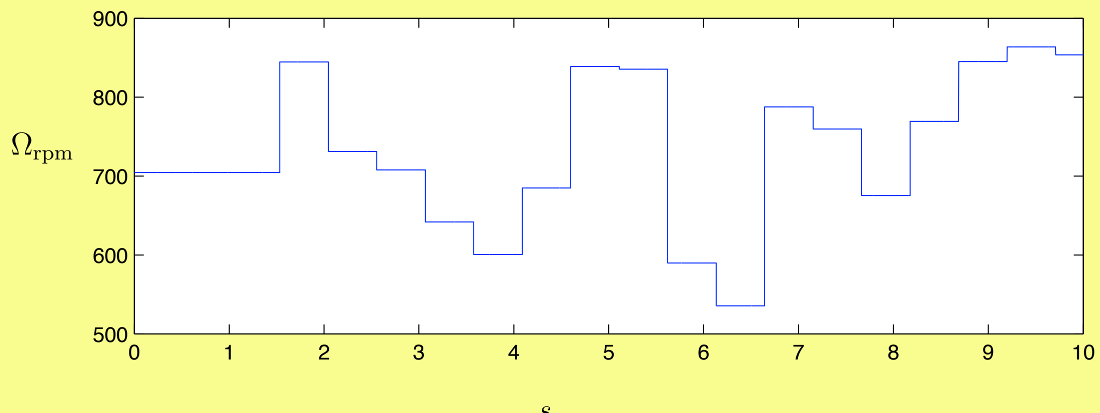
$$
\tau = \frac{2\pi\alpha}{\Omega}, \qquad \Omega = \overline{\Omega}(1 + q\zeta(t))
$$

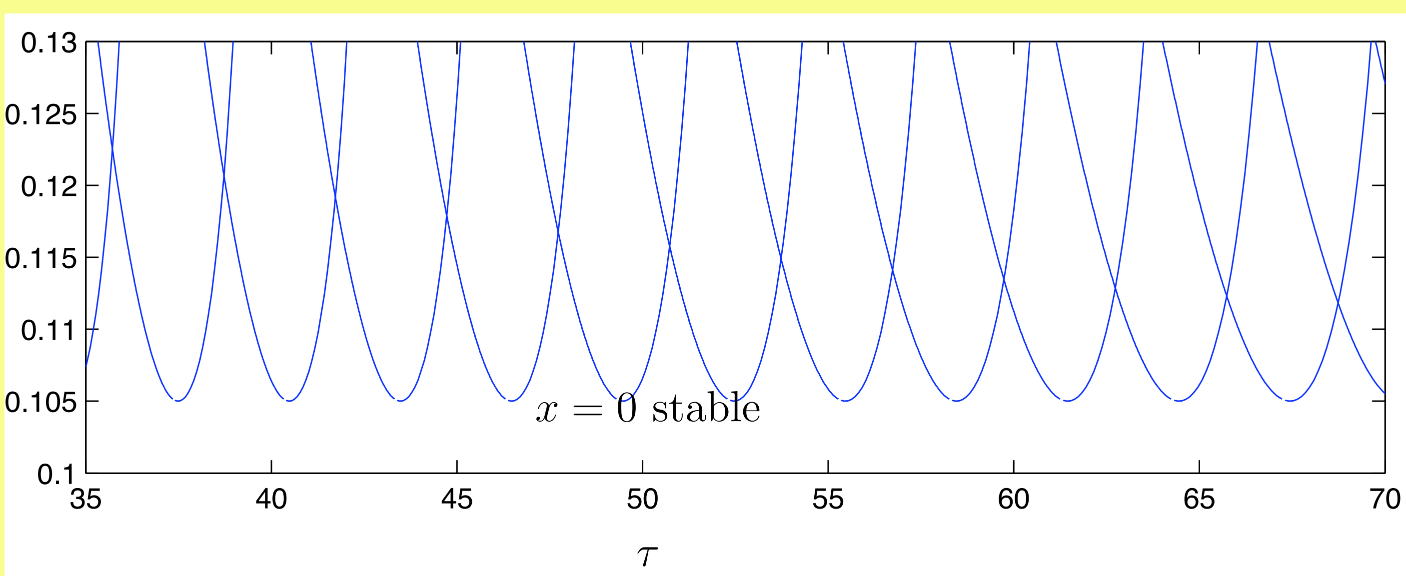
takes uniform values at regular time intervals (L)

MRSSV, (Yilmaz, et al, 2003)



For parameters in region where x=0 is unstable:



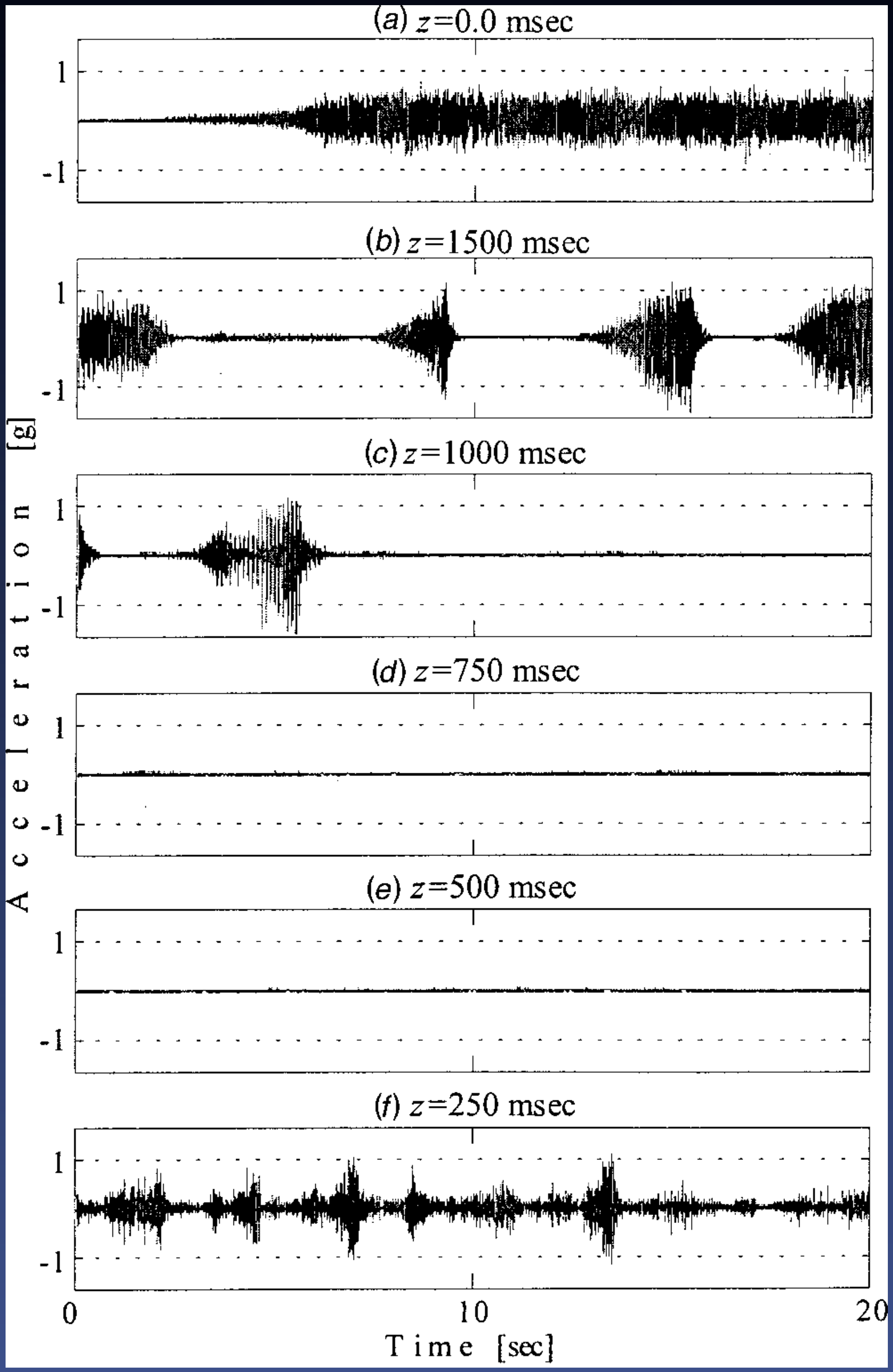


Random changes in delays: suppressing oscillations

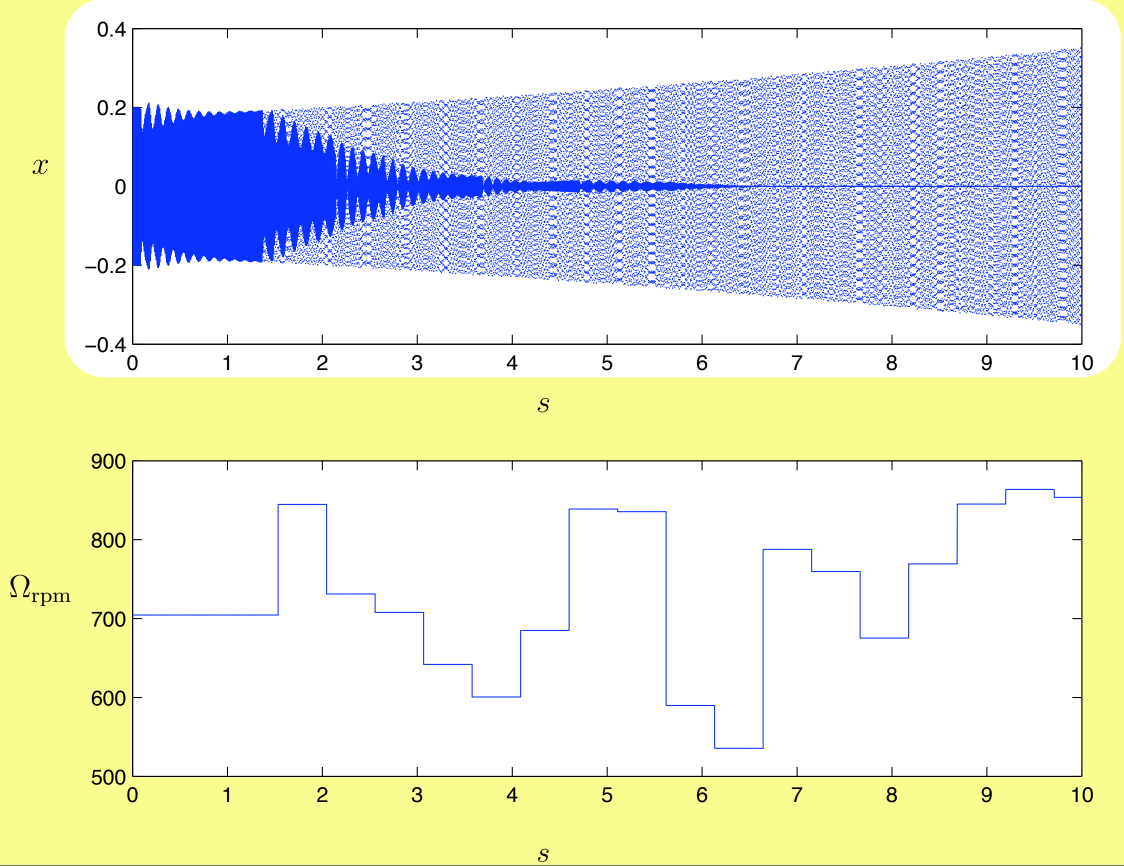
Results for variation of delay w/o variation of material parameters

Without CR: eigenvalues of discretized problem (Yilmaz, 2003)

Sinusoidal variation of delay: Must be tuned precisely for damping (Namachchivaya, et al, 2003)



Random switching of speed (delay) damps coherent oscillations (w/o variation in material parameters)



- Noise-induced order via transients
- Switching between exponential damping and slow growth of CR
- Apparent controlled oscillations/quiescence is a sequence of transients

Envelope equations for oscillations:

$$\begin{pmatrix} dA \\ dB \end{pmatrix} = \begin{pmatrix} \psi_A(A,B) \\ \psi_B(A,B) \end{pmatrix} dT + \frac{\delta}{\epsilon} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} d\xi_1(T) \\ d\xi_2(T) \end{pmatrix},$$

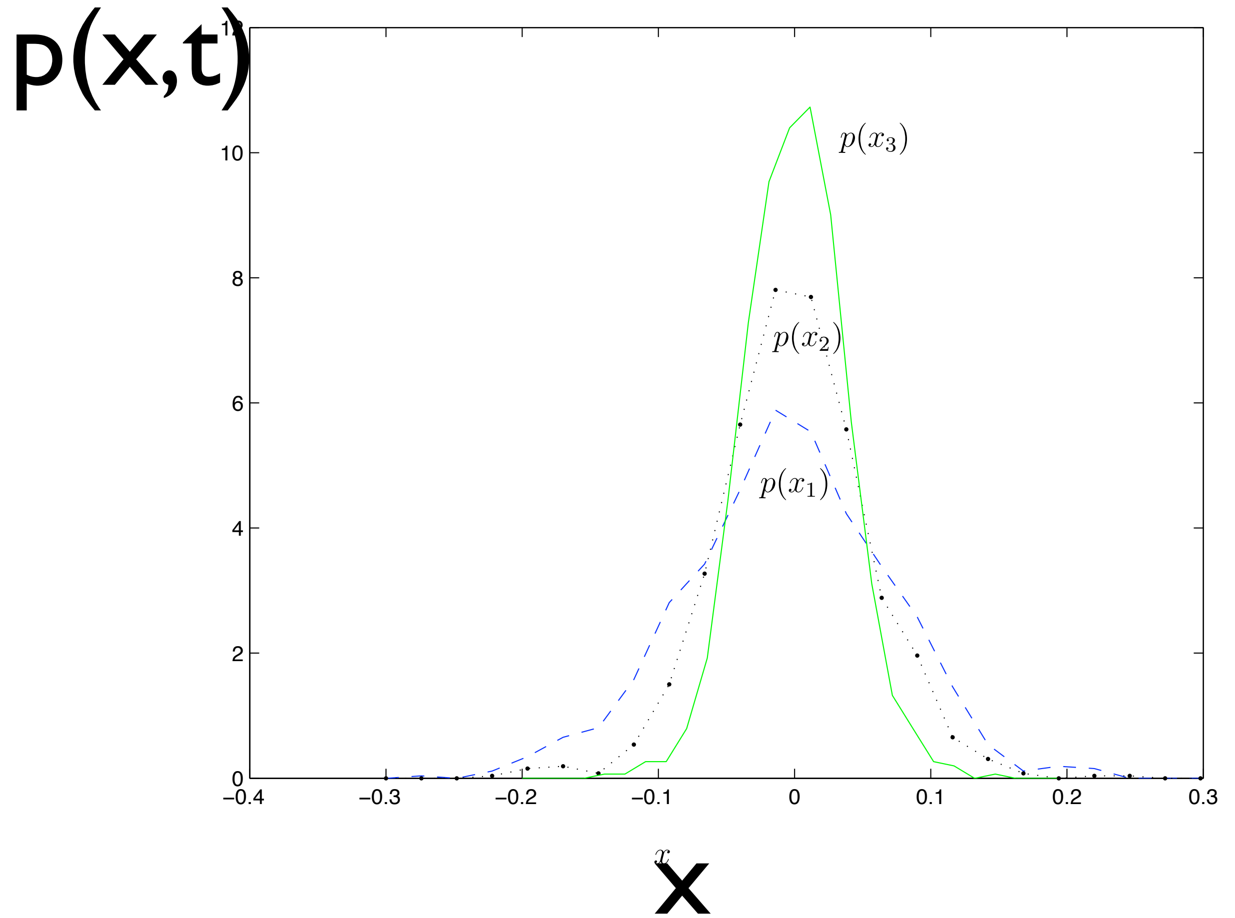
$$\begin{aligned} \psi(A,B) &= c_1^c \left[g_1(\omega,\kappa) \frac{A(T-\epsilon^2\tau) - A(T)}{\epsilon^2} + g_2(\omega,\kappa) \frac{B(T-\epsilon^2\tau) - B(T)}{\epsilon^2} \right] \\ &+ c_{12}[h_1(\omega,\kappa)A(T) + h_2(\omega,\kappa)B(T)] \end{aligned}$$

K. 2006

Time scale of coherence resonance: T
Time scale of oscillations: t
Time interval for varying delay: L

Variance of resonant mode on intermediate scale:
Valid for L sufficiently large

Both noise sources: Coherence resonance sustains oscillations, variable delay suppresses oscillations



Consider oscillations on intermediate time scale

If amplitude is O-U:

$$
\begin{aligned}
dA &= -rA\,dT + \frac{\delta}{\epsilon}dW(T) \\
\operatorname{Var}A(T) &= \operatorname{Var}[A(0)]e^{-2T} + \frac{\delta^2}{2r\epsilon^2}[1 - e^{-2rT}] \\
&= \frac{\delta^2}{2r\epsilon^2}[1 - e^{-2rT}] + O(\delta^2)\,.
\end{aligned}
$$

Variance increases with time until stationary

Envelope equations for oscillations:

$$\begin{pmatrix} dA \\ \\ dB \end{pmatrix} = \begin{pmatrix} \psi_A(A,B) \\ \\ \psi_B(A,B) \end{pmatrix} dT + \frac{\delta}{\epsilon} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} d\xi_1(T) \\ \\ d\xi_2(T) \end{pmatrix},$$

$$\begin{aligned} \psi(A,B) &= c_1^c \left[g_1(\omega,\kappa) \frac{A(T-\epsilon^2\tau) - A(T)}{\epsilon^2} + g_2(\omega,\kappa) \frac{B(T-\epsilon^2\tau) - B(T)}{\epsilon^2} \right] \\ &+ c_{12}[h_1(\omega,\kappa)A(T) + h_2(\omega,\kappa)B(T)] \end{aligned}$$

K, 2006, K. 2009

Time scale of coherence resonance: T
Time scale of oscillations: t
Time interval for varying delay: L

$$t \ll L \ll 1/\epsilon^2.$$

Variance of resonant mode on intermediate scale:
Valid for L sufficiently large

Other models with feedback

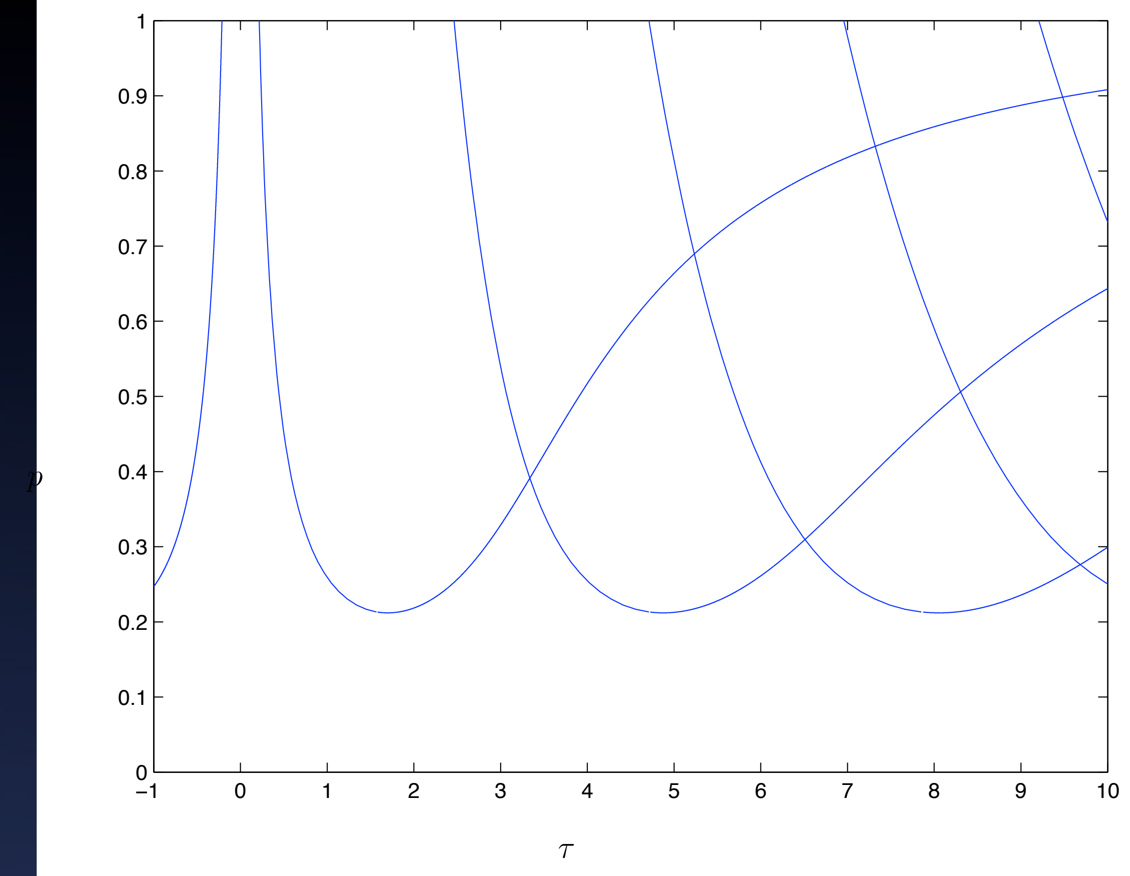
$$
\begin{aligned}
x_t &= y \\
y_t &= -2\kappa x_t - x - px(t-\tau)
\end{aligned}
$$

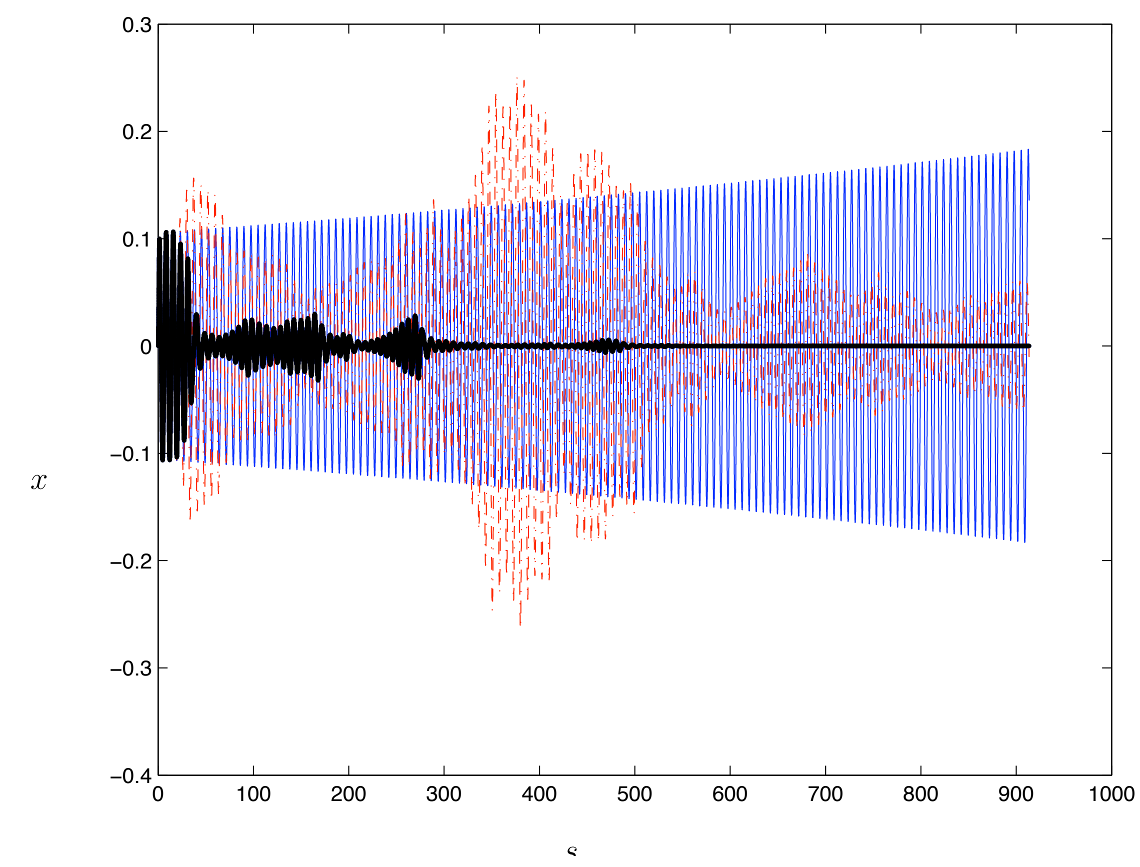
Illustrating feedback in substructure modeling: hybrid experimental-numerical model

Delays due to signal processing, computation, etc.

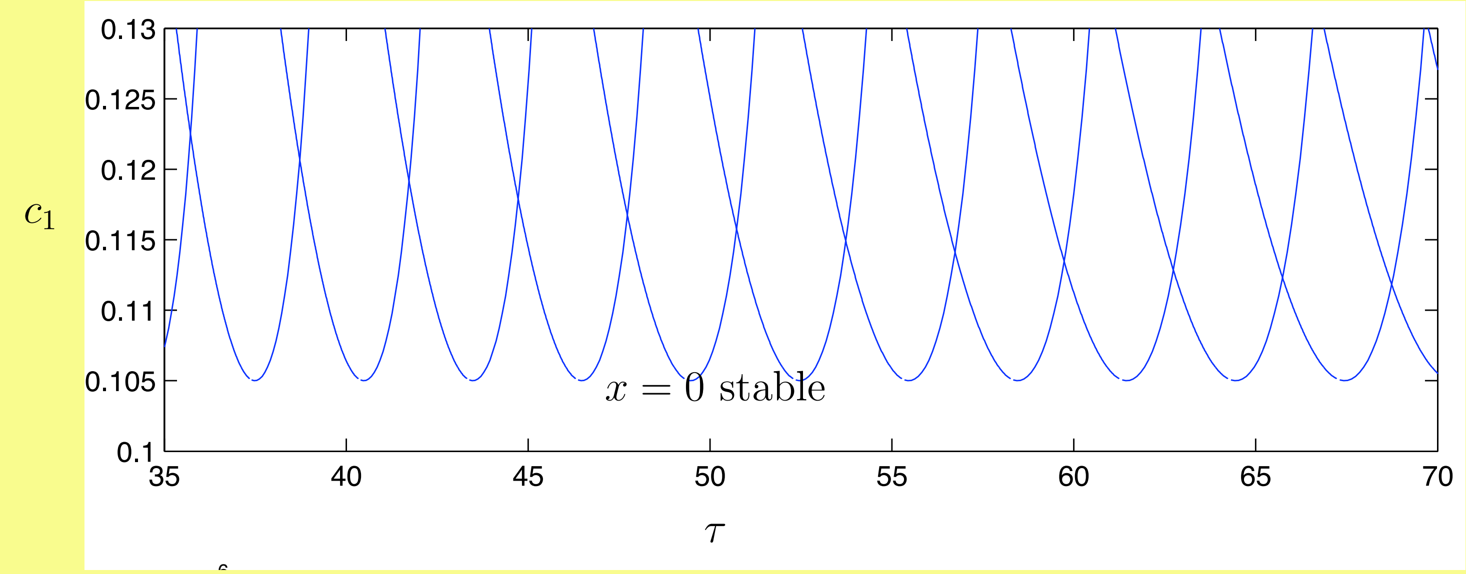
Wallace, et al ASME, 2005.

Variable delay again stabilizing (discrete time intervals)

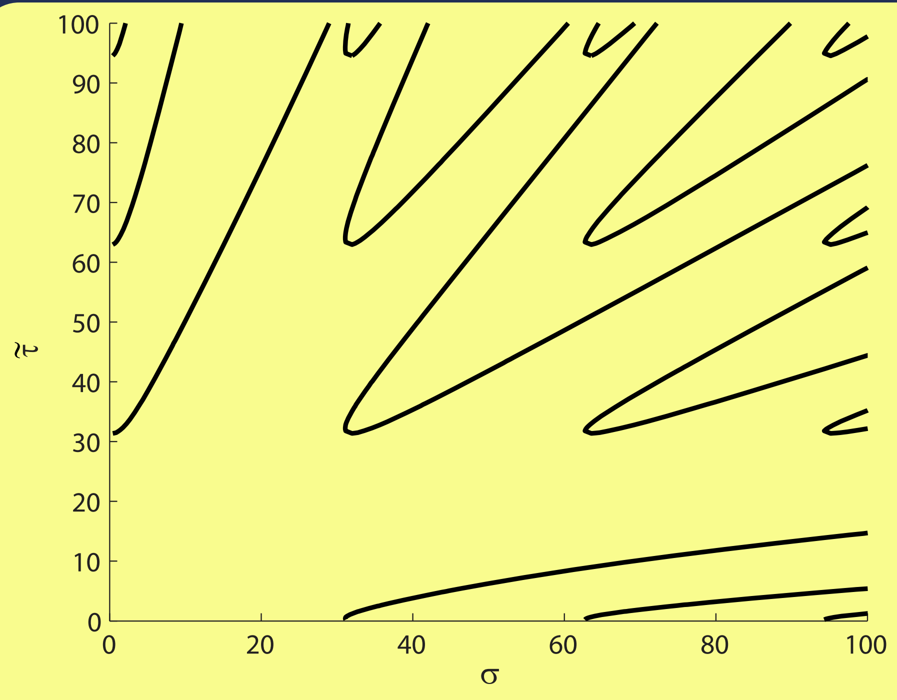




Complicated stability regions not uncommon for DDE's: Controlled container crane, Erneux, Kalmar-Nagy, 2008



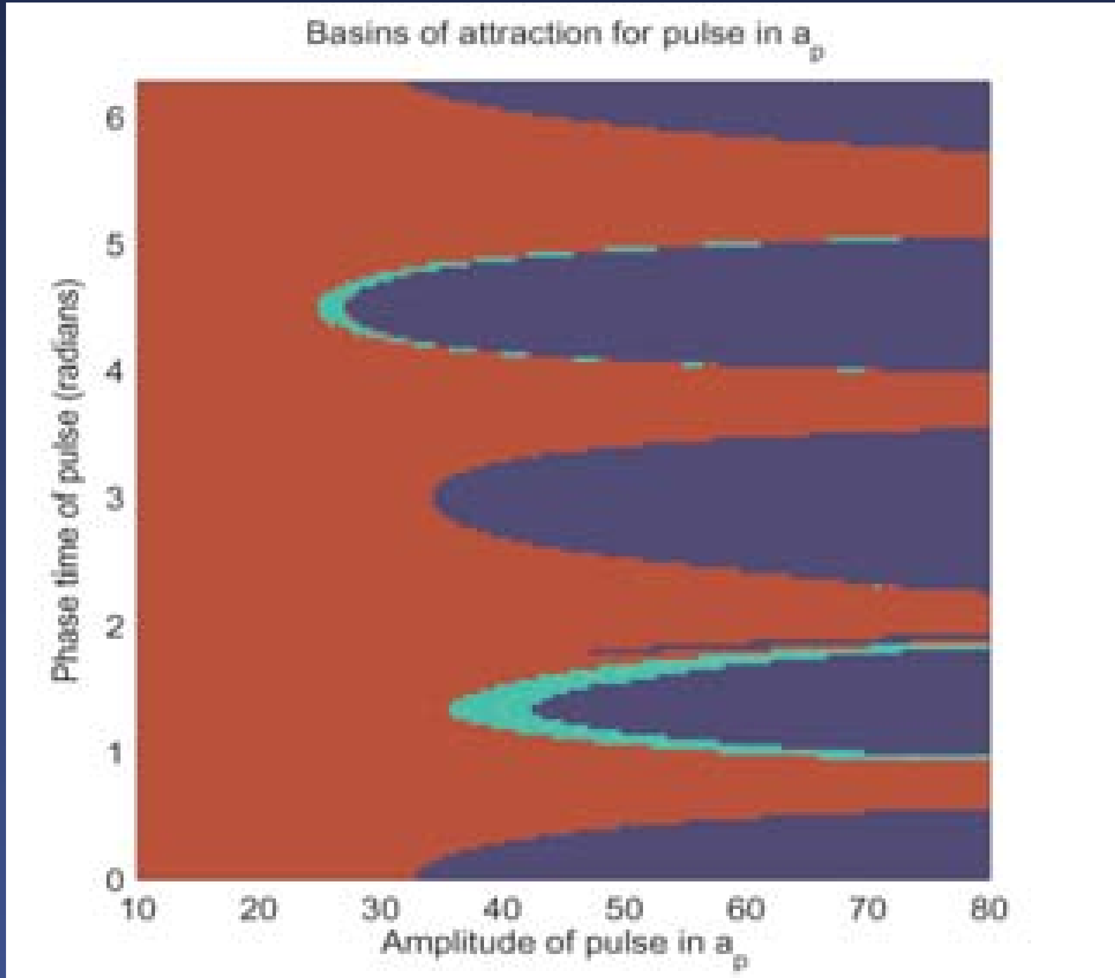
Immune response to leukemia treatment: delays for cell division/interaction



Niculescu, et al, 2008

Delayed response for therapies:

Colijn, Mackey, 2007



Hematopoietic system:

Stem cells
$$\frac{dq}{dt} = -qb_1h_q(q) + b_1\mu_1 q_1 h_q(q_1) - q\left\{b_2h_n(n) + b_3h_p(p) + b_4h_r(r)\right\},$$

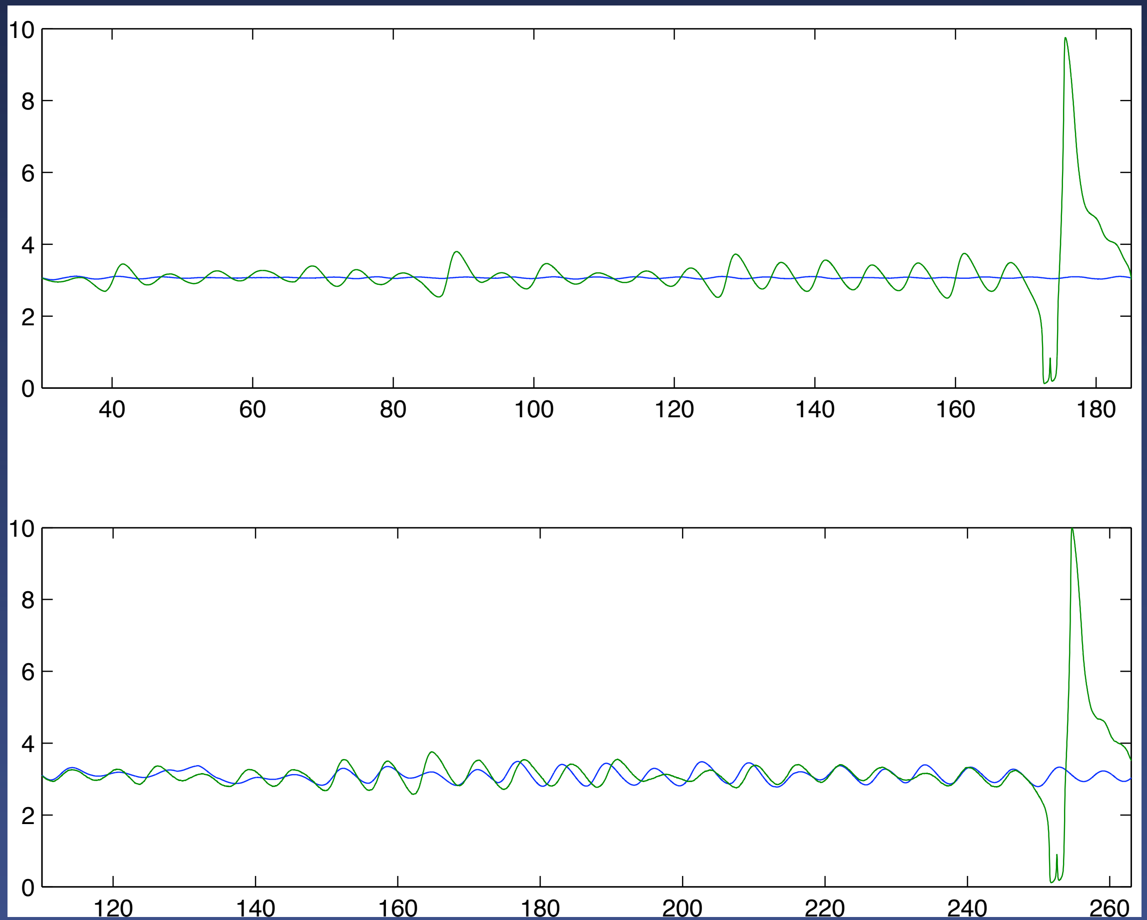
Neutrophils
$$\frac{dn}{dt} = -\gamma_n n + a_n b_2 q_{\tau_{nm}} h_n(n_{\tau_{nm}}),$$

Platelets
$$\frac{dp}{dt} = -\gamma_p p + a_p b_3 \left\{q_{\tau_{pm}} h_p(p_{\tau_{pm}}) - \mu_3 q_{\tau_{psum}} h_p(p_{\tau_{psum}})\right\},$$

Erythrocytes
$$\frac{dr}{dt} = -\gamma_r r + a_r b_4 \left\{q_{\tau_{rm}} h_r(r_{\tau_{rm}}) - \mu_4 q_{\tau_{rsum}} h_r(r_{\tau_{rsum}})\right\},$$

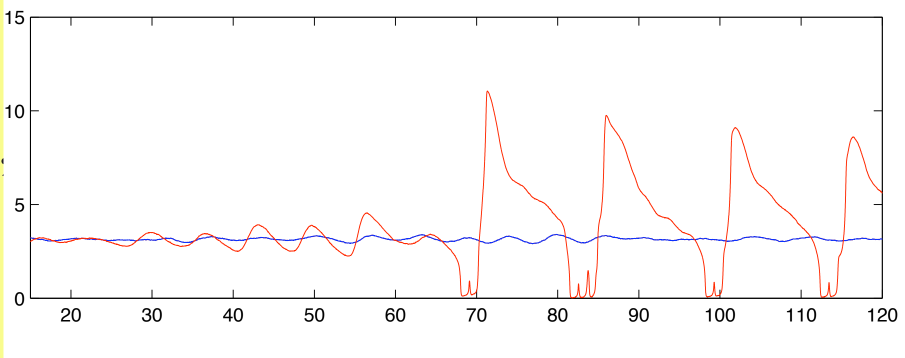
Colijn, Mackey, 2007

Fluctuations in proliferation times and in lower cell population levels can drive transitions to large oscillations (disease)

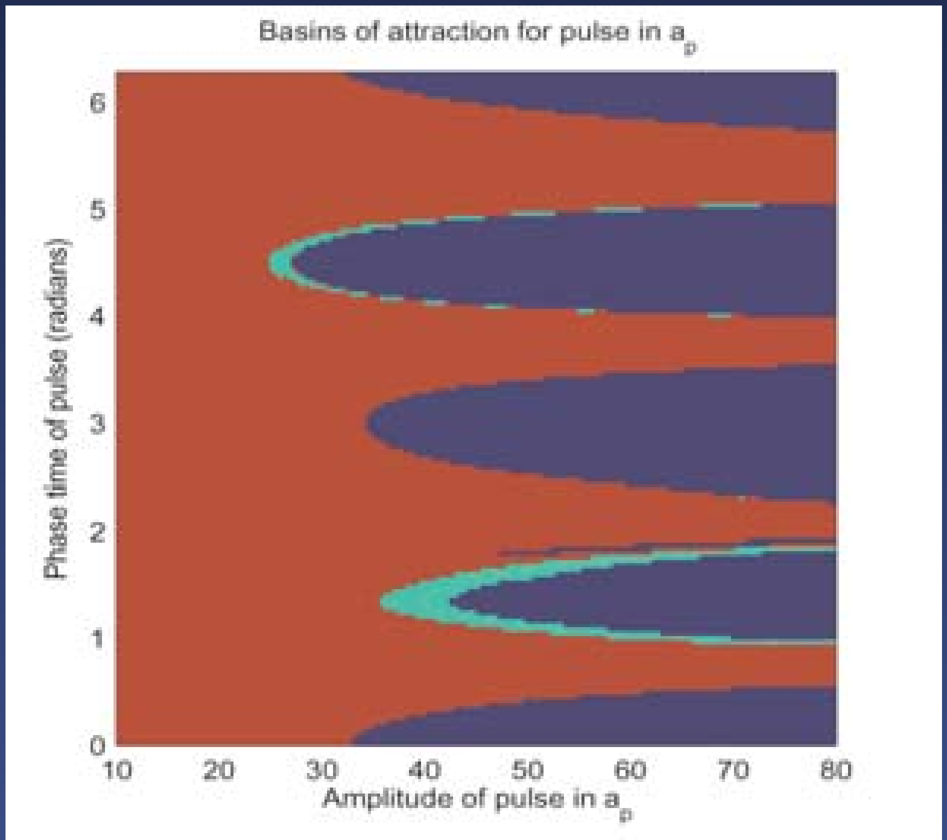


Hematopoietic system:

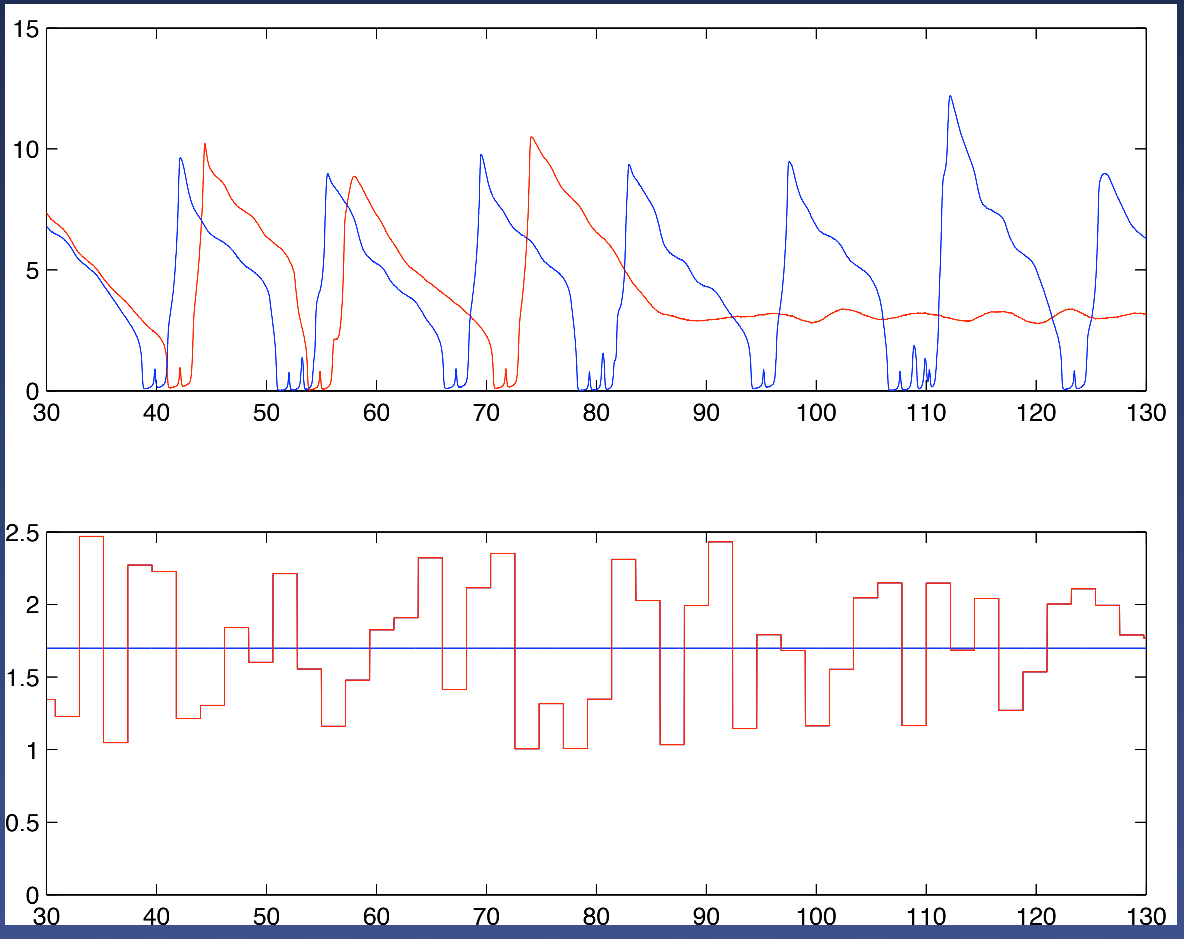
Stem cells
$$\frac{dq}{dt} = -qb_1h_q(q) + b_1\mu_1q_1h_q(q_1) - q\left\{b_2h_n(n) + b_3h_p(p) + b_4h_r(r)\right\},$$
$$\frac{dn}{dt} = -\gamma_n n + a_nb_2q_{\tau_{nm}}h_n(n_{\tau_{nm}}),$$
Platelets
$$\frac{dp}{dt} = -\gamma_p p + a_pb_3\left\{q_{\tau_{pm}}h_p(p_{\tau_{pm}}) - \mu_3q_{\tau_{psum}}h_p(p_{\tau_{psum}})\right\},$$
$$\frac{dr}{dt} = -\gamma_r r + a_rb_4\left\{q_{\tau_{rm}}h_r(r_{\tau_{rm}}) - \mu_4q_{\tau_{rsum}}h_r(r_{\tau_{rsum}})\right\},$$



Platelet amplification



Variable delay (production)



Colijn, Mackey, 2007

Infectious disease modeling: Stochastic vs. deterministic

- Quasi-regular recurrence

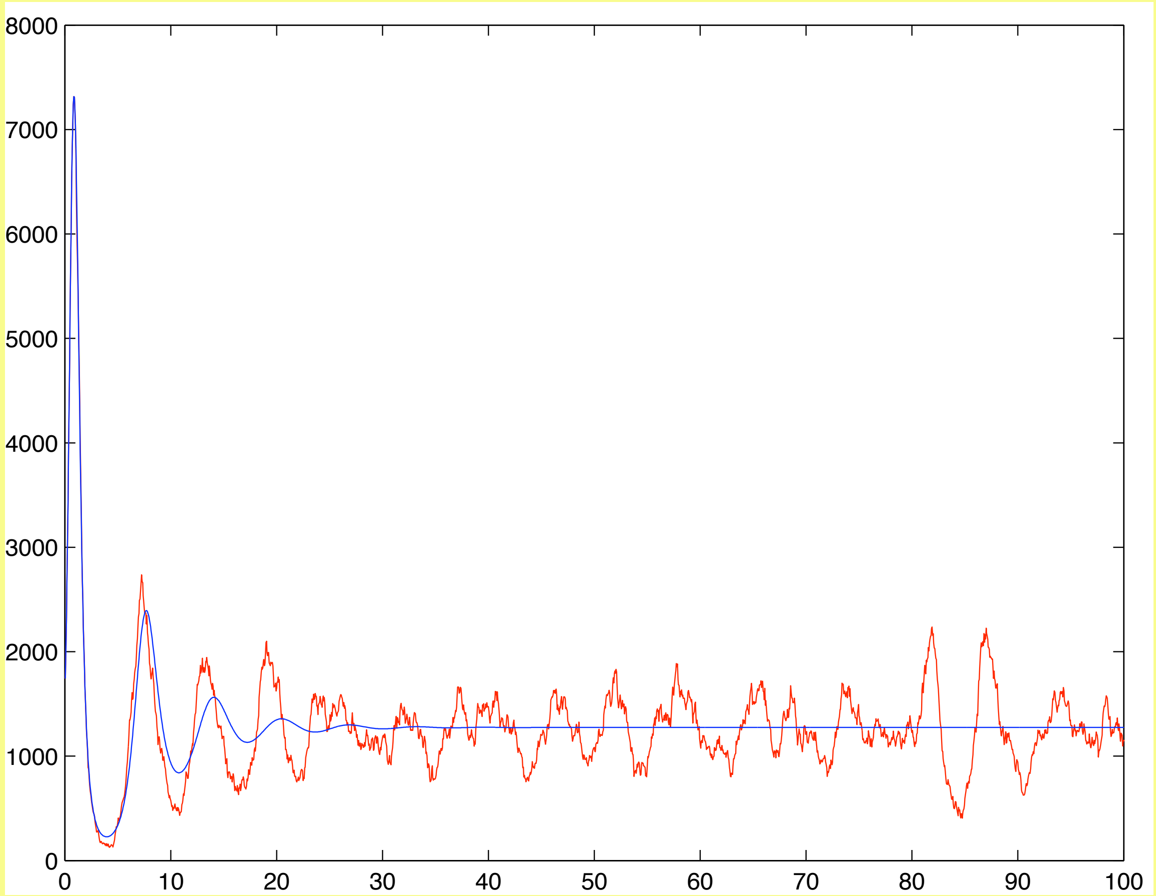
Two criteria:

1. Multiple time scales

2. Balance of noise and deterministic dynamics

S= Susceptibles, I = Infected, R= Recovered

Infected Population



t

SIR:

$$\epsilon^2 = \frac{R_0}{2}\sqrt{\frac{\mu}{\mu+\gamma}\frac{1}{R_0-1}} \ll 1,$$

$$\frac{\delta^2}{2\epsilon^2} = \frac{\mu+\gamma}{4N\mu}\left(1+\frac{R_0+1}{R_0-1}+2\frac{\mu+\gamma}{\mu(R_0-1)}\right) \ll 1.$$

μ = birth/death rate

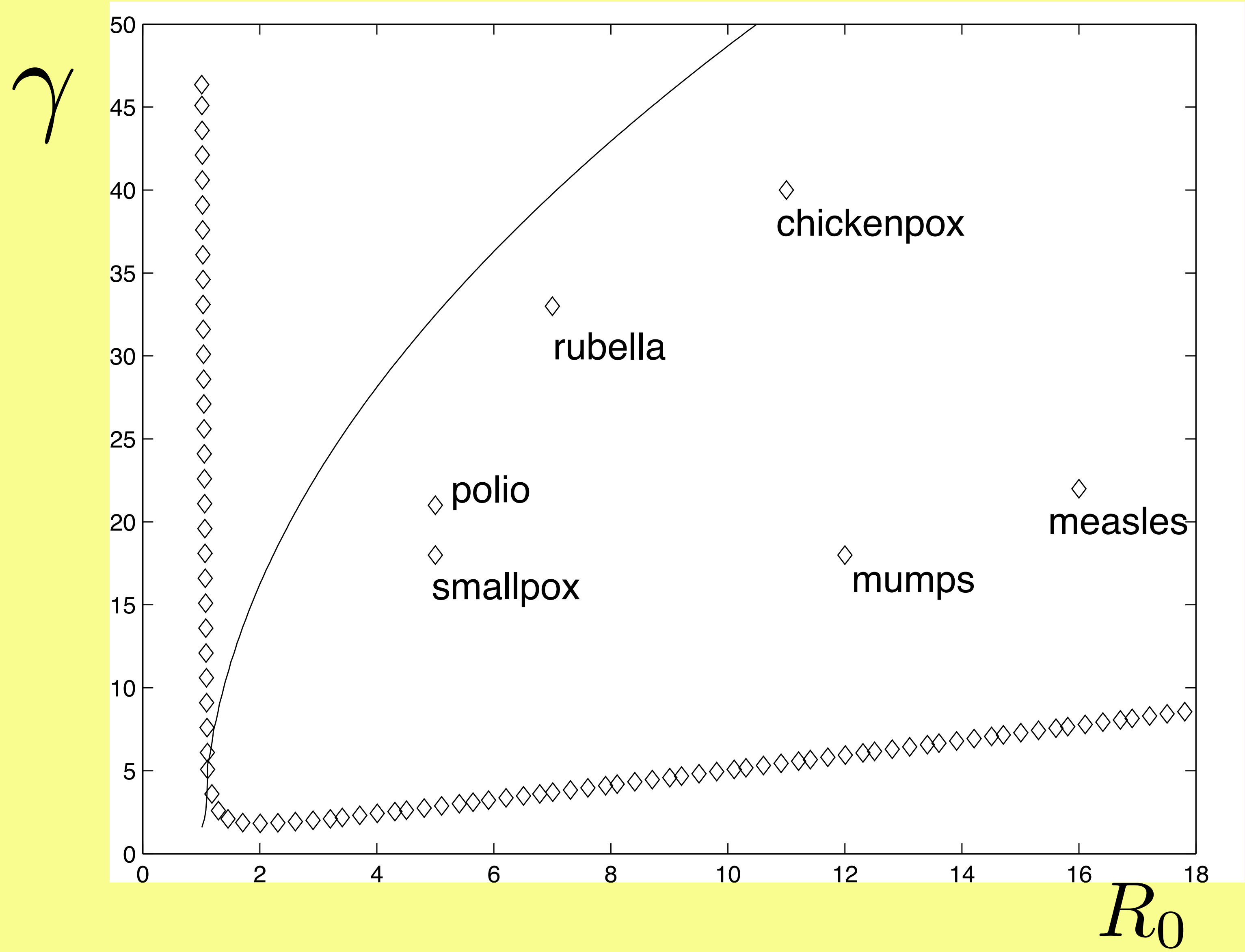
γ = recovery rate $(1/D)$

β/N = the infection rate per pair of susceptibles and infectives.

$R_0 = \beta/(\gamma+\mu)$ = avg. secondary infections/infected individual

First criterion: Multiple scales

Second criterion: Balance of noise/ dynamics



Cts approximation to interacting individuals:

$$dS = \left[\mu(N - S) - \frac{\beta}{N}SI\right] dt + g_1(S,I)dW_1 + g_2(S,I)dW_2$$

$$dI = \left[-\gamma I + \frac{\beta}{N}SI\right] dt + g_2(S,I)dW_2 + g_3(S,I)dW_3$$

Approx to Poisson increments: appear small

Focus on fluctuations about equilibrium:

$u = (S - S_{\mathrm{eq}})/S_{\mathrm{eq}}, v = (I - I_{\mathrm{eq}})/I_{\mathrm{eq}}, t \to \Omega t$

$$\begin{pmatrix} du \\ dv \end{pmatrix} = \left[\begin{pmatrix} -2\epsilon^2 & -b \\ 1/b & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + \text{nonl. terms } (uv)\right] dt + G_1(u,v)\, d\mathbf{W_1} + G_2(u,v)\, d\mathbf{W_2} + G_3(u,v)\, d\mathbf{W_3}$$

Full model vs. multiscale

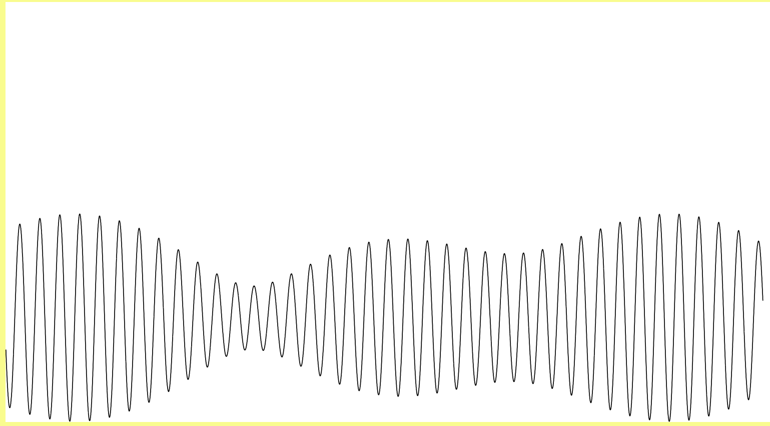
$$\begin{pmatrix} u \\ v \end{pmatrix} \sim A(T) \begin{pmatrix} b\cos t \\ \sin t \end{pmatrix} + B(T) \begin{pmatrix} b\sin t \\ -\cos t \end{pmatrix}$$

A, B are Gaussian (O-U processes): $dA = -AdT + c\frac{\delta(g_j)}{\epsilon}dW(T)$

with P. Greenwood and L. Gordillo, JTB, 2007

Modulation equations for amplitudes of oscillations:

$A(T)\cos\omega t + B(T)\sin\omega t$



T is a "slow" time : $T = \epsilon^2 t$, $\epsilon \ll 1$.

Stochastic vdPol-Duffing model:
Stochastic bifurcations/Lyapunov exponents

L. Arnold, Namachchivaya, Baxendale, 80's, 90's

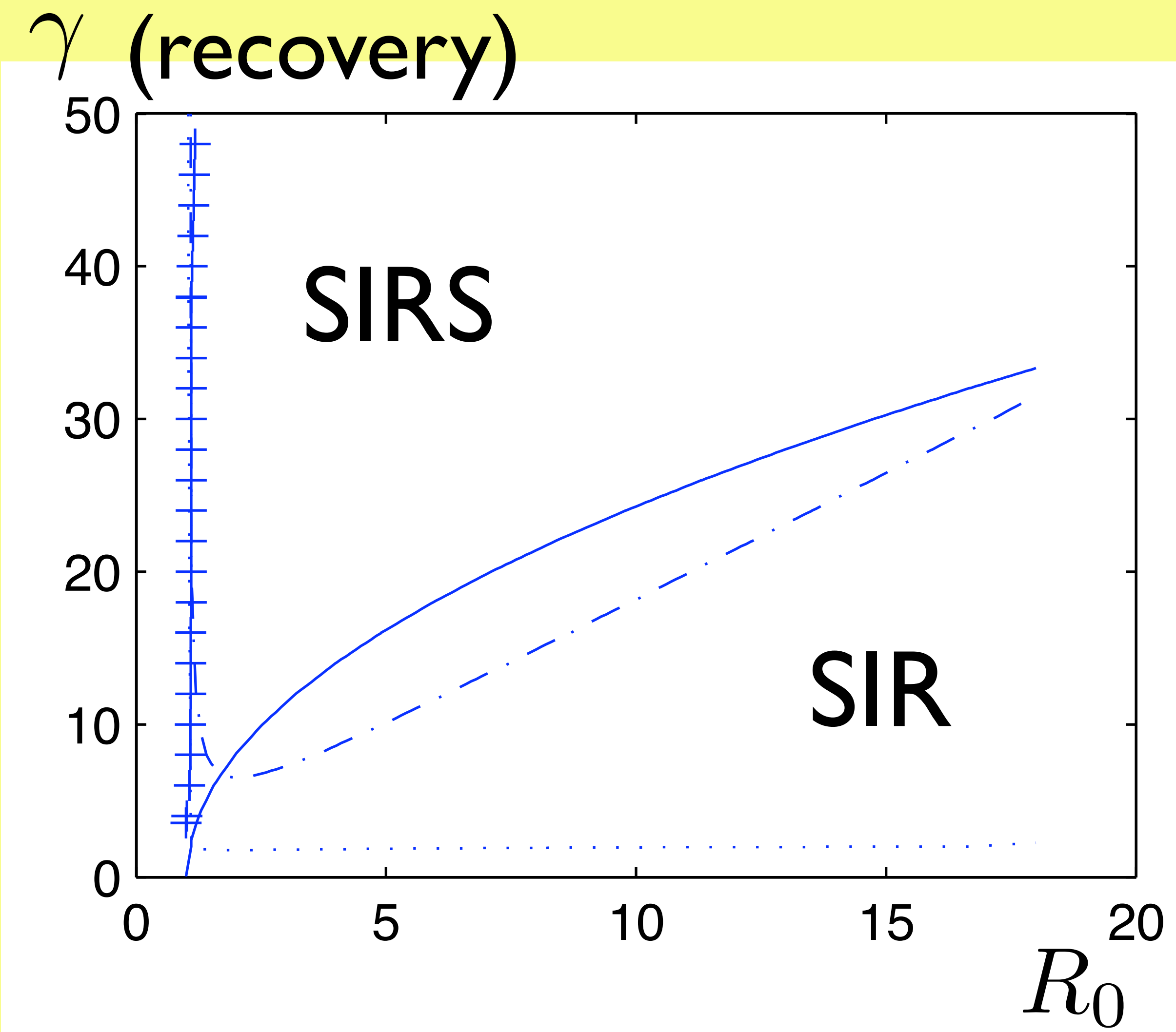
What information do we have from criteria for CR?

$$\epsilon^2 = \frac{R_0}{2}\sqrt{\frac{\mu}{\mu+\gamma}\frac{1}{R_0-1}} \ll 1,$$

First criterion: Multiple scales

$$\frac{\delta^2}{2\epsilon^2} = \frac{\mu+\gamma}{4N\mu}\left(1+\frac{R_0+1}{R_0-1}+2\frac{\mu+\gamma}{\mu(R_0-1)}\right) \ll 1.$$

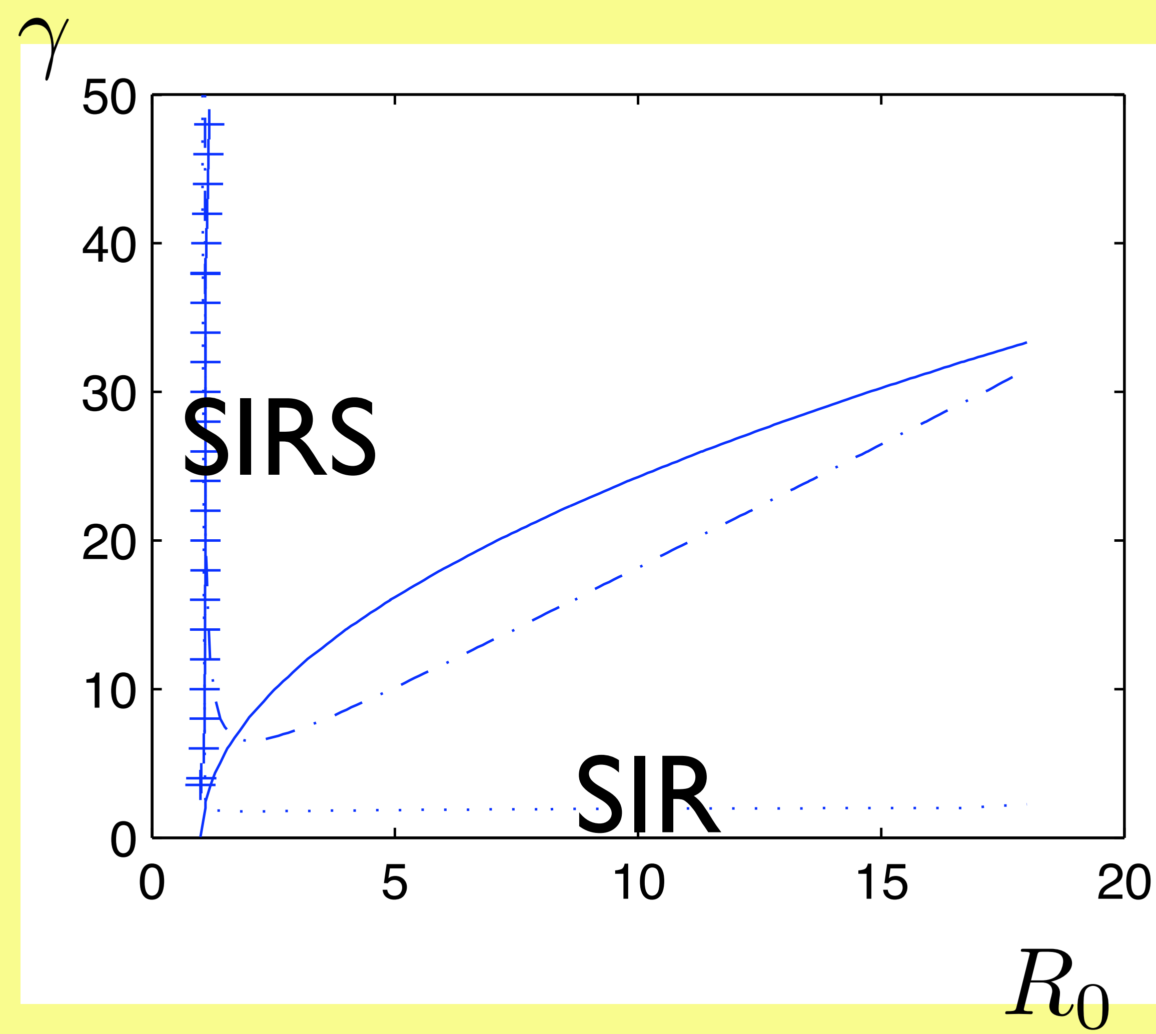
Second criterion: Balance of noise/dynamics



(# secondary infections)

SIRS: With loss of immunity

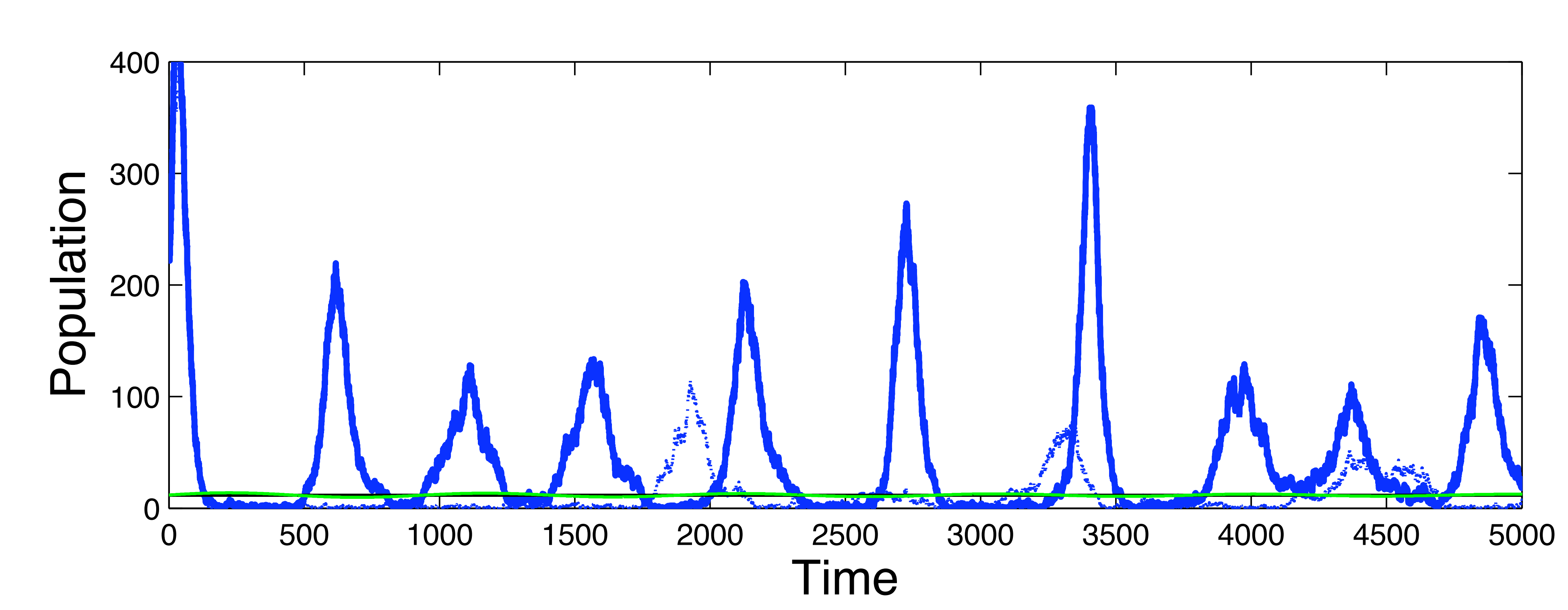
Potential for large/nonlinear fluctuations on criteria edge



Nonlinear amplitude equations

Stochastic effects for low levels

Cholera: SIRS + bacteria



CRII in infectious disease:

Fluctuations in interacting populations not necessarily “small”, even for large populations

Loss of immunity increases range of CR

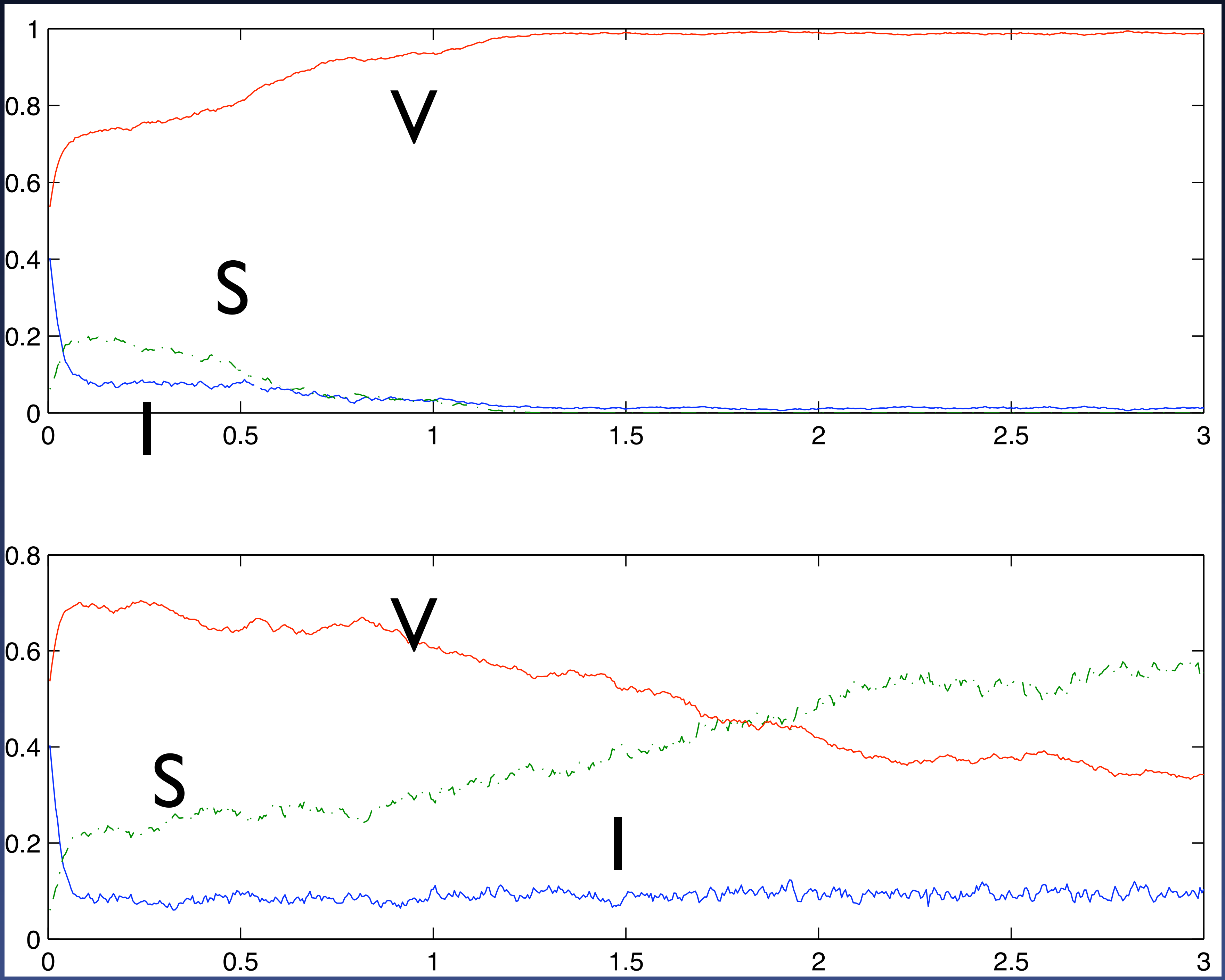
Hyper-infective strains

CR in Networks

Hidden time scales and state/time dependent potentials: Success of vaccinations

Initial period: short

Intermediate period: stochastic variation competes with quasi-steady behavior

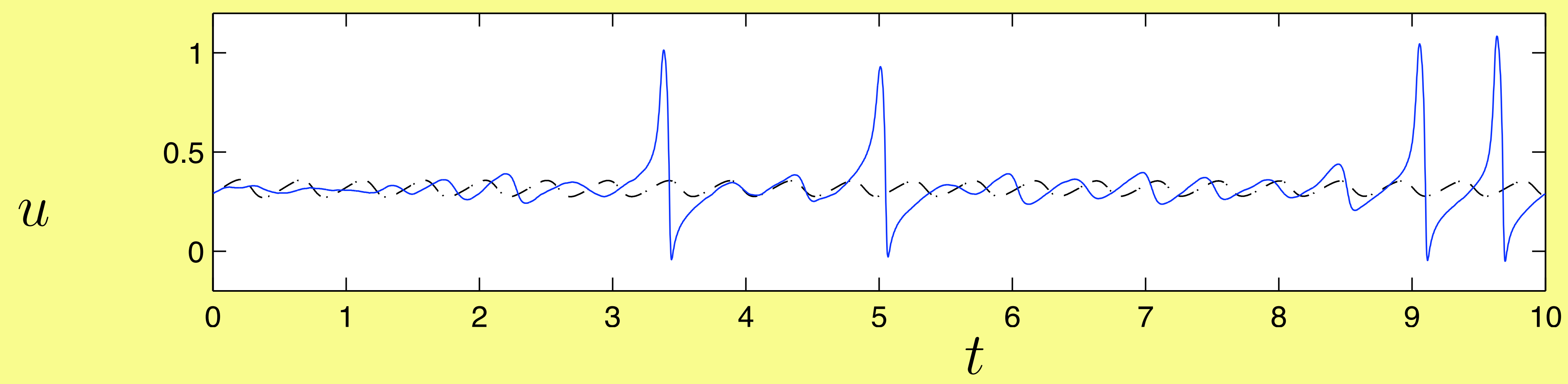


R. Lopez, B. Dembele, P. Greenwood, K. 2007

Prevalence of CR:

- Noise “stabilized” transients
- Importance of multiple time scales
- Reduced systems for different scales
- Discrete and cts modeling significant

Mixed mode oscillations: CR I and II



Other routes: noise + coupling in Type II:

N. Yu, K., YX Li, 2008