The multiple scales of lightning and a moving boundary problem

Ute Ebert[1,2]

[1] CWI (Centrum voor Wiskunde en Informatica) Amsterdam, The Netherlands

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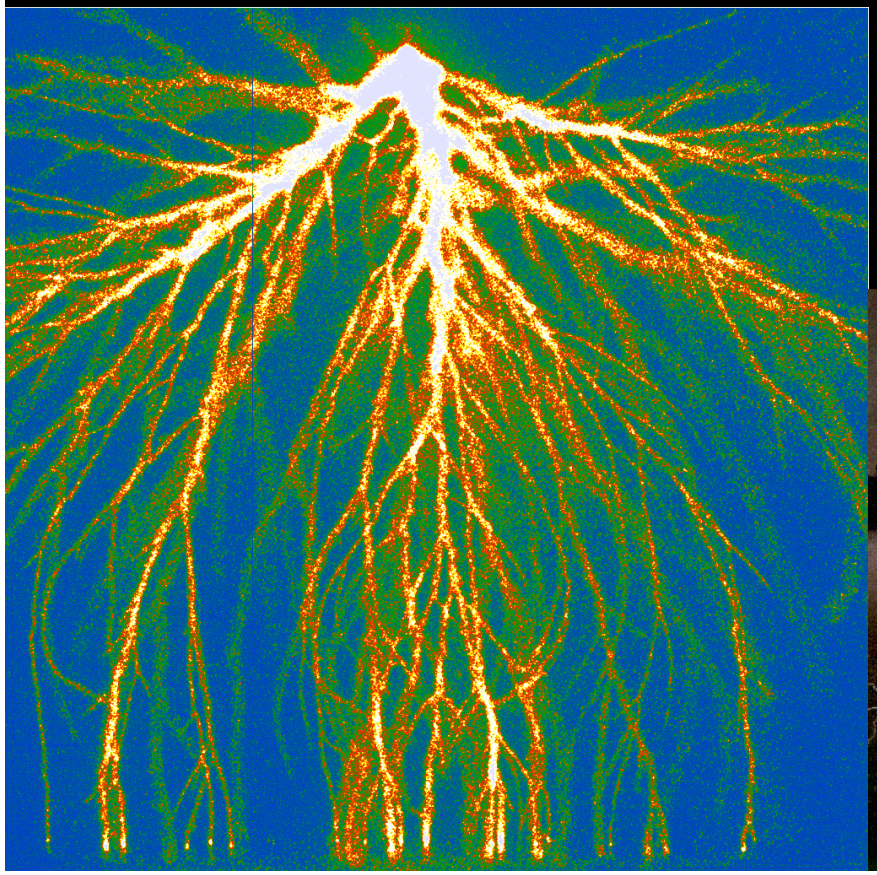

TexPoint fonts used in EMF

The multiple scales of lightning and a moving boundary problem

Ute Ebert[1,2]

Theory:
[1] CWI Amsterdam (0.8)

Willem Hundsdorfer (staff)
Manuel Arrayas
Fabian Brau
Wouter Brok
Chao Li
Alejandro Luque
Bernard Meulenbroek
Carolynne Montijn
Margreet Nool
Valeria Ratushnaya
Andrea Rocco
Gideon Wormeester

Experiment:
[2] Eindhoven Univ. Tech. (0.2)

Eddie van Veldhuizen (staff)
Tanja Briels
Sander Nijdam
Hans Winands
Vuong Nguyen
van Deursen (staff)

+ many collaborations
in applied physics and
electrical engineering

and

Benny Davidovitch (U Mass)
Gianne Derks (Surrey, UK)
Chiu-Yen Kao (Ohio)
Sergey Pancheshnyi (Toulouse)
Wim van Saarloos (Leiden)
Lothar Schäfer (Essen-Duisburg)
Davis Sentman (Fairbanks, Alaska)
Saleh Tanveer (Ohio)
Yoav Yair (Tel Aviv)

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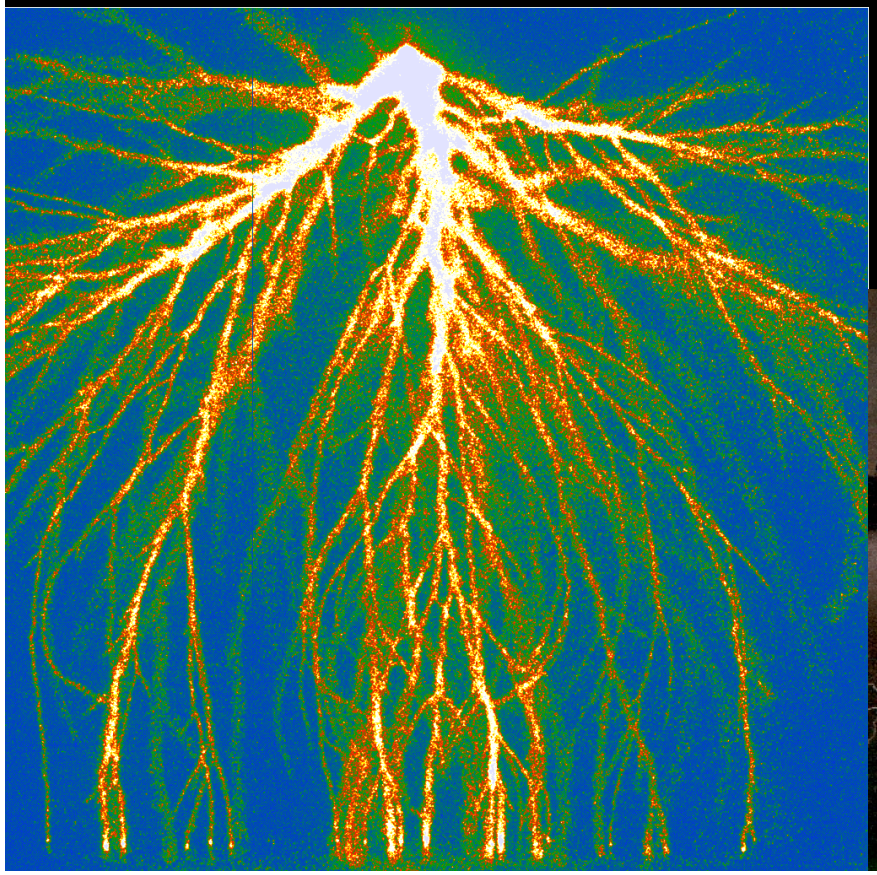
The multiple scales of lightning and a moving boundary problem

Observations/Experiments

Multiscale simulations

Boundary approximation

Results boundary problem



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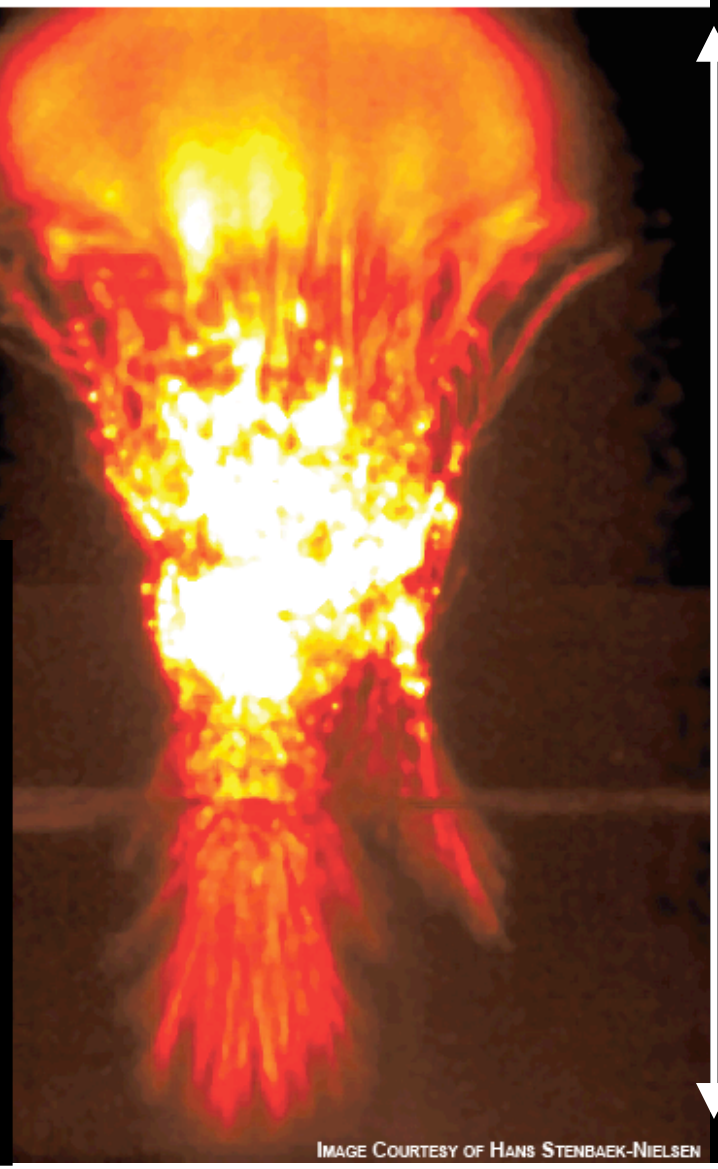
Chapman Conference on the Effects of Thunderstorms and Lightning in the Upper Atmosphere

Penn State University, State College, PA, USA
May 10–14, 2009

Topics:

- Observations of Transient Luminous Events
- Theory of Transient Luminous Events
- ELF/VLF Effects of Lightning and Transient Luminous Events
- Energetic Radiation from Lightning and Terrestrial Gamma Ray Flashes



40 km

Image Courtesy of Hans Stenbaek-Nielsen

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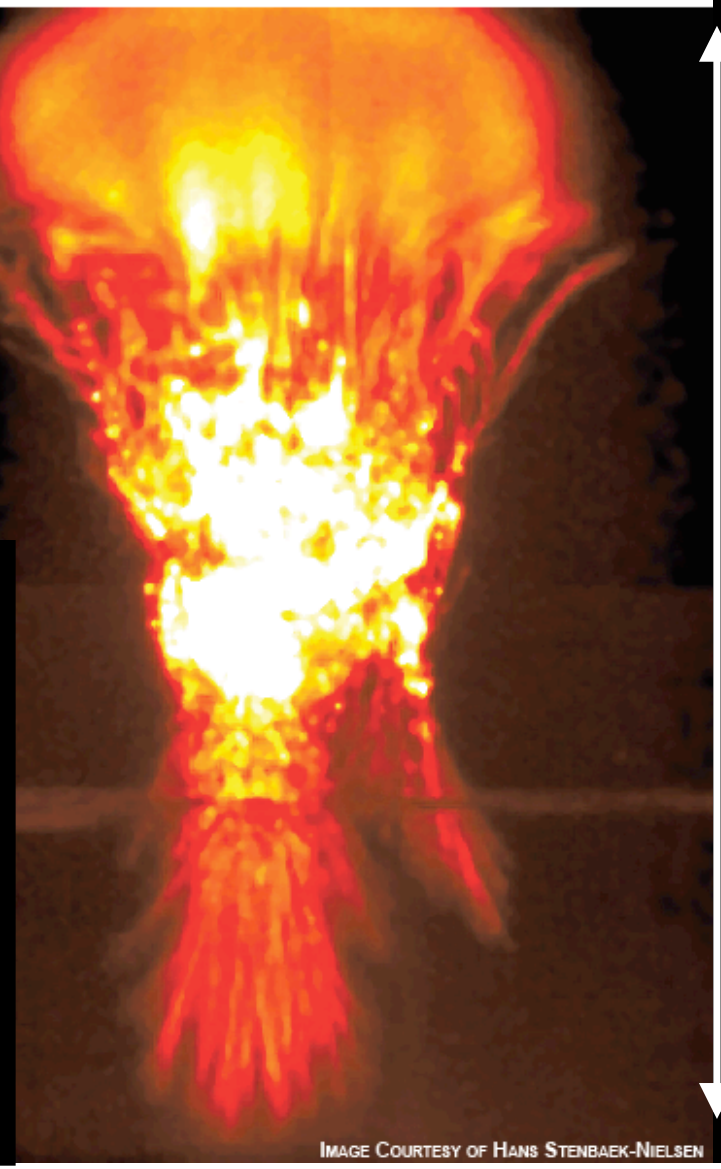

Image Courtesy of Hans Stenbaek-Nielsen

40 km

Transient luminous events

Terrestrial Gamma-Ray Flashes

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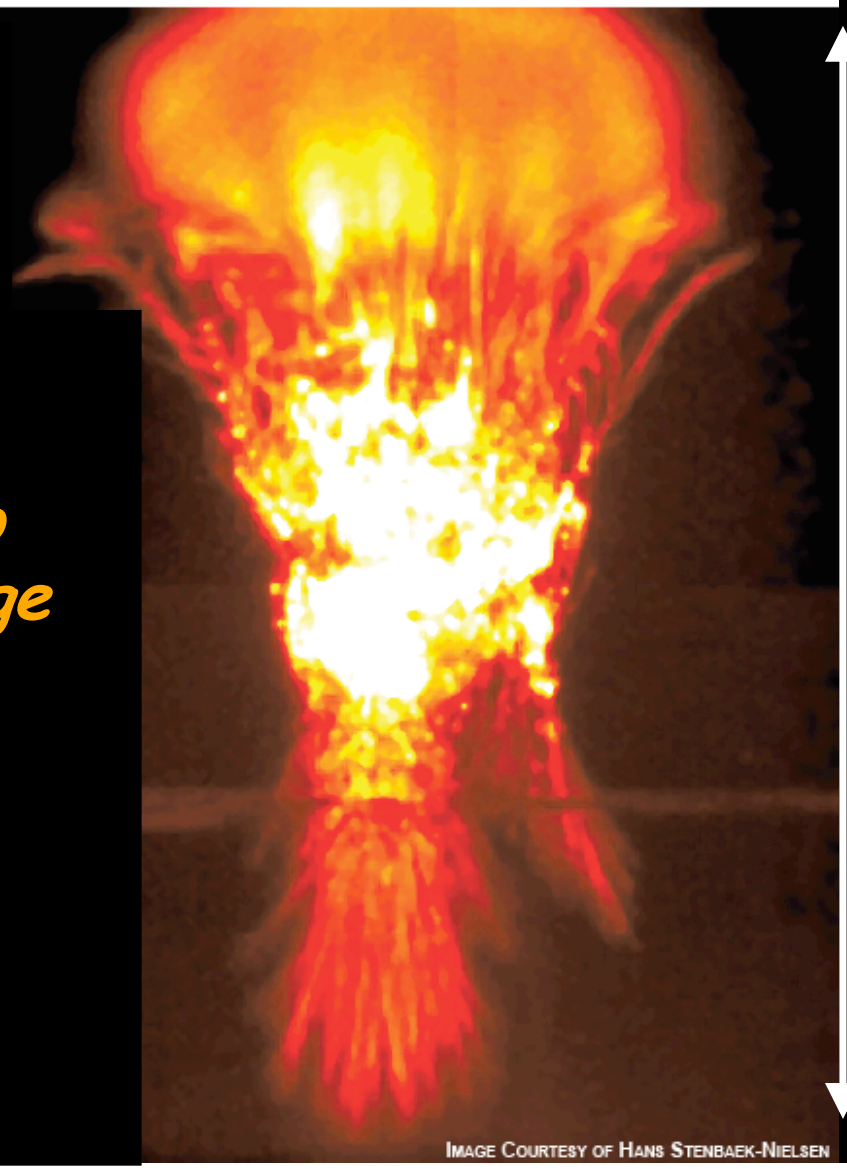

Challenge: Relate lightning to gas discharge physics!

40 km

Transient luminous events

Terrestrial Gamma-Ray Flashes

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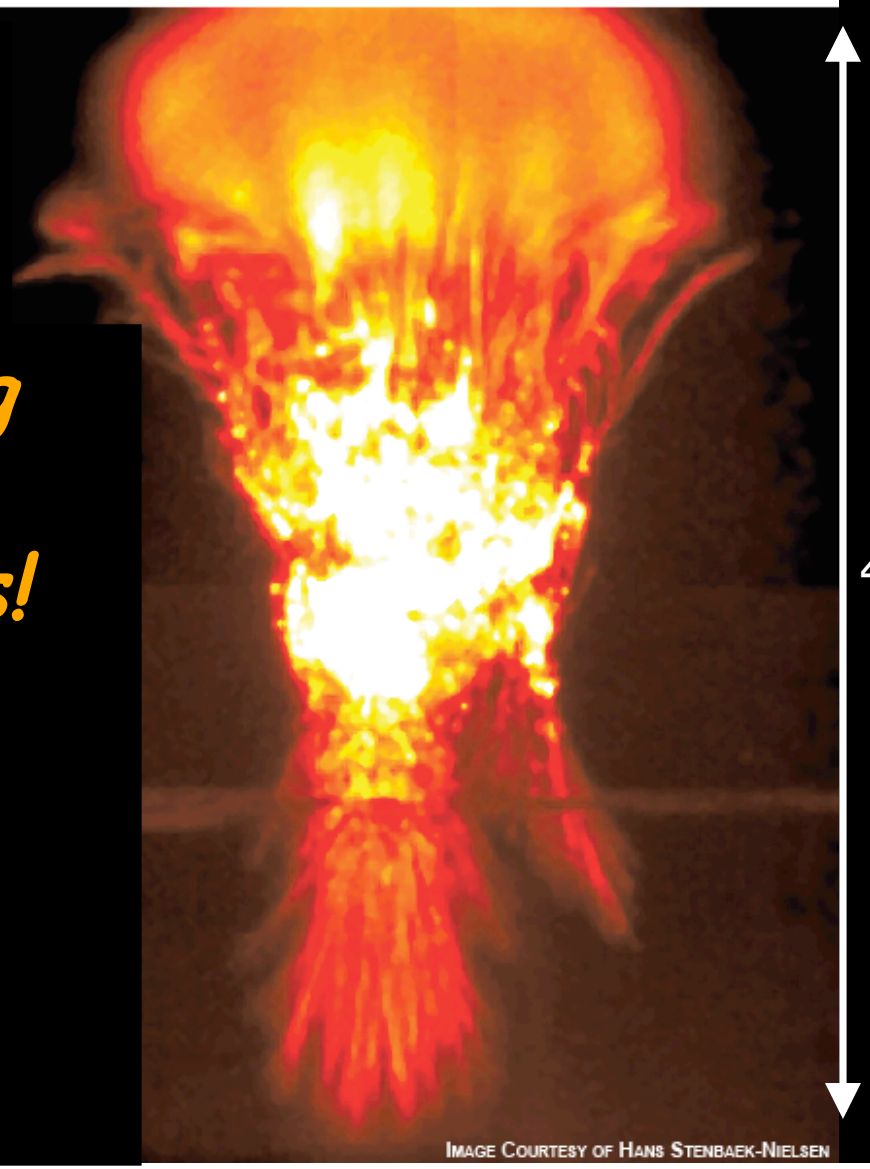


Nothing but fingers!

40 km

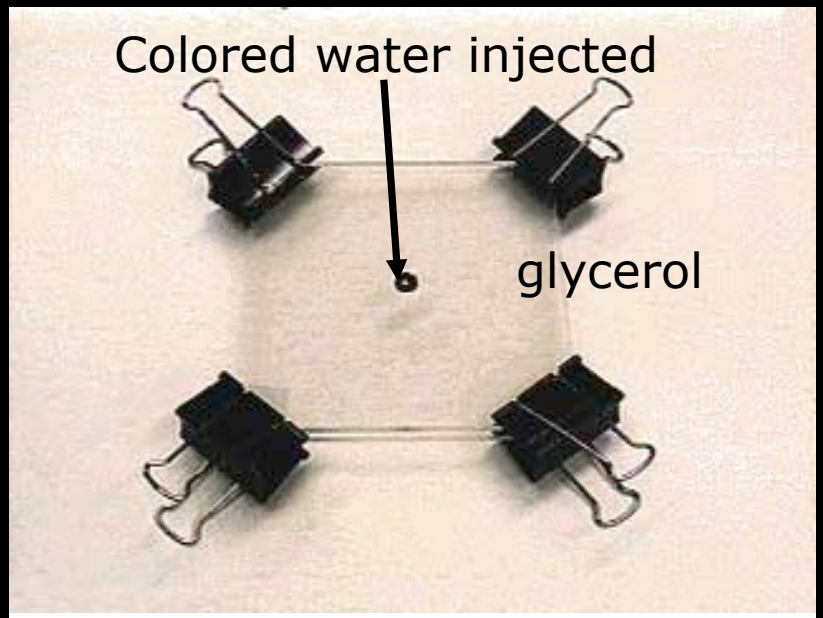
Transient luminous events

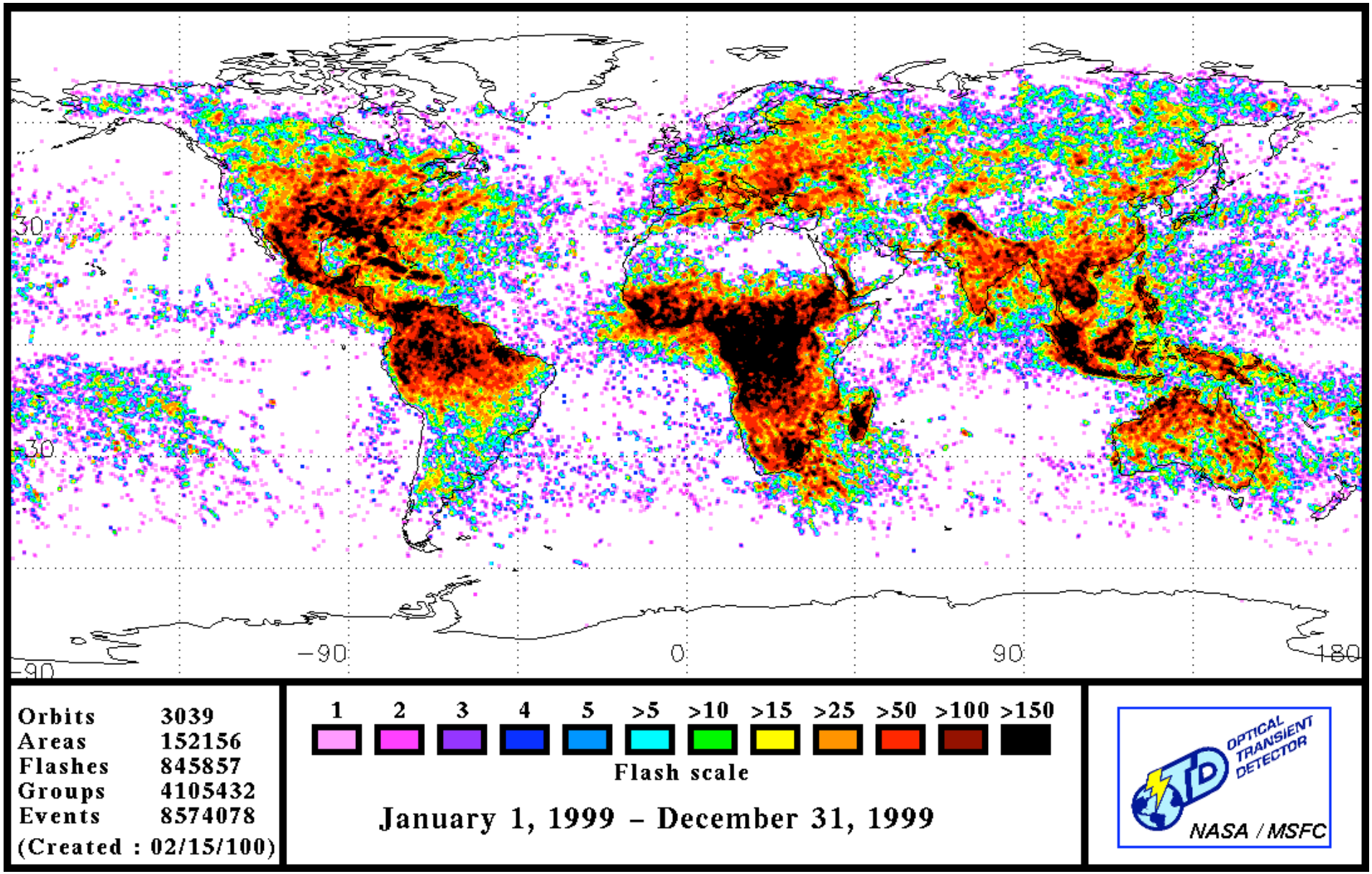
Terrestrial Gamma-Ray Flashes

NFERENCE ON THE EFFECTS OF THUNDERSTORMS
LIGHTNING IN THE UPPER ATMOSPHERE
PENN STATE UNIVERSITY, STATE COLLEGE, PA, USA
MAY 10–14, 2009

CHAPMAN CONFERENCE

Nothing but fingers: (here: Hele-Shaw flow)

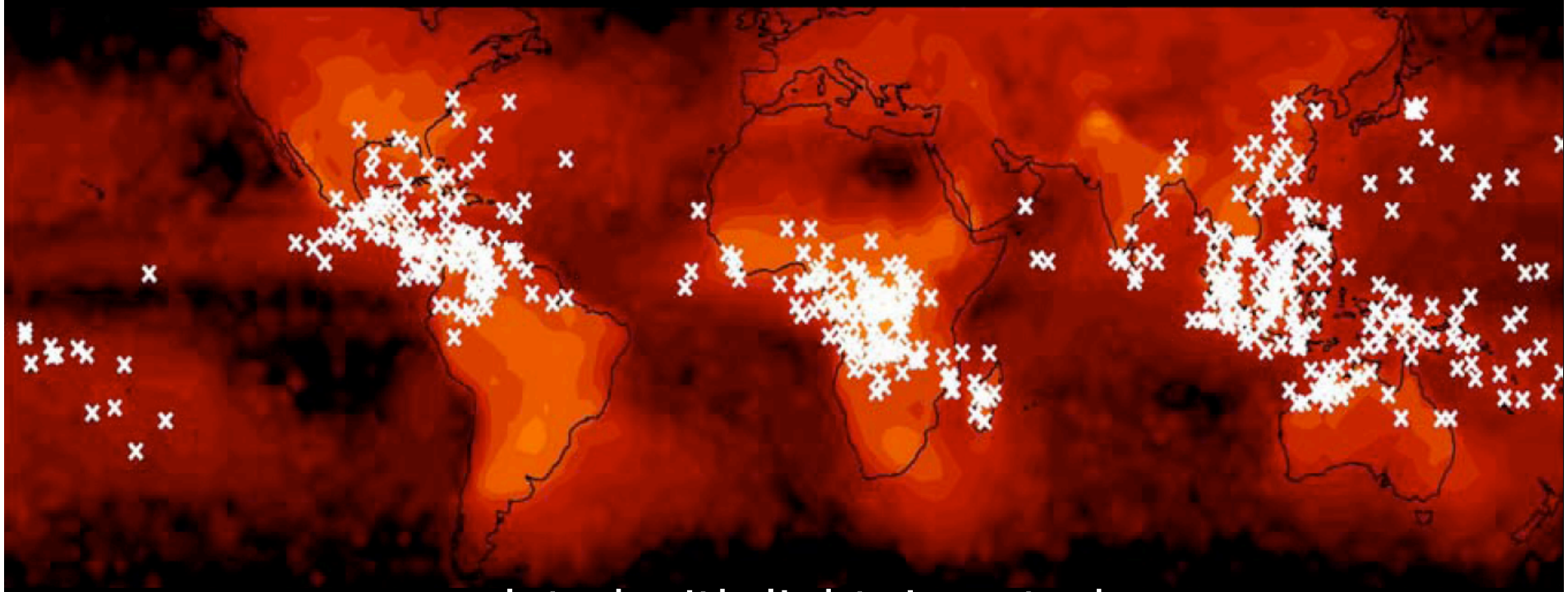




- **ca. 45 lightning flashes/second worldwide**
- **important source of ozone and NO_x (green house gases)**

Terrestrial Gamma-Ray Flashes, > 50/day

[discovered 1994, here RHESSI satellite data 2006]



correlated with lightning strokes

There are puzzles in cosmology,
but do we understand our own earth?

Terrestrial Gamma-Ray Flashes, > 50/day

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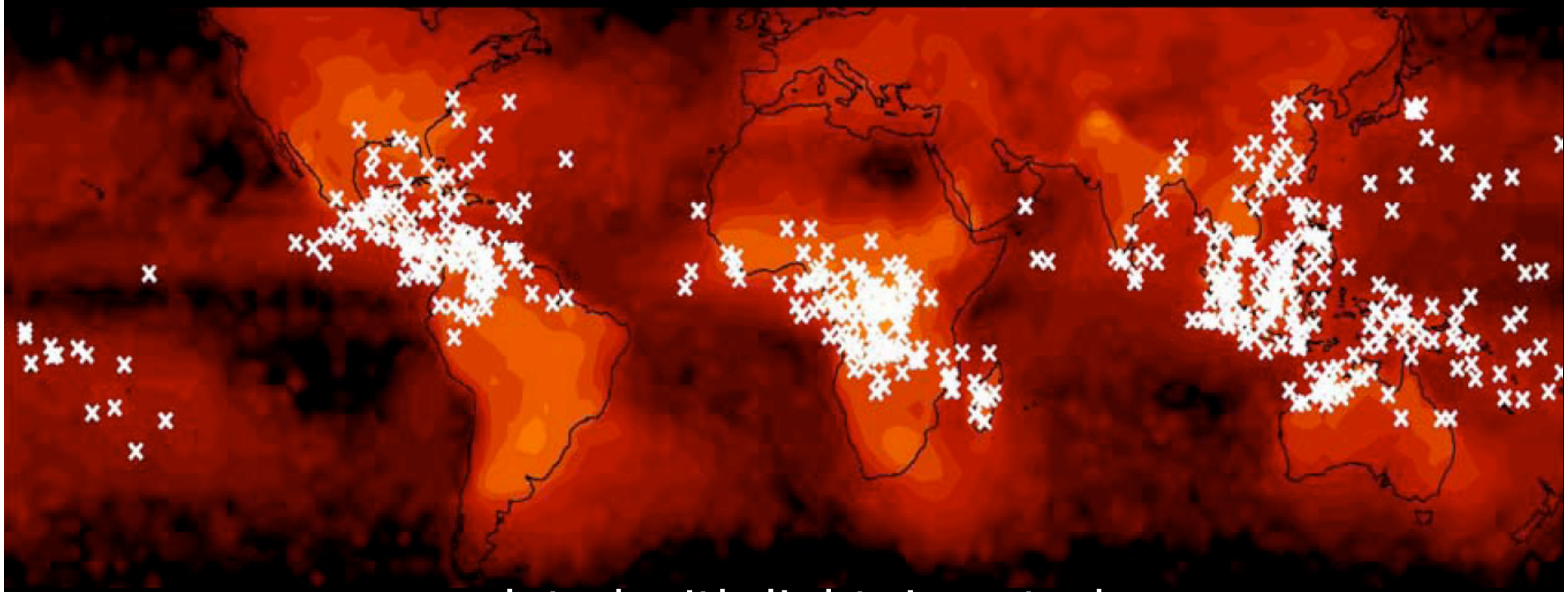
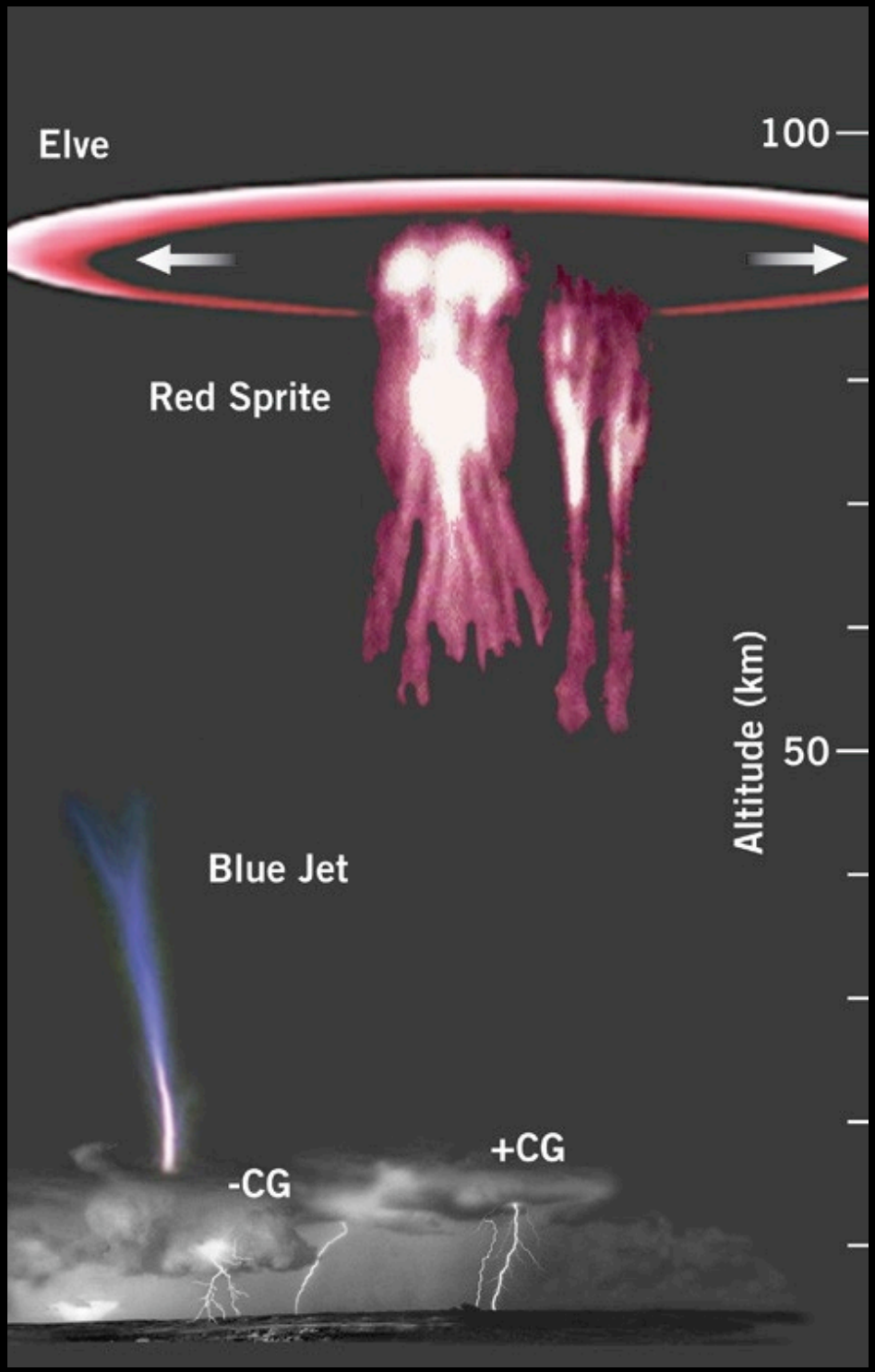


correlated with lightning strokes

There are puzzles in cosmology,
but do we understand our own earth?

Theory: avalanches versus fingers

ASIM-mission of ESA to international space station in 2013



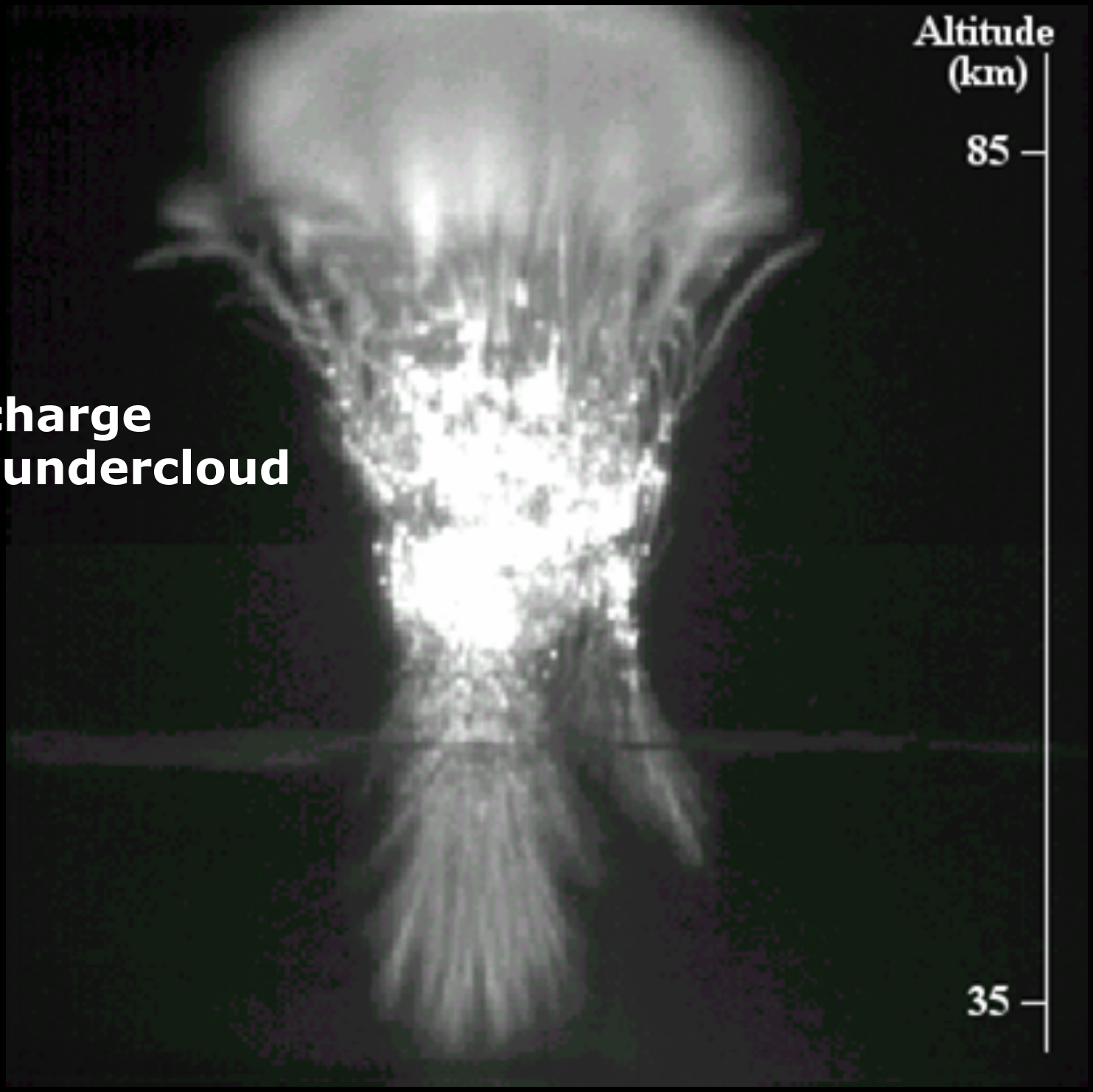
Transient luminous events:
Elves, sprites, jets*

correlated
with lightning strokes

Predicted in 1925,
observed since 1989.

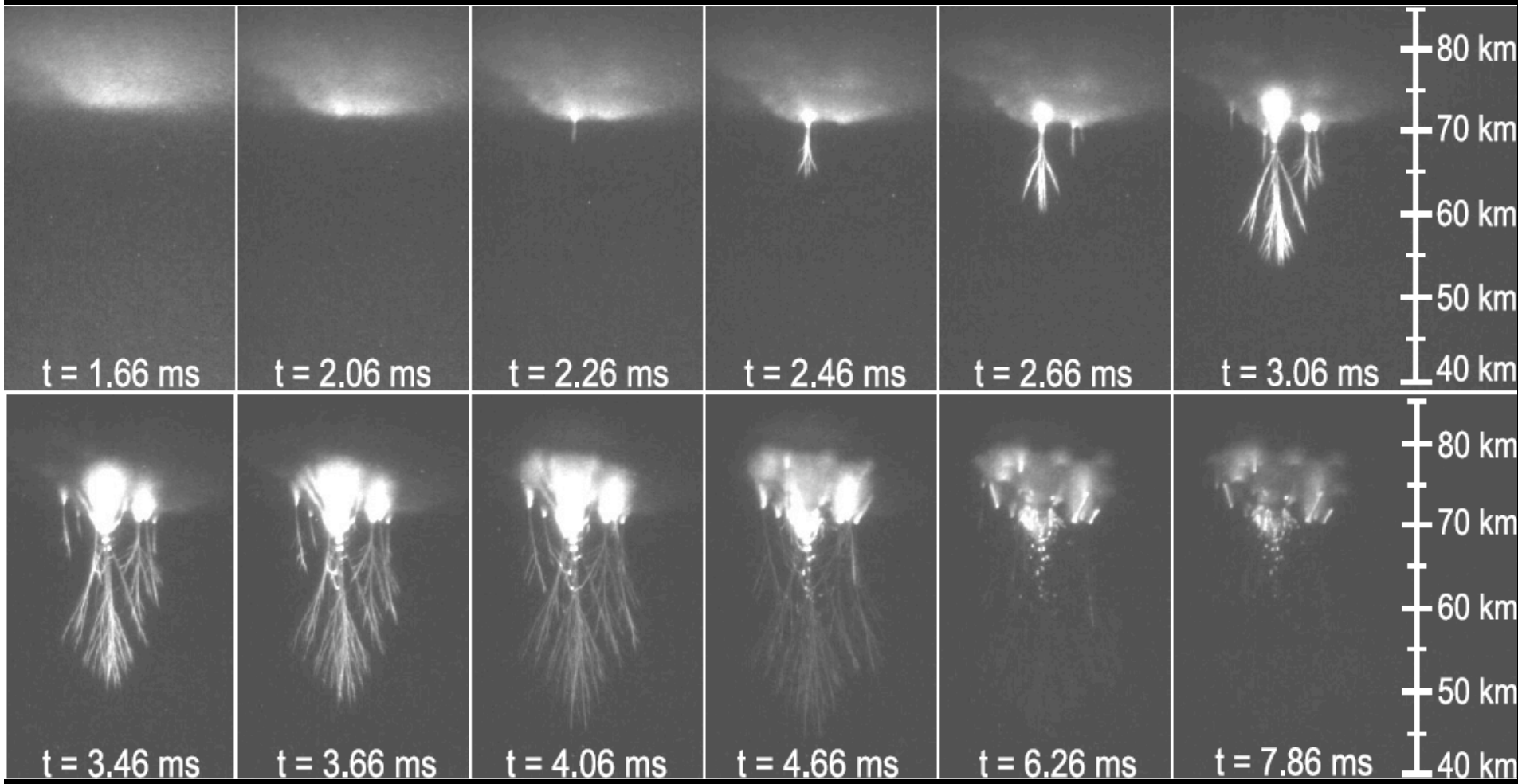
*) Shakespeare,
Mid-Summer Night's Dream

Sprite discharge above a thundercloud



Submillisecond imaging of sprites

[Cummer et al., Geophys. Res. Lett. 2006]



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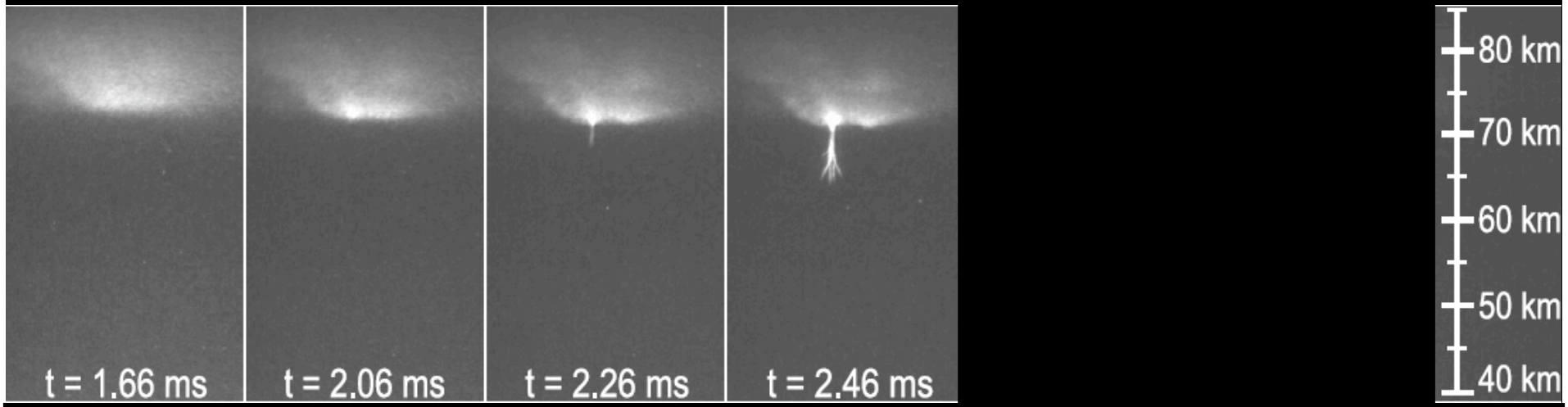


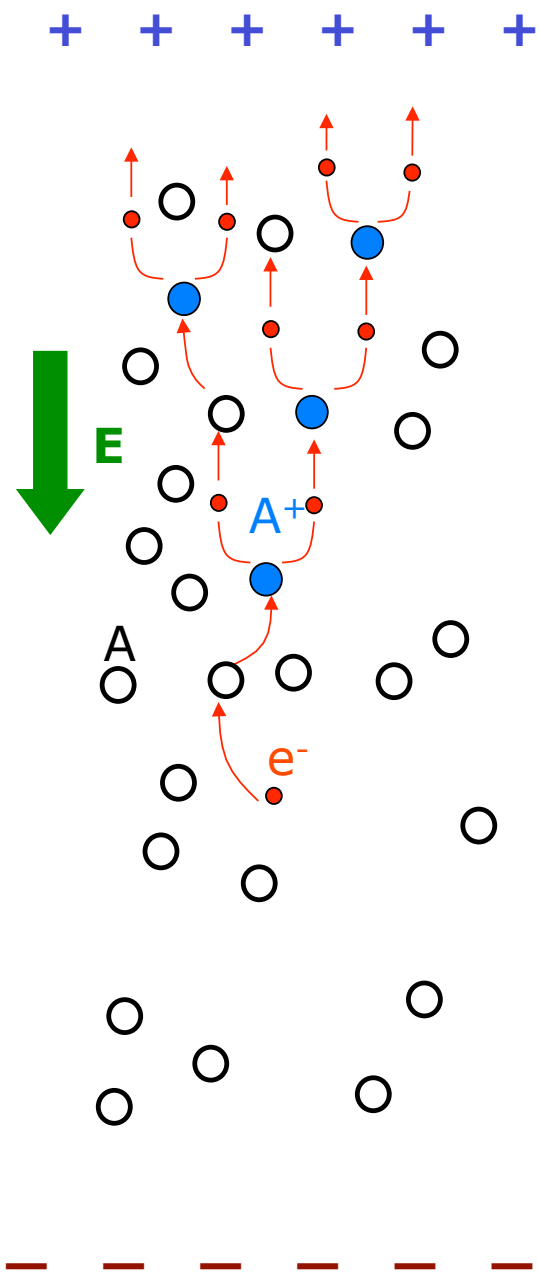
Sprite halo and sprite inception?
Velocity v? Diameter d?

Simulations with realistic profiles for air density and ionization density reproduce sprite halo and the first sprite finger shooting out of it. [Luque, Ebert, subm]

Which model to use?

Fast processes in the ionization front:

Electrons drift and diffuse in local **E**-field. Elastic, inelastic and ionizing collisions with neutral molecules. Degree of ionization < 10^{-4}.

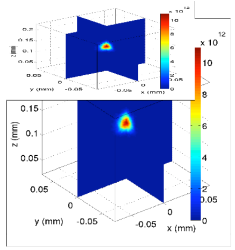


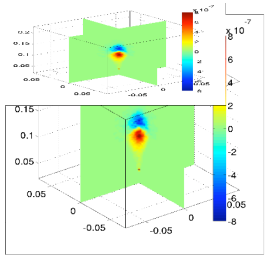
3D particle simulation at E_0=100kV/cm

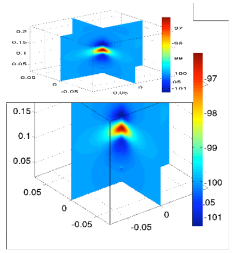
Electron density | Charge density | Electric field

T=0.18ns

Avalanche (linear):

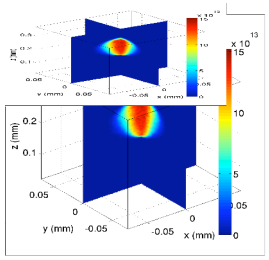


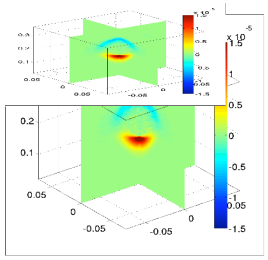


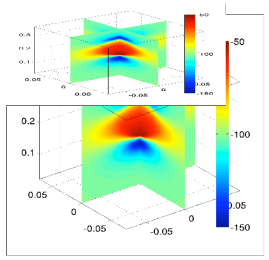


T=0.36ns

Streamer (space charge effects):







[Li et al, IEEE Trans Plasma Sci 2008]

Fast processes in the ionization front:

Electrons drift and diffuse in local **E**-field.
Elastic, inelastic and ionizing collisions with neutral molecules. Degree of ionization < 10^{-4}.

→ **Minimal streamer model**
(for negative streamers in pure N_2 or Ar):

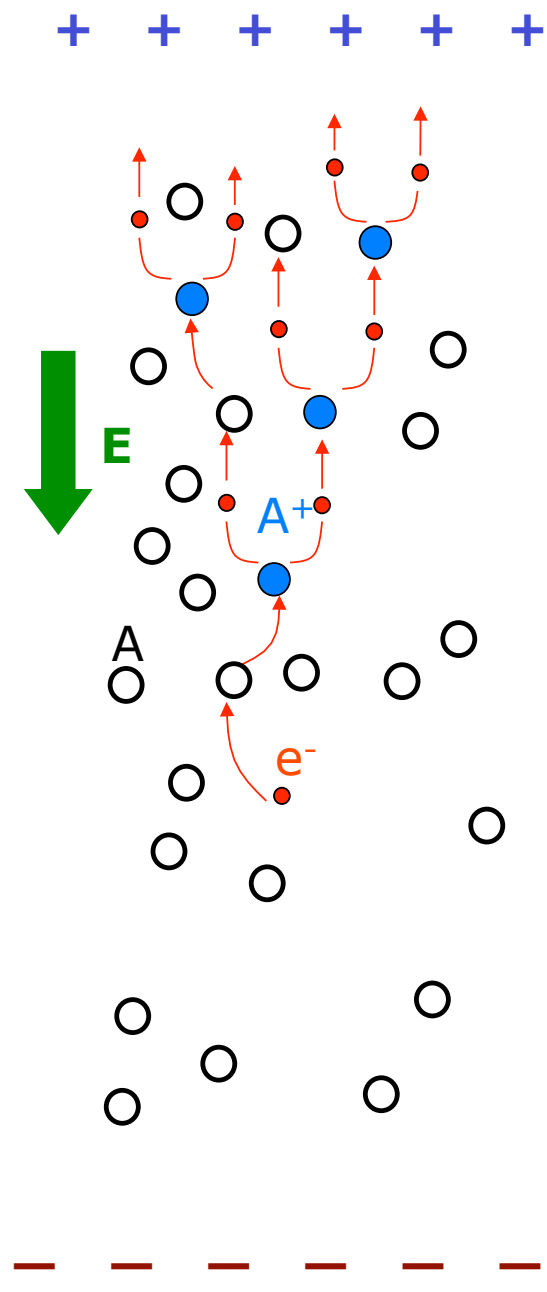
Continuity equations for densities of electrons σ and positive ions ρ, coupled to the electrostatic field **E.**

$$\partial_t \rho = D \nabla^2 \rho + \nabla \cdot (E \rho) + \rho f(|E|);$$

$$\partial_t \sigma = \rho f(|E|);$$

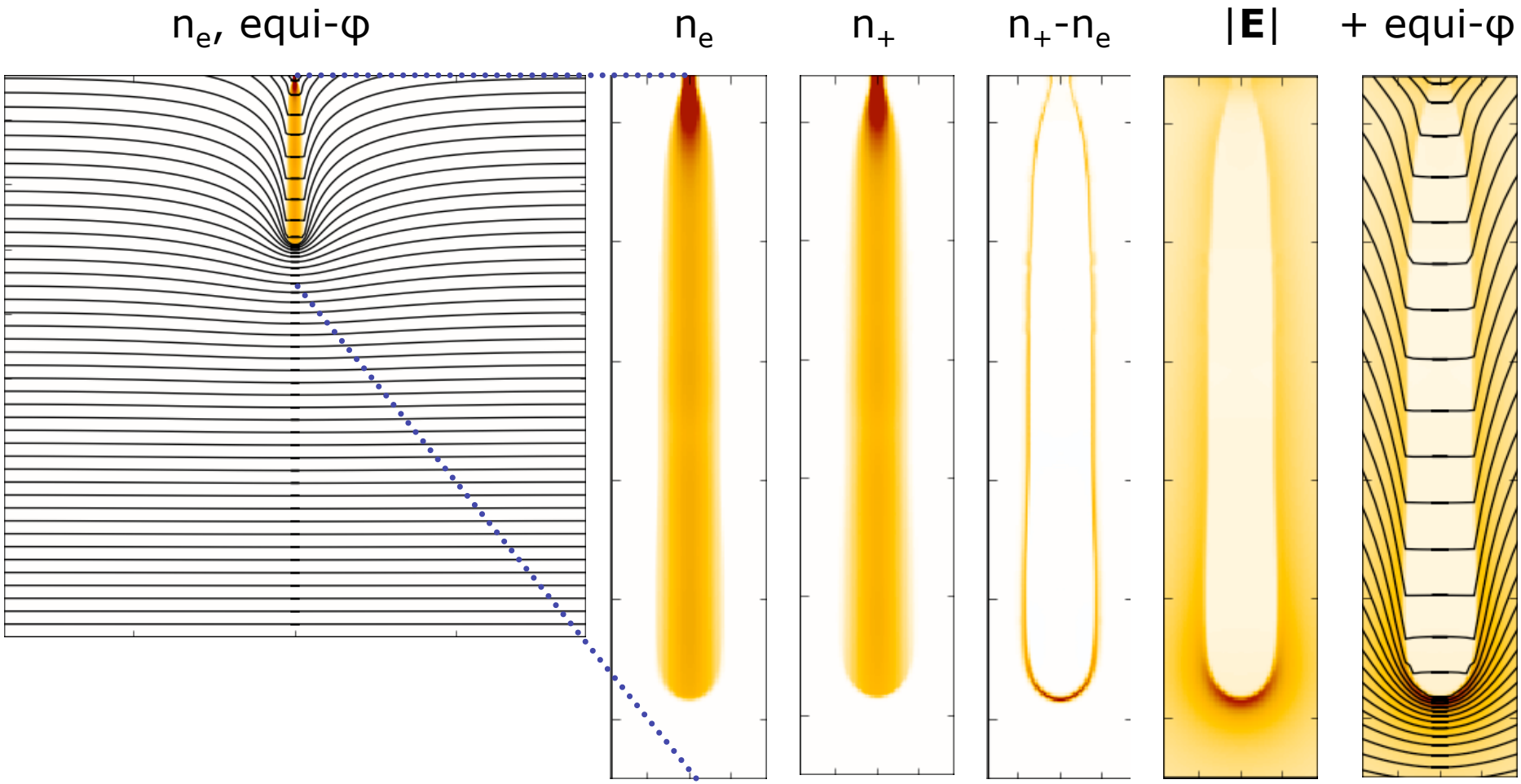
$$\Delta \phi = \sigma - \rho, \qquad E = -\nabla \phi;$$

$$f(x) = |x| e^{-1/|x|}; \qquad D = 0.1:$$



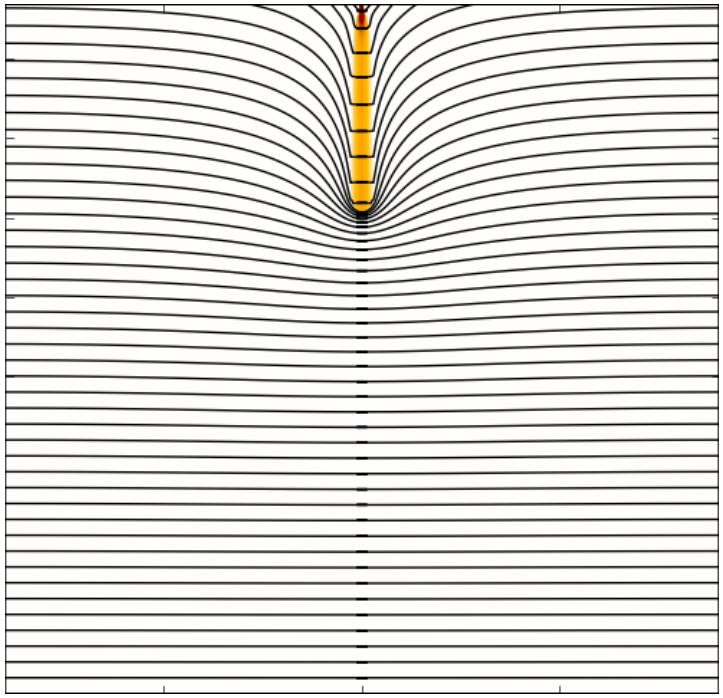
Positive streamer in air

Complete system: Zoom:

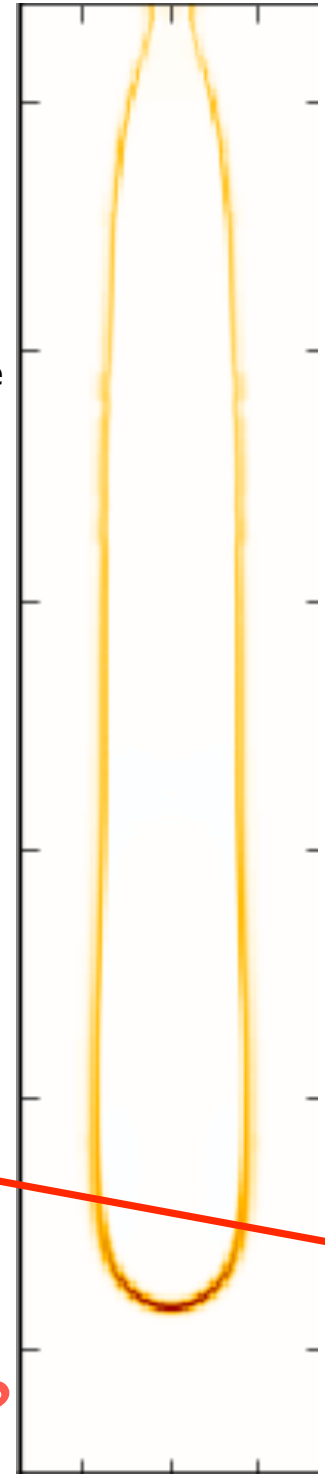


Local field enhancement allows the streamer finger to propagate

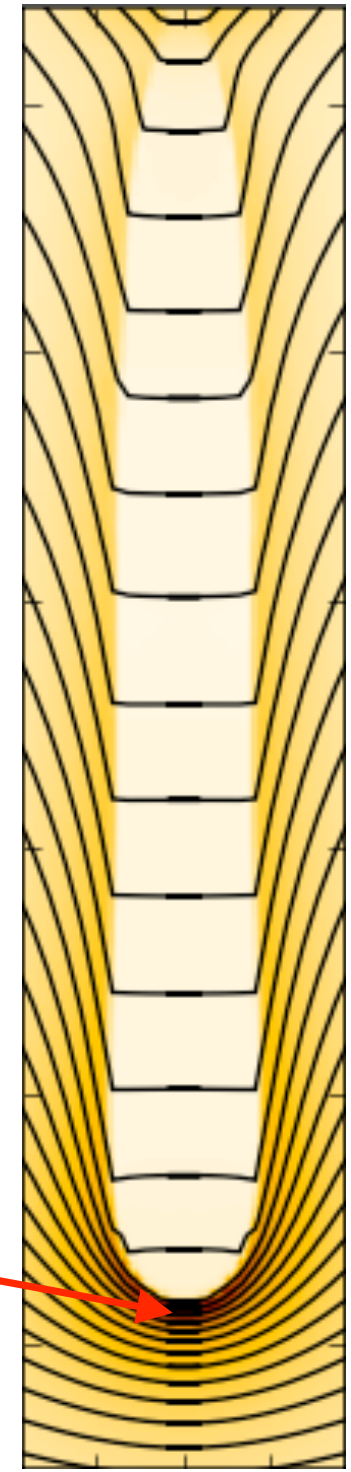
Positive streamer in air



n_+-n_e



|**E**|,
equi-φ



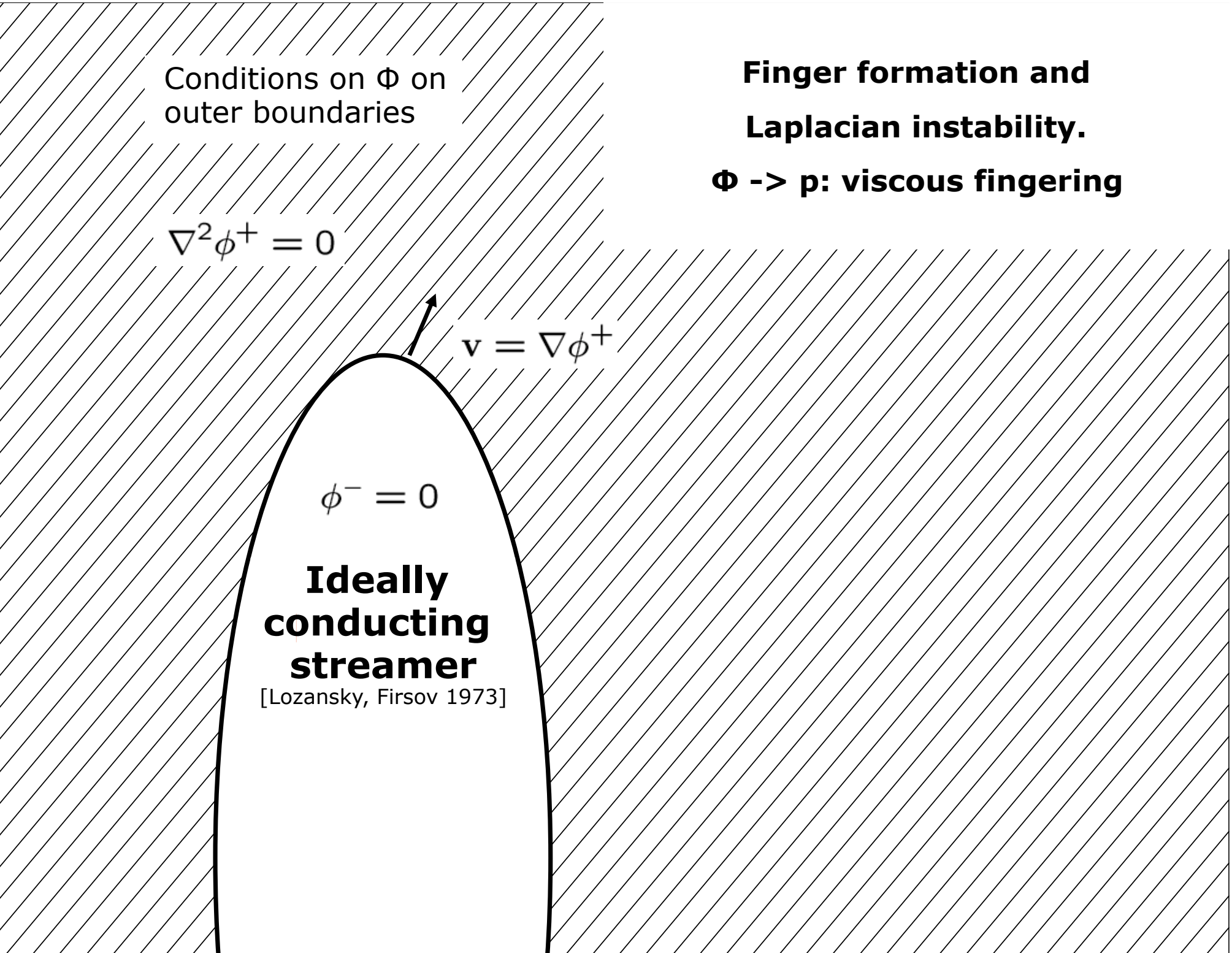
Electron acceleration in enhanced field

-> X-rays and terrestrial gamma-rays?

Finger formation and

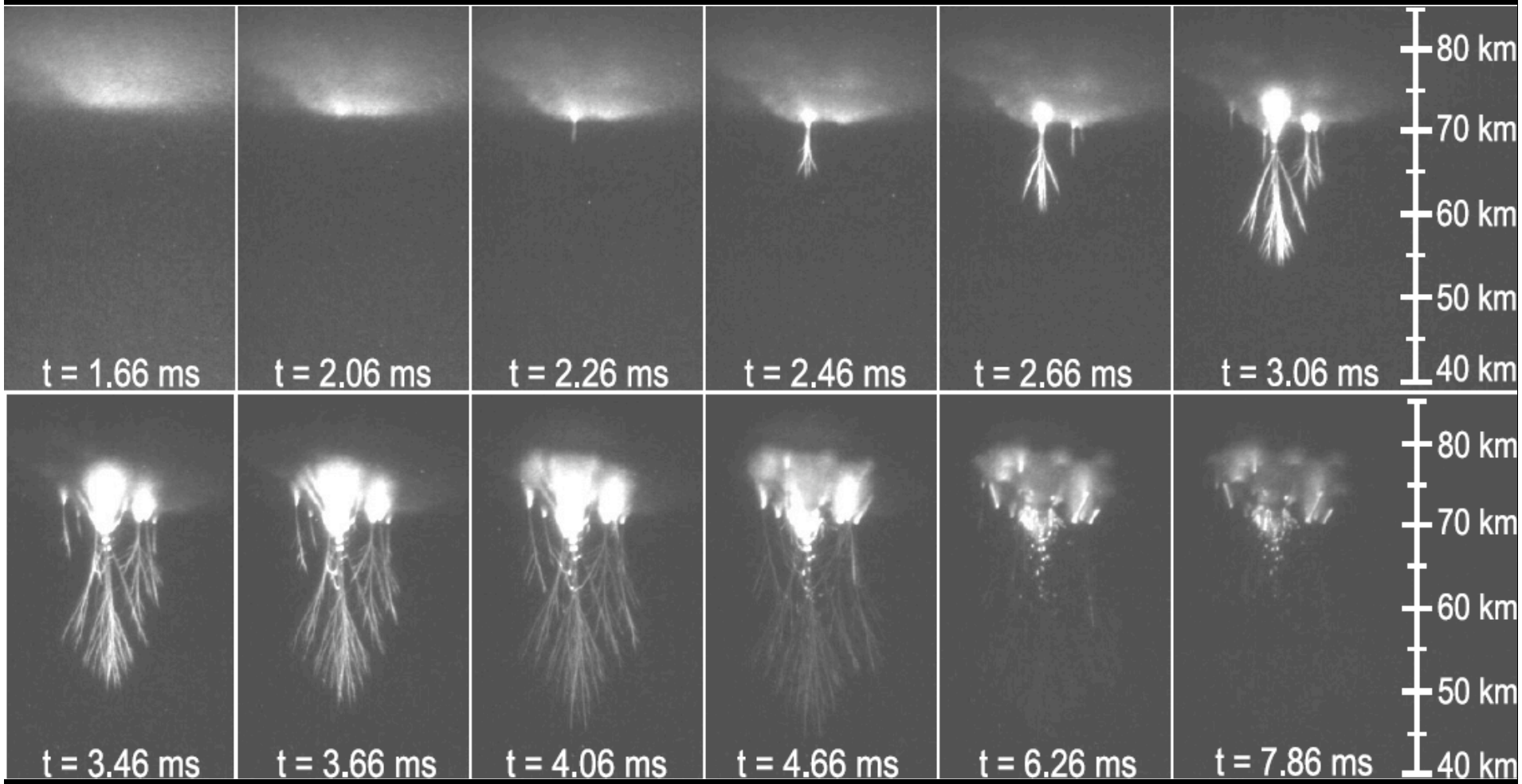
Laplacian instability.

Φ -> p: viscous fingering



Submillisecond imaging of sprites

[Cummer et al., Geophys. Res. Lett. 2006]

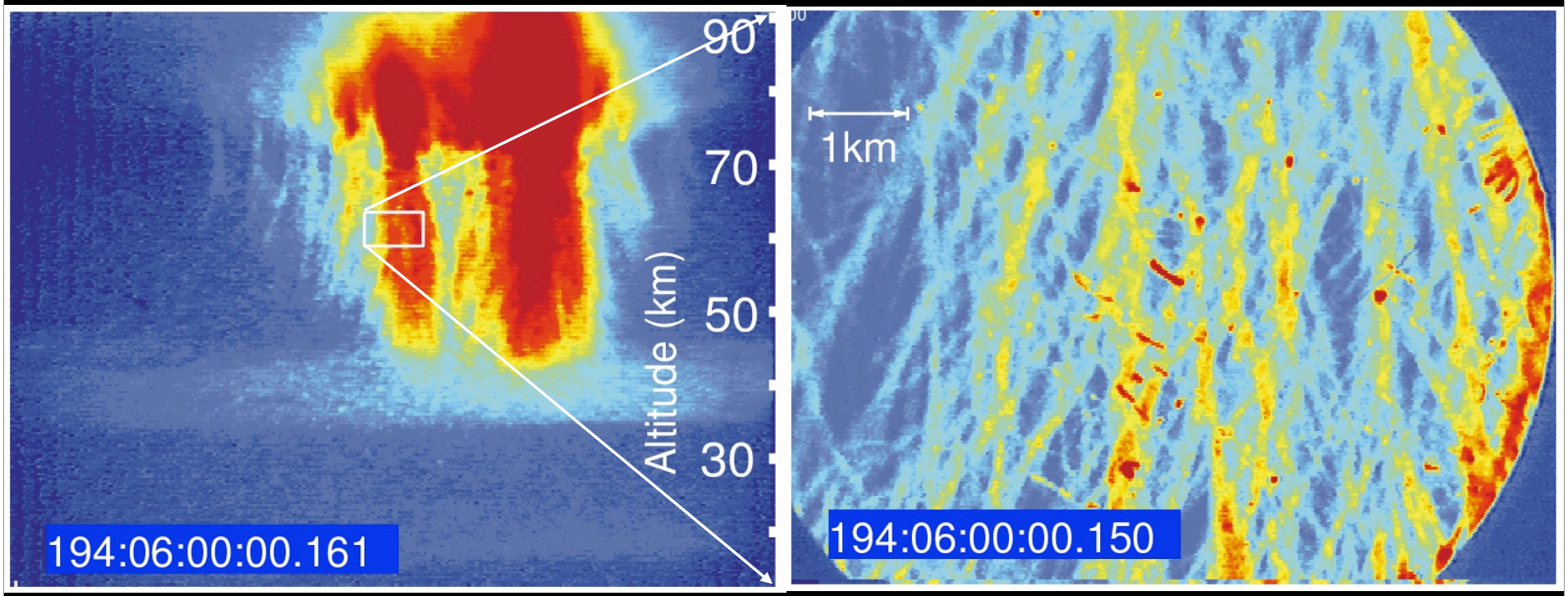


A closer
experimental
inspection
of sprites,

taking them
down to earth.

Telescopic images of sprite discharges

[Gerken et al., Geophys. Res. Lett. 2000]



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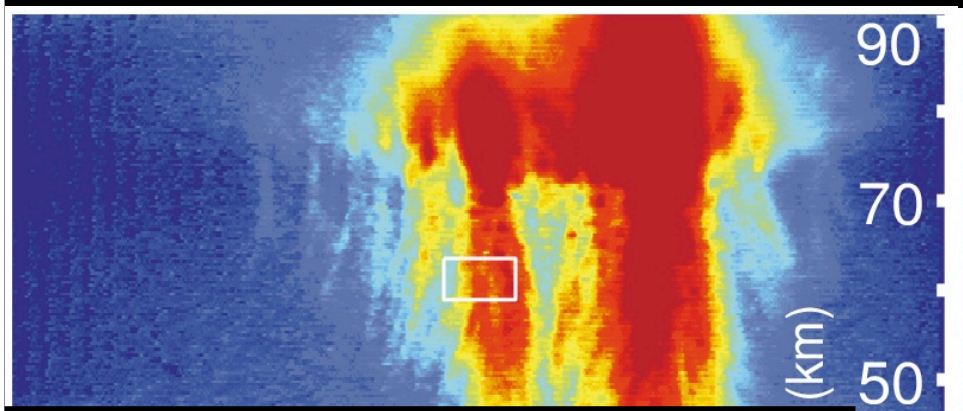
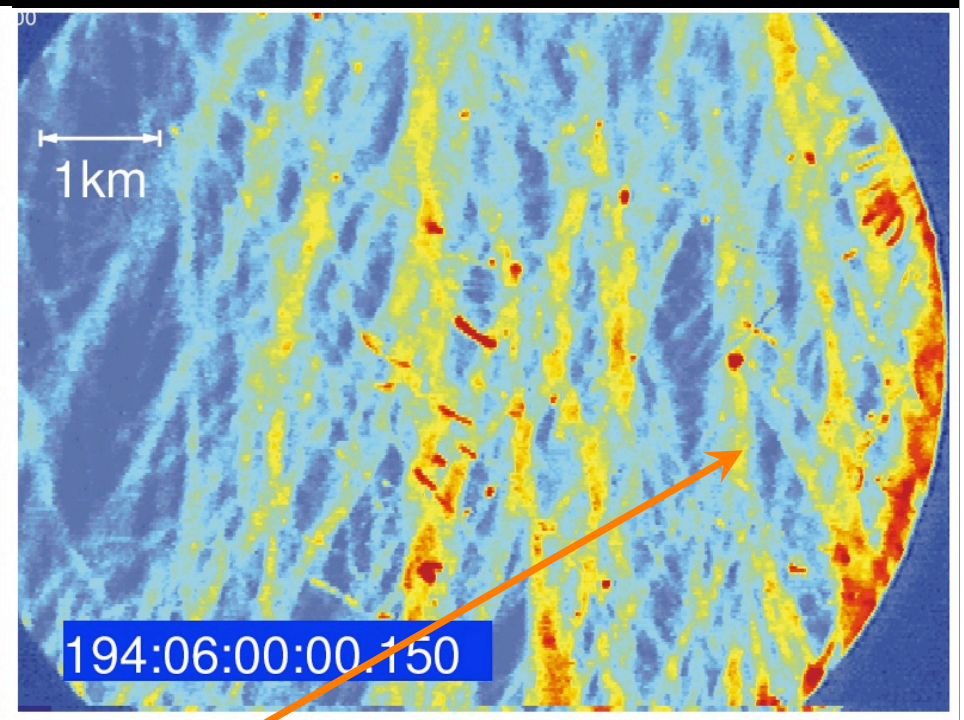
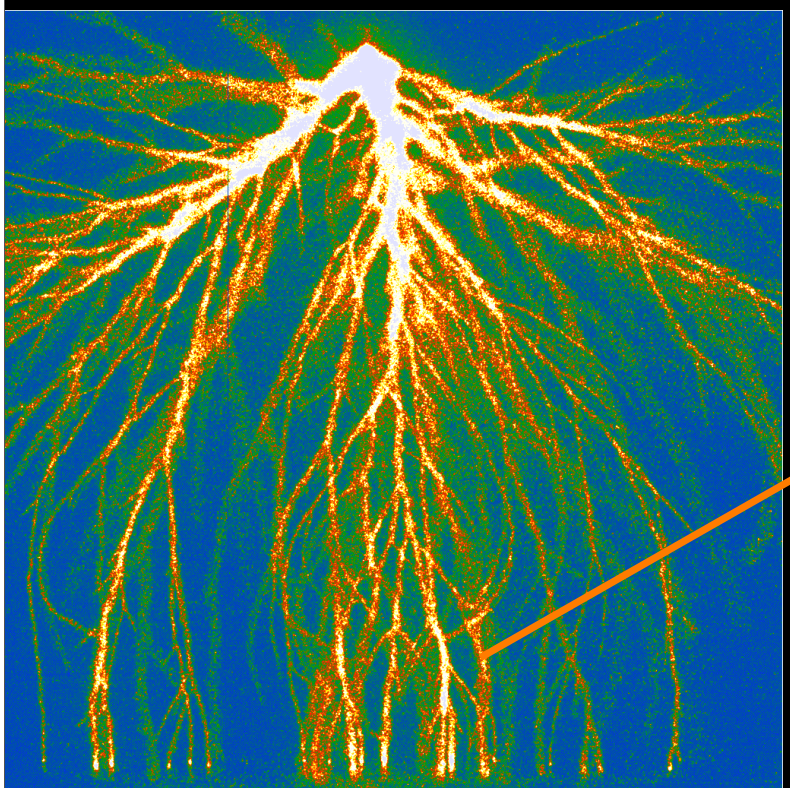




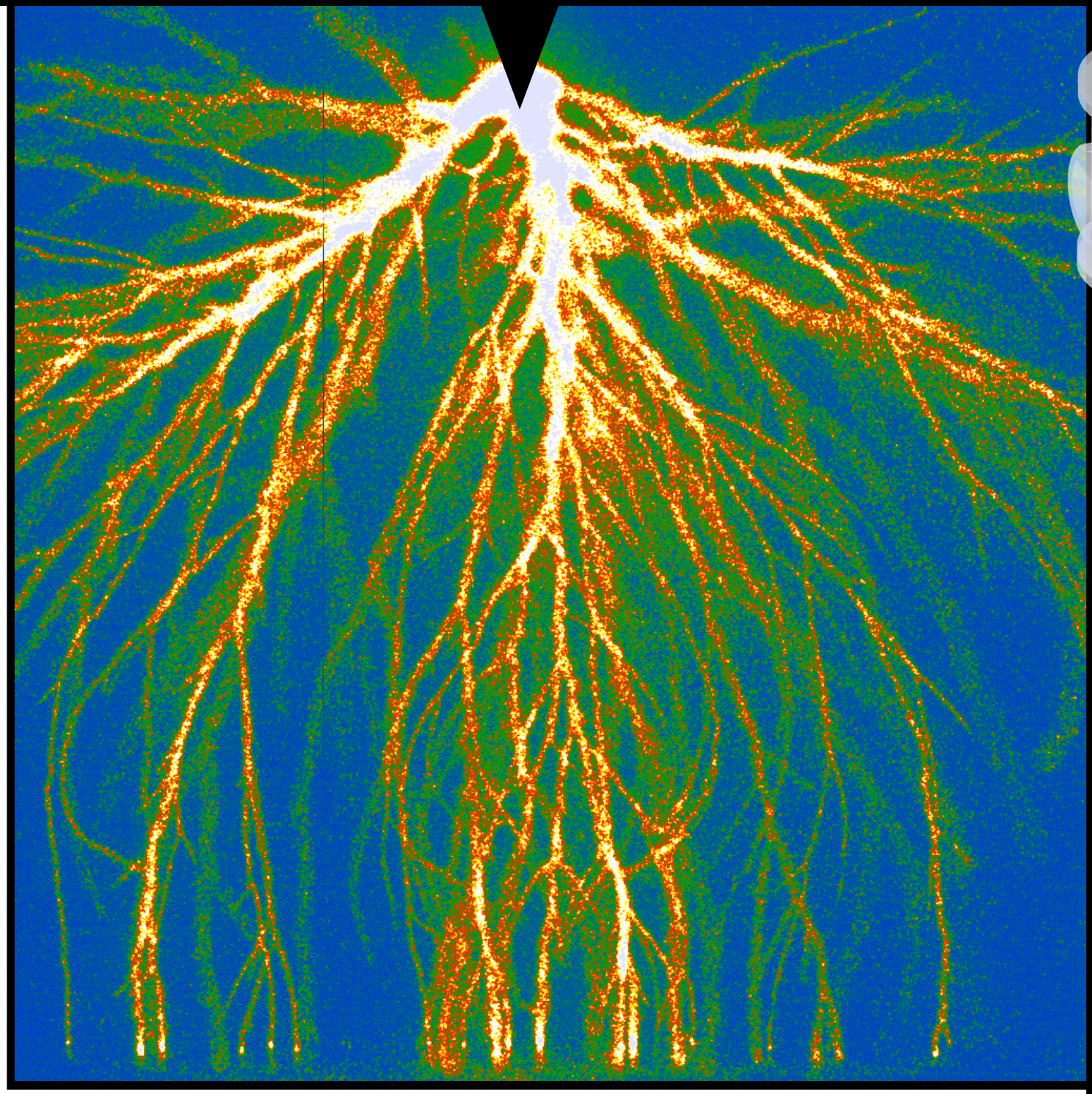


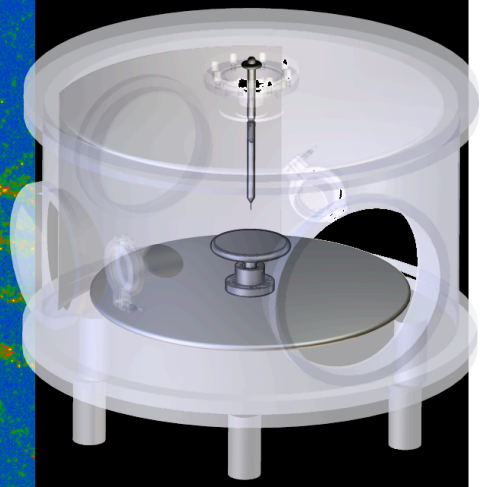
4 cm
1 bar

Approximate similarity between different gas densities,

better than theory predicts.

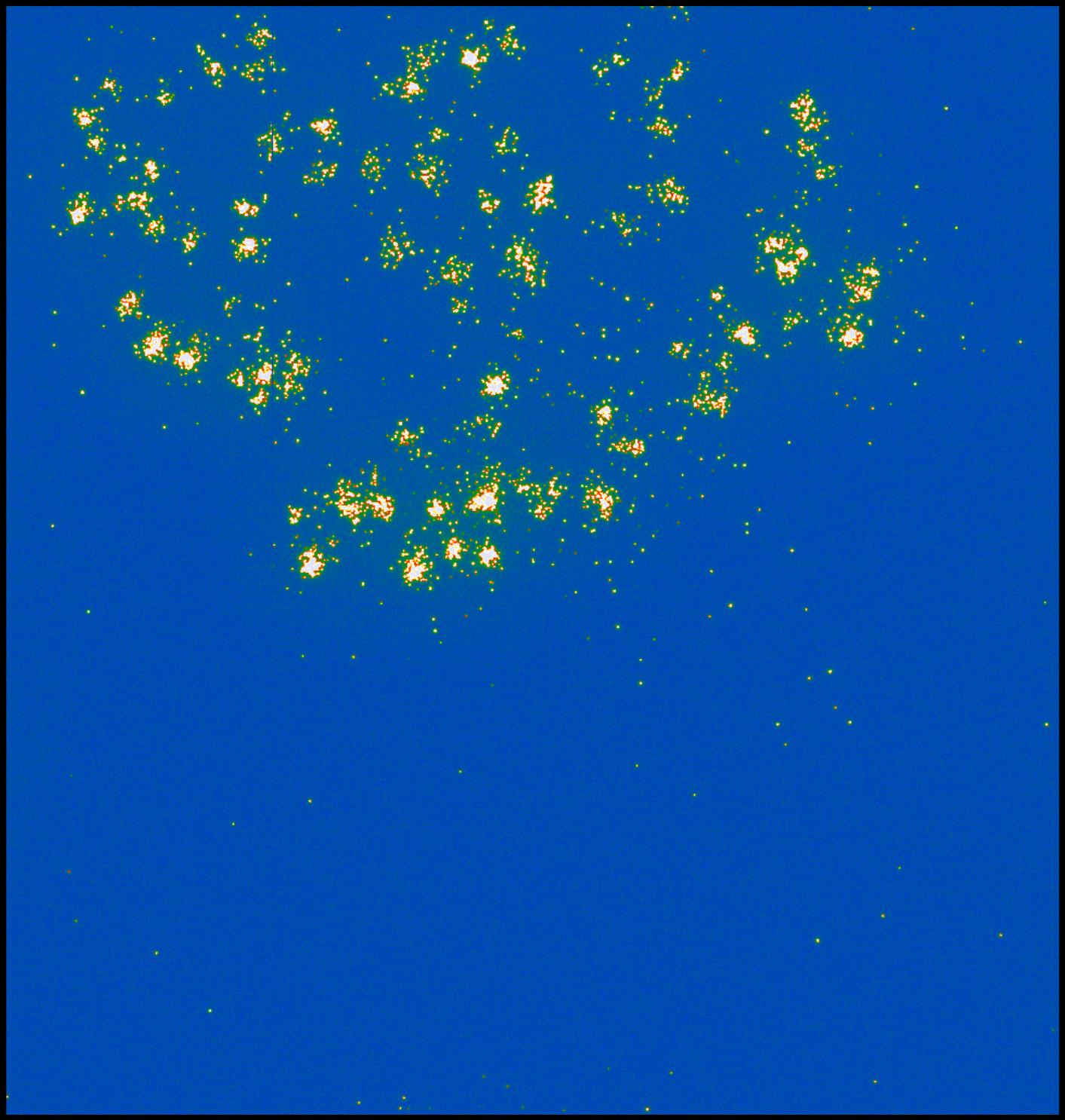
[Briels et al, J Phys D 2008]





Air,
+28 kV
on 40 mm,

exposure
0<t<300 ns



Air,
+28 kV
on 40 mm,

exposure
46<t<47 ns

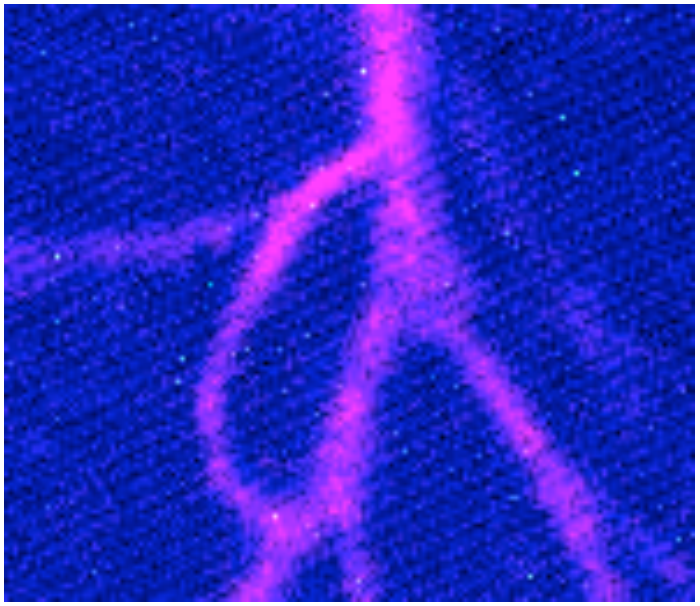
Air, 1 bar, +28 kV pulse on point above, 40 mm gap to plate below

exposure: 1 ns (46 ns < t < 47 ns) | 10 ns (50 ns < t < 60 ns) | 50 ns (50 ns < t < 100 ns) | 300 ns (0 ns < t < 300 ns)

[Ebert et al., PSST 06, Briels et al., J Phys D 2006]

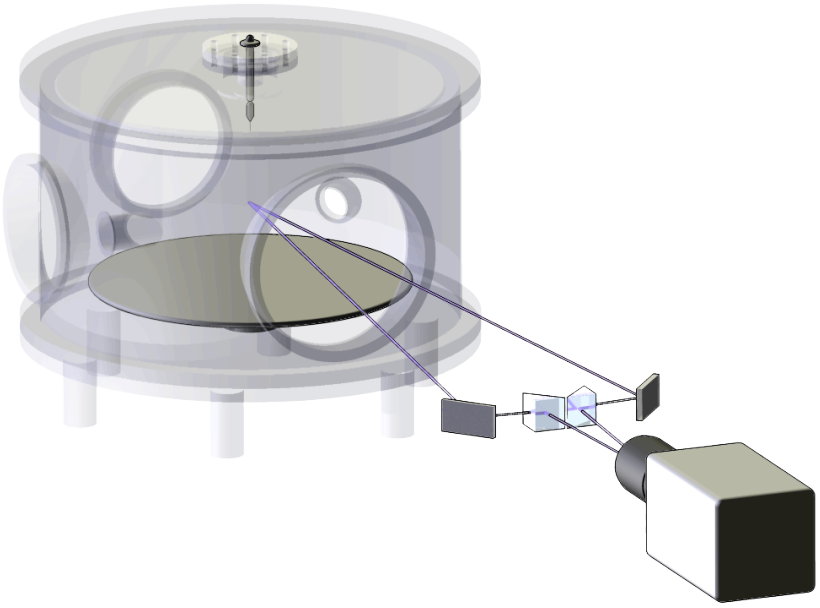
Images:

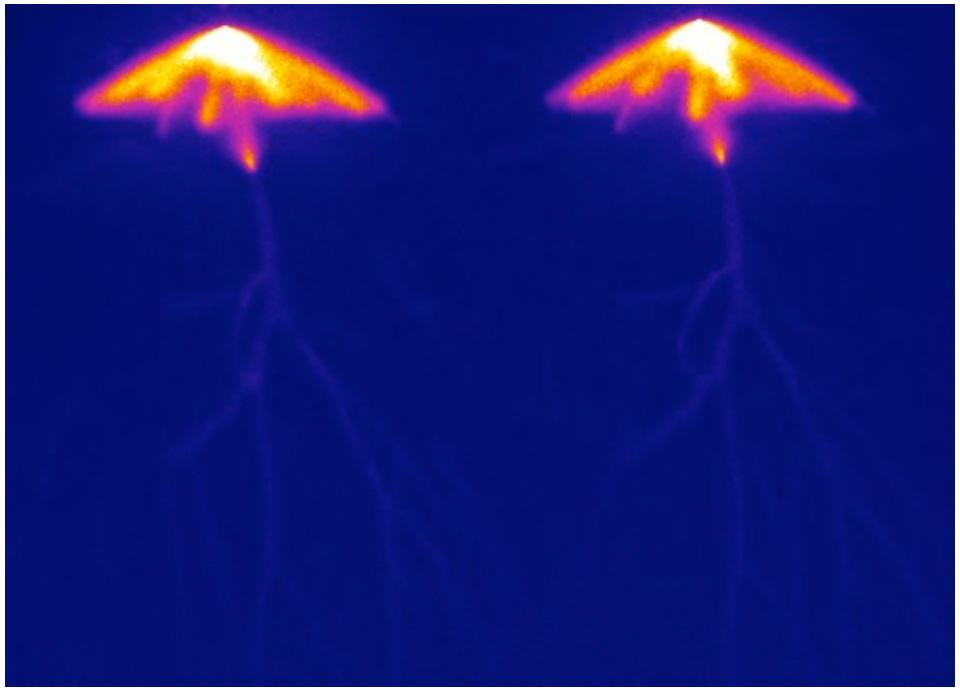
2D projection of 3D physics



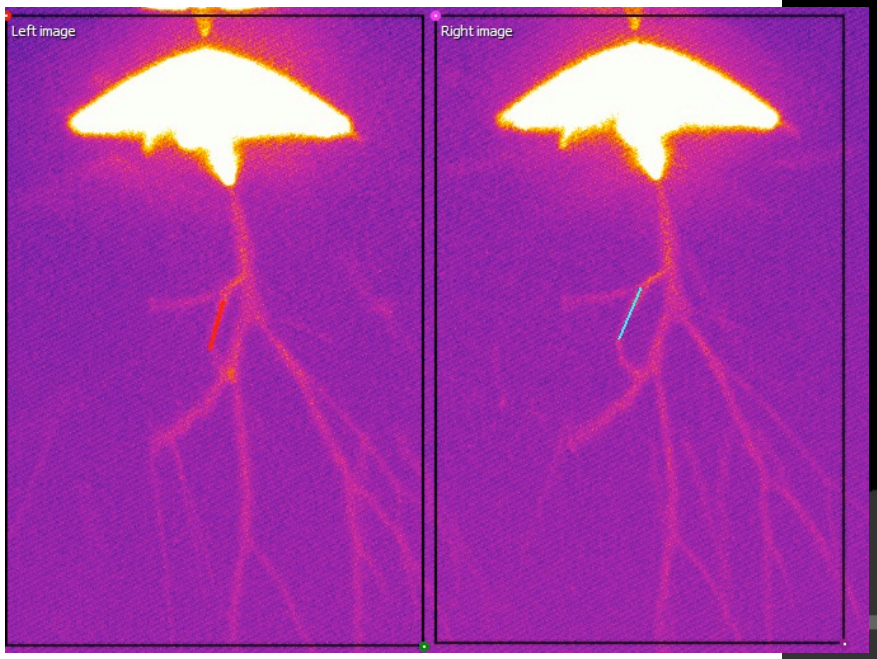
Is this a loop or not?

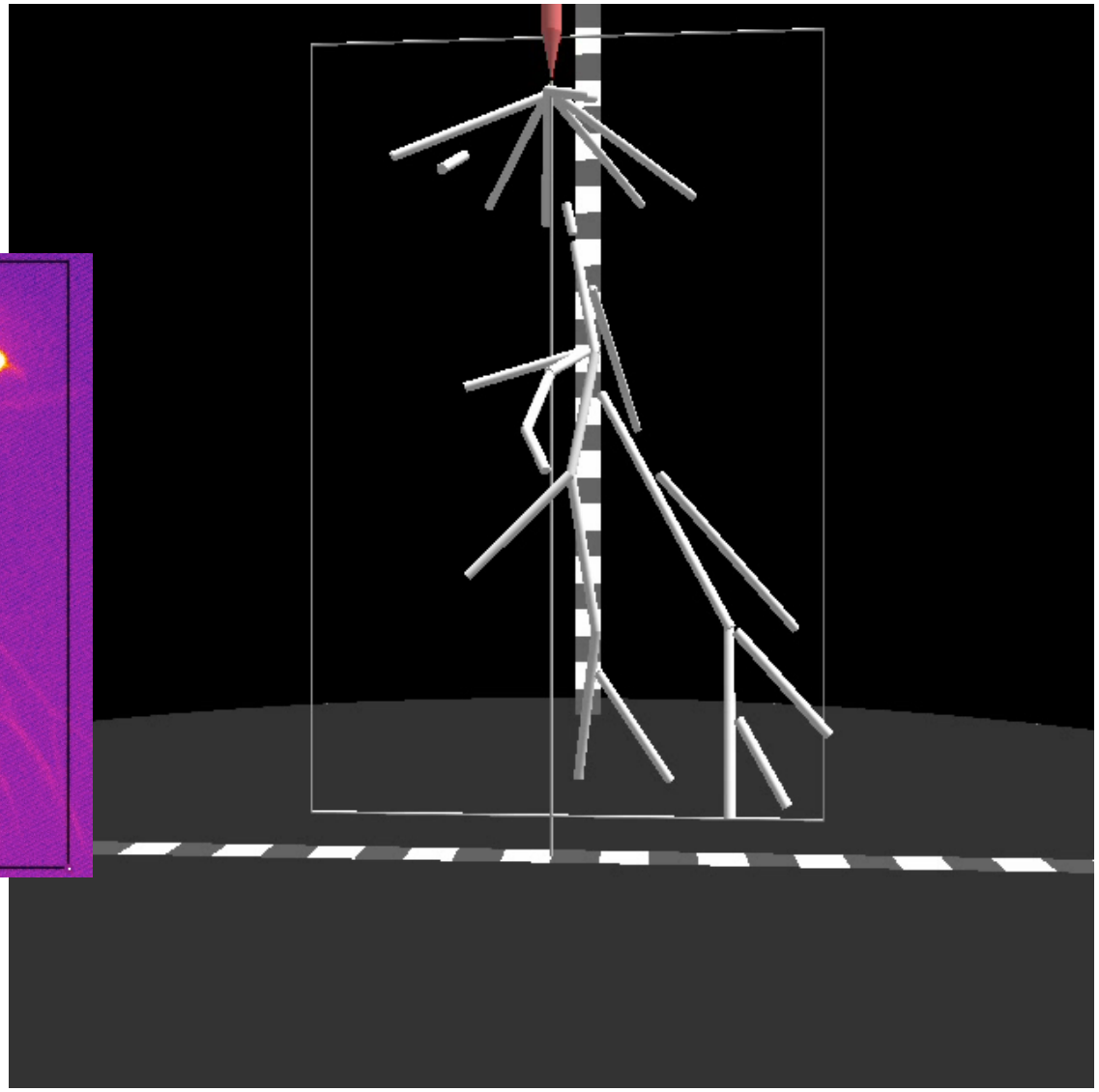
→ Stereoscopic imaging:





3D reconstruction:

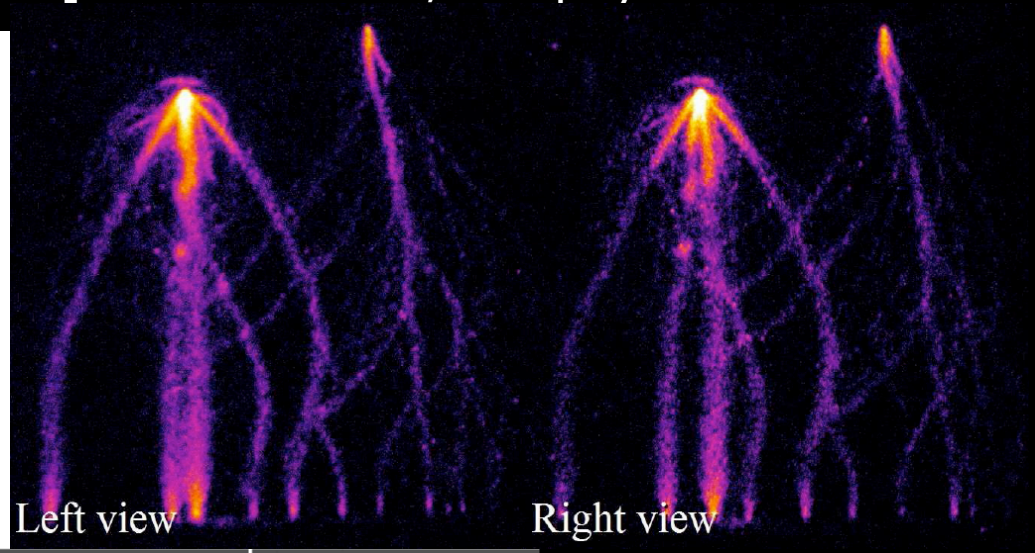


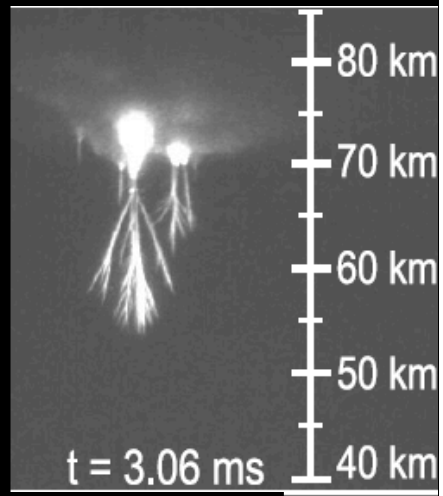


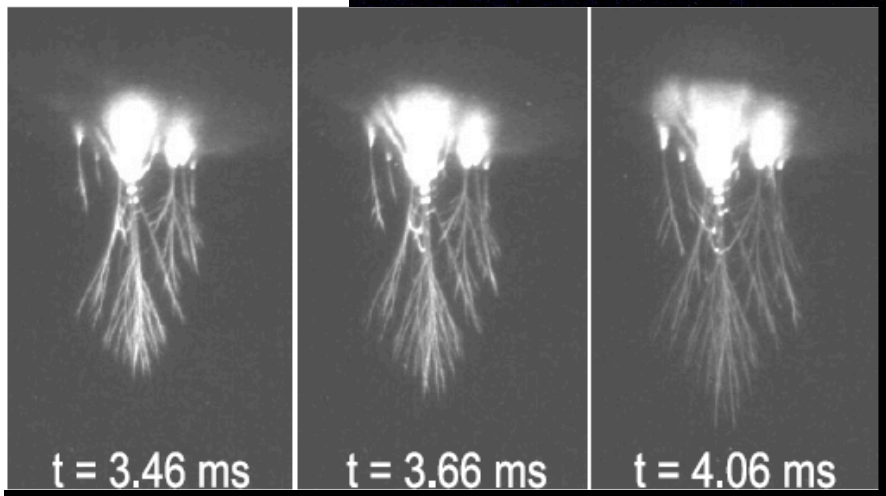
- **Branching angles** [Nijdam et al., Appl Phys Lett 2008]
- **Reconnections** [Nijdam et al., J. Phys. D 2009 incl. 3D movies]

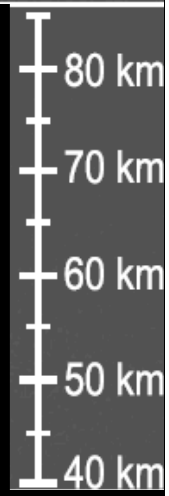
Submillisecond imaging of sprites

[Cummer et al., Geophys. Res. Lett. 2006]



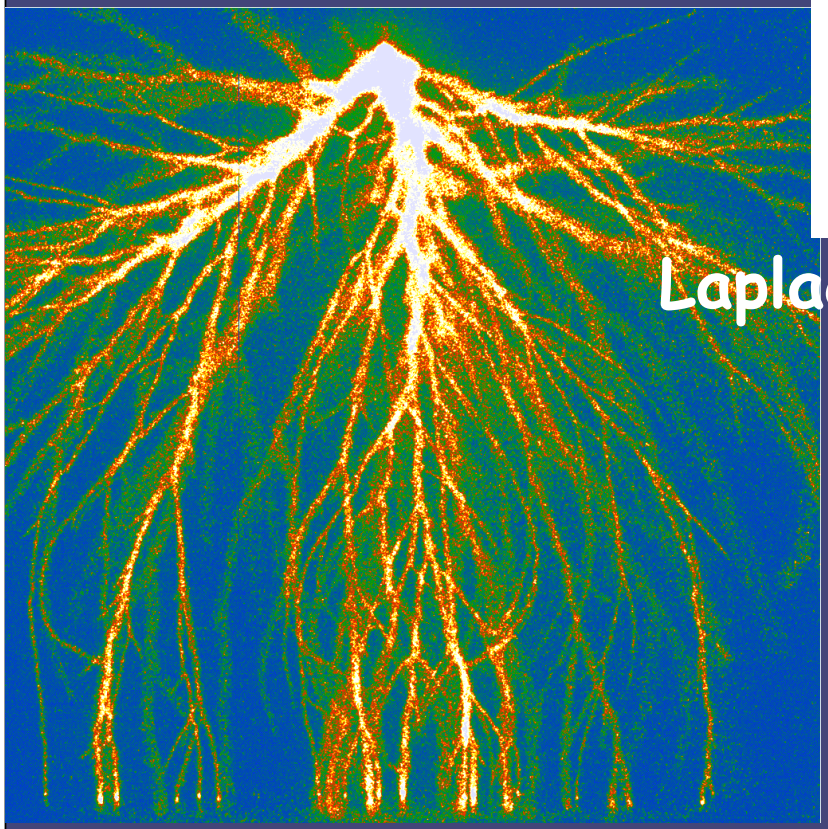


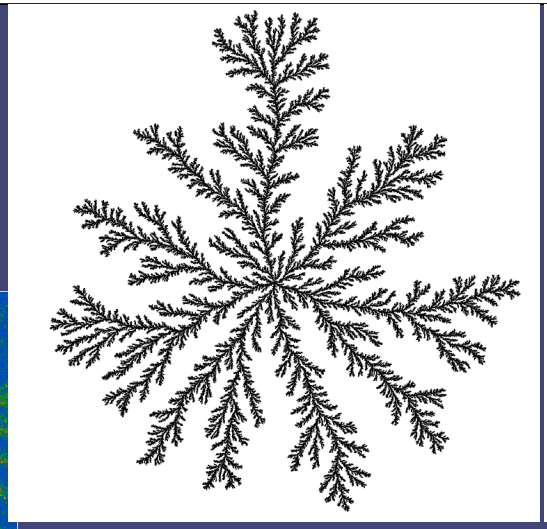


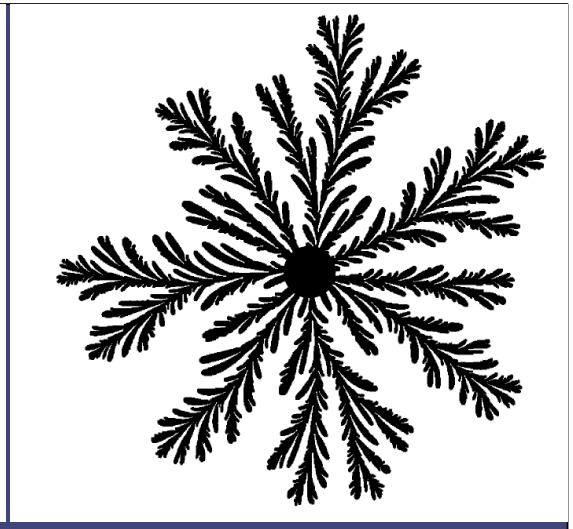


Experiments on streamer reconnection
[Nijdam et al., J Phys D 2009],

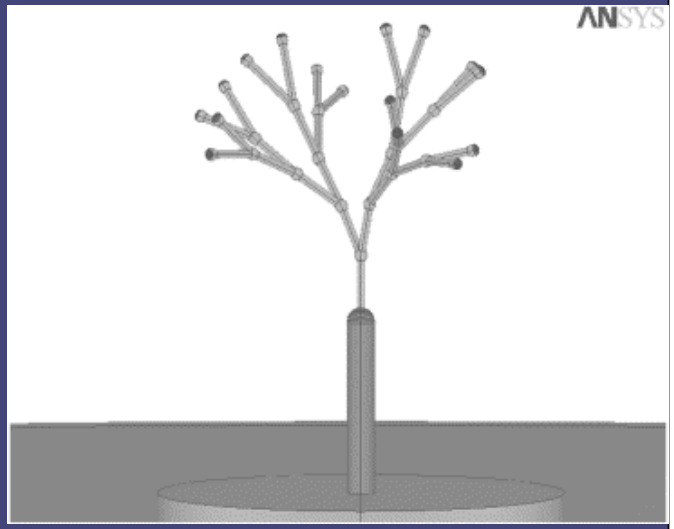
also simulations [Luque et al., Phys. Rev. Lett. 2008].







Laplacian growth models?
Fractals?



Too simple! Patterns are not fractal.
Rich inner multiscale structure!

Work your way up the scales!

Fast processes in the ionization front:

Electrons drift and diffuse in local **E**-field.
Elastic, inelastic and ionizing collisions with neutral molecules. Degree of ionization < 10^{-4}.

→ **Minimal streamer model**
(for negative streamers in pure N_2 or Ar):

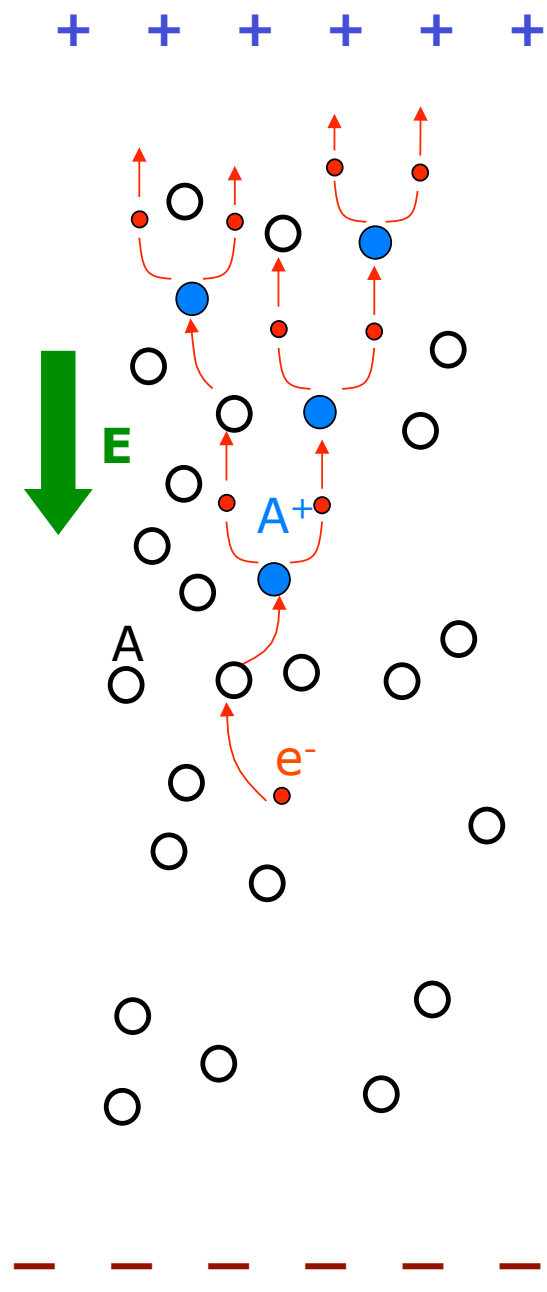
Continuity equations for densities of electrons σ and positive ions ρ, coupled to the electrostatic field **E.**

$$\partial_t \rho = D \nabla^2 \rho + \nabla \cdot (E \rho) + \rho f(|E|);$$

$$\partial_t \sigma = \rho f(|E|);$$

$$\Delta \phi = \sigma - \rho, \qquad E = -\nabla \phi;$$

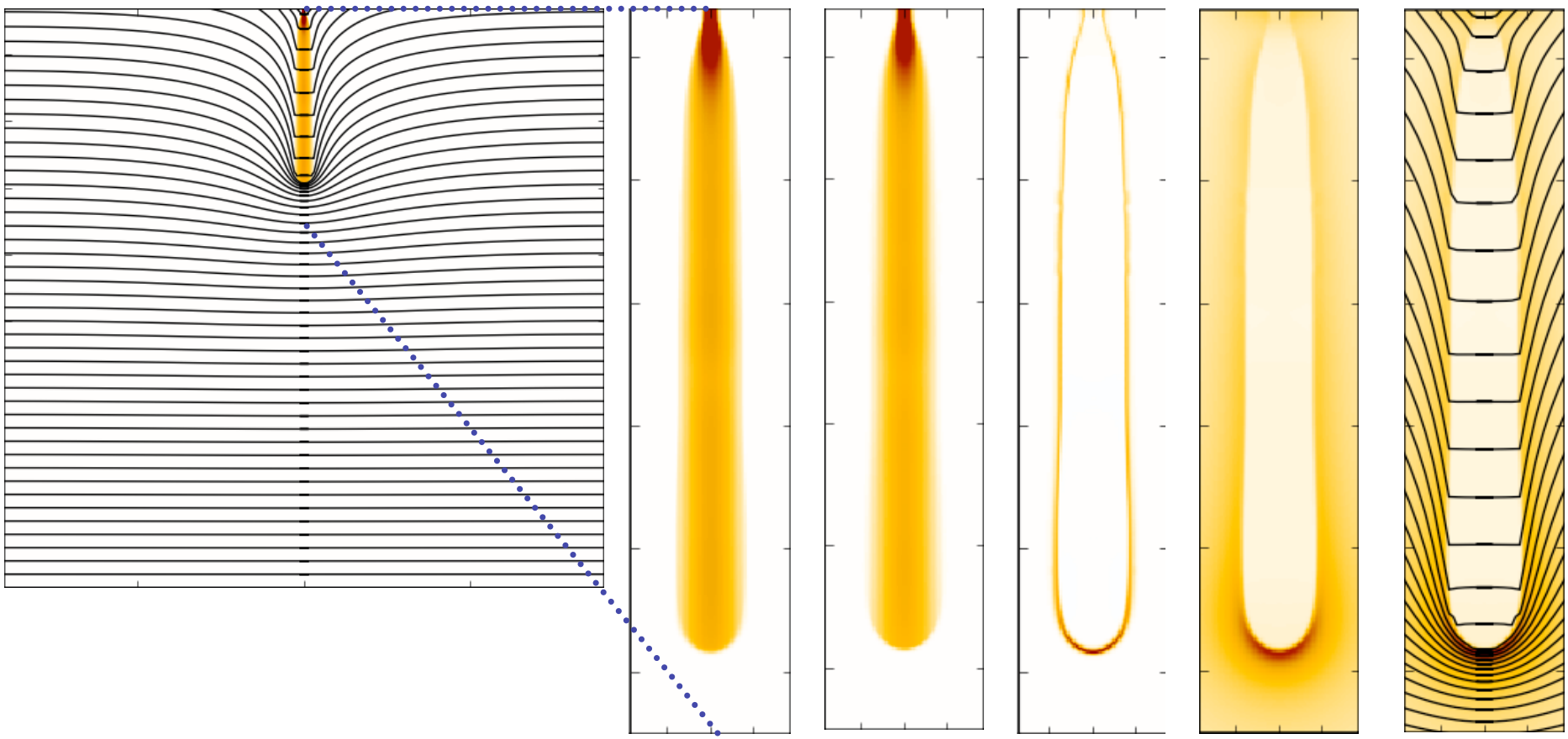
$$f(x) = |x| e^{-1/|x|}; \qquad D = 0.1:$$



Positive streamer in air

Complete system: Zoom:

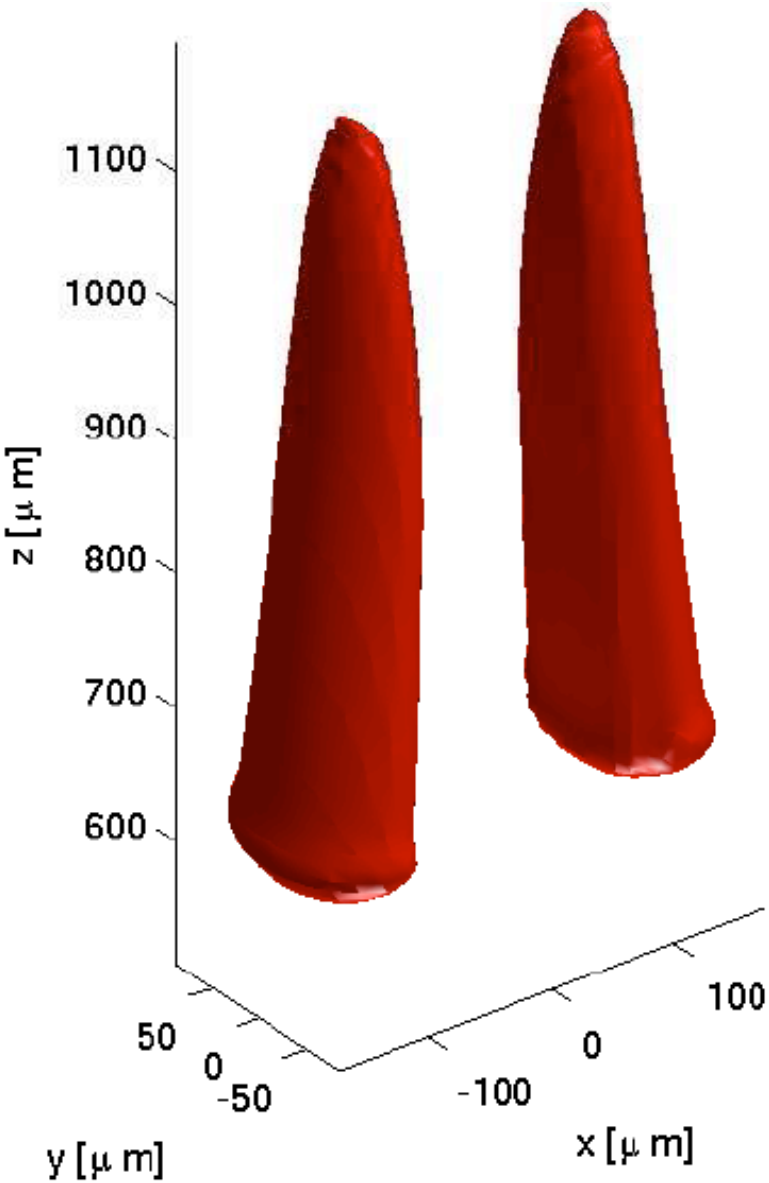
n_e, equi-φ n_e n_+ n_+-n_e |**E**| + equi-φ



Computations with adaptive grid refinement [Montijn et al., Luque et al.]

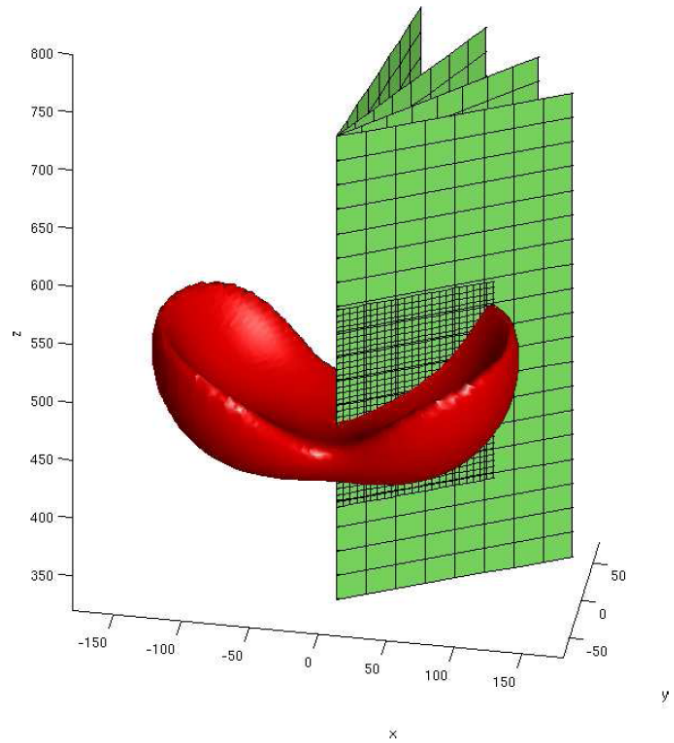
2 interacting streamers in 3D:

Surfaces of equal electron density



Electrostatic repulsion versus attraction through photoionization

Quasispectral method for the Poisson equation

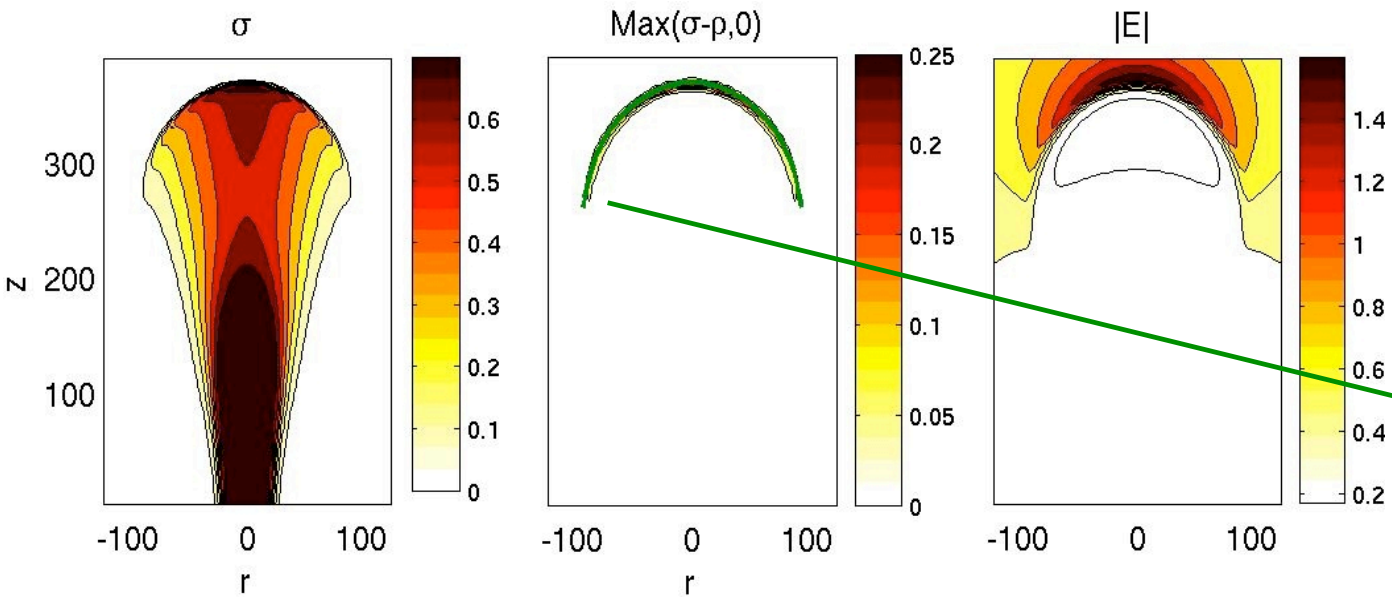


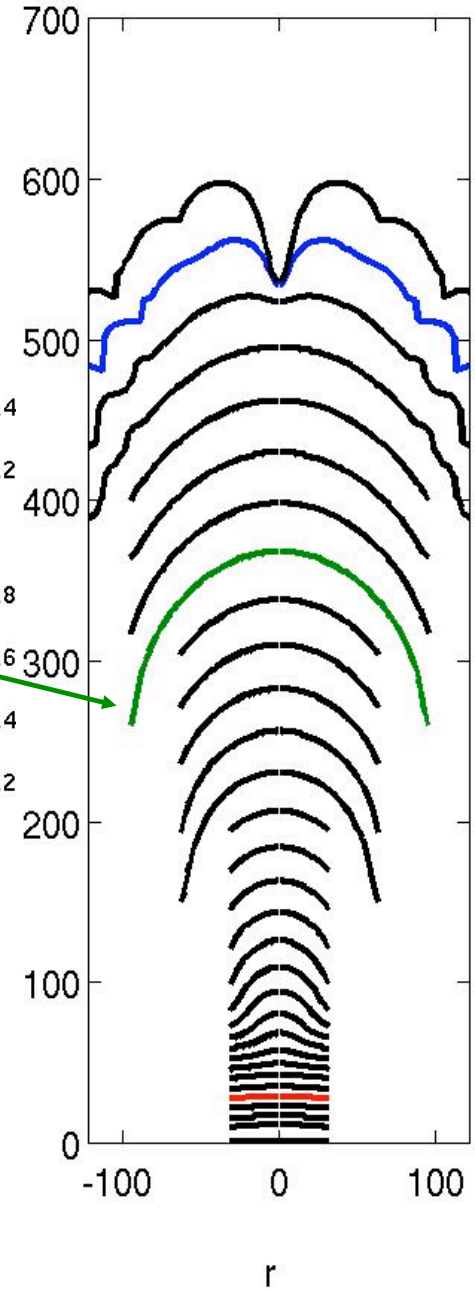
[Luque et al., Phys Rev Lett 08, Research Highlight Nature 08]

Negative streamer in strong field

Propagating streamer:

Electrons **Space charge** **|Field|**

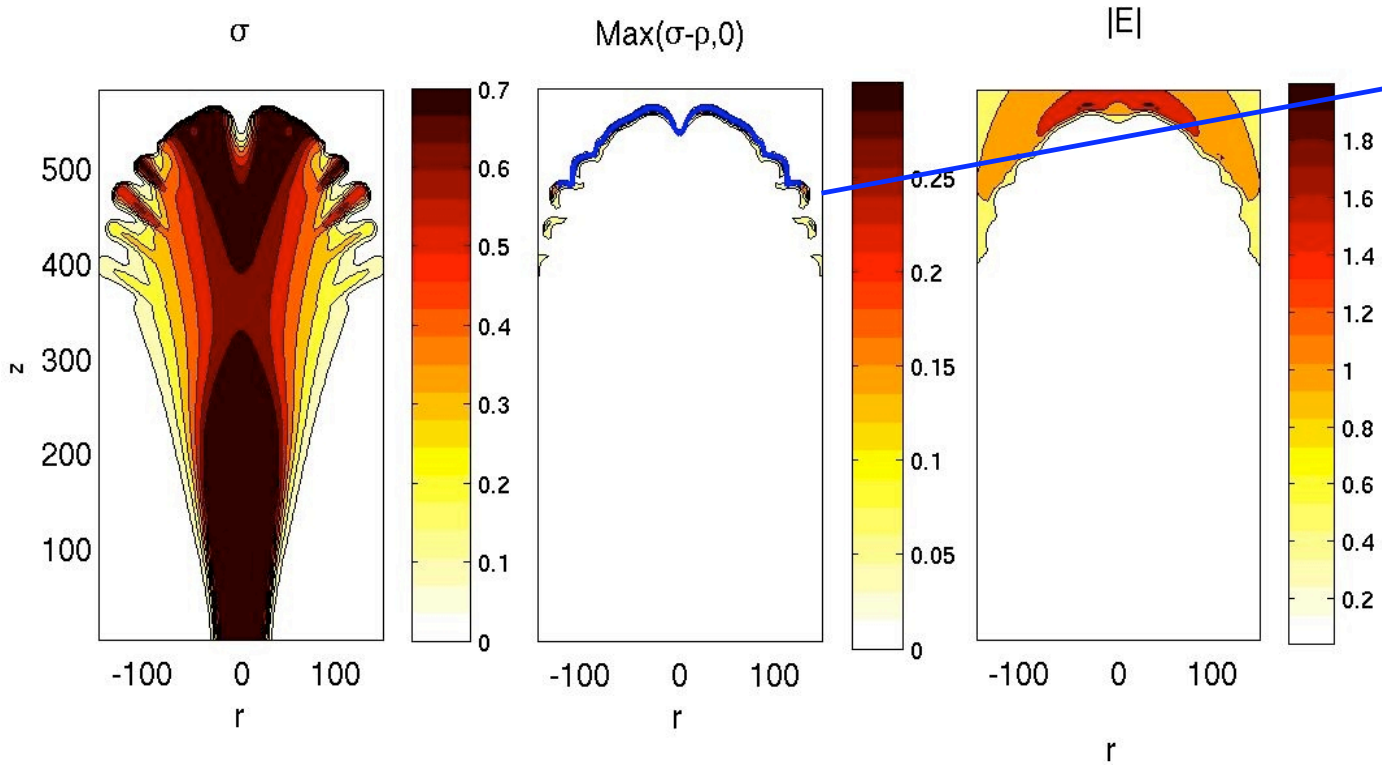


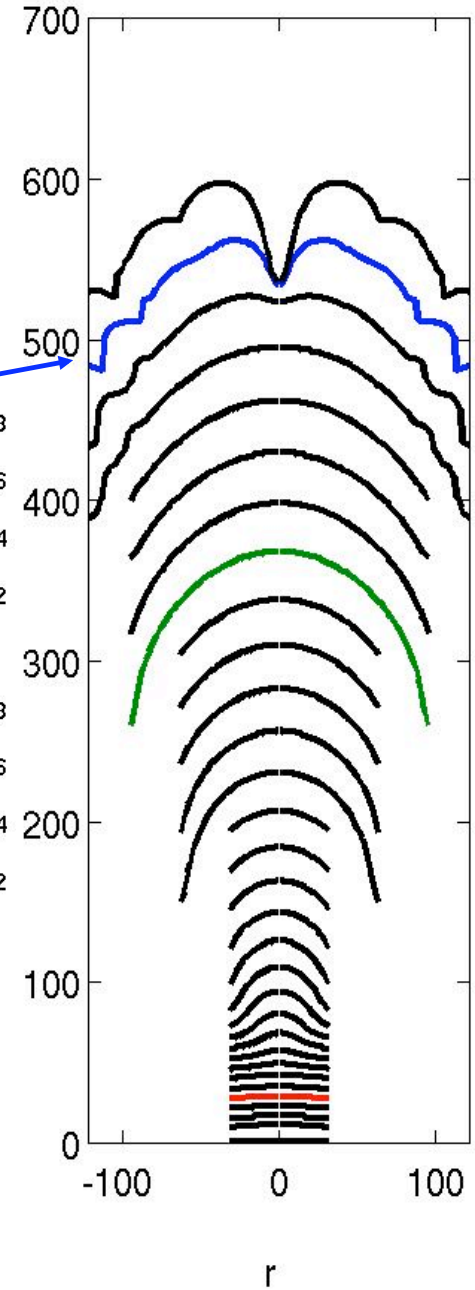


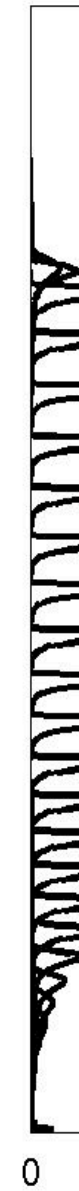
Negative streamer in strong field

Branching streamer:

Electrons **Space charge** **|Field|**





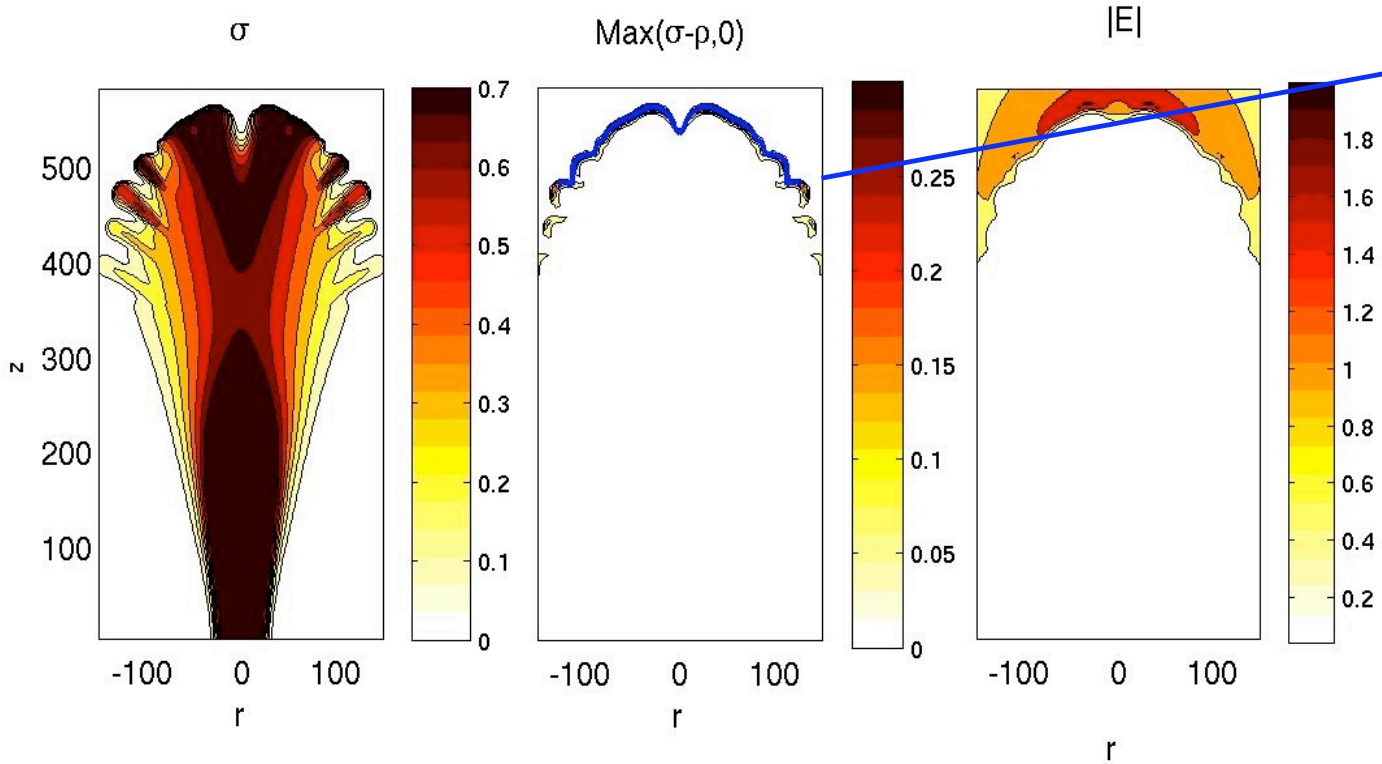


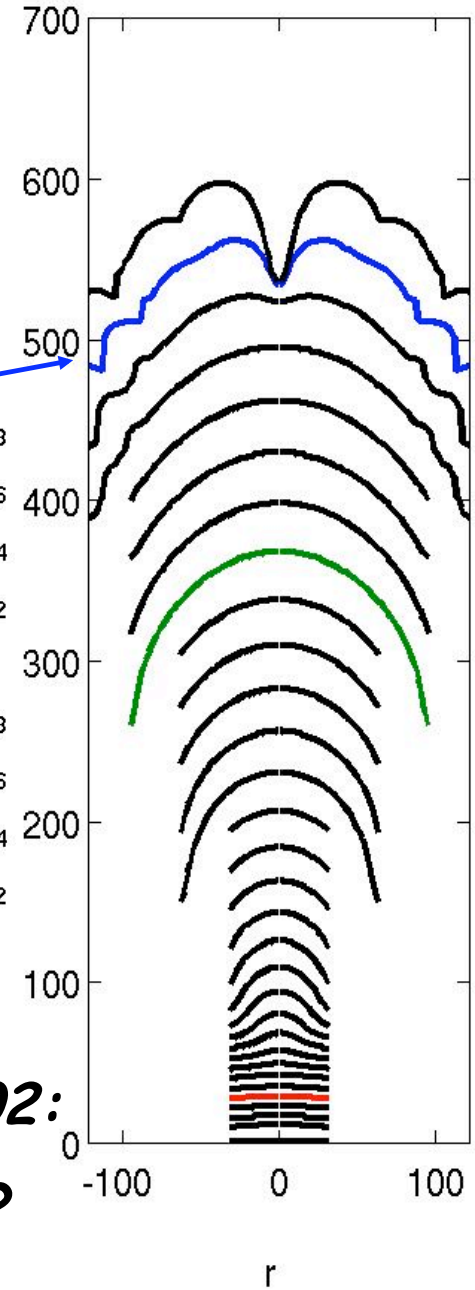
[Arrayas et al, PRL 2002], [Rocco, et al., PRE 2002],
[Montijn et al., PRE 2006], [Luque et al., APL 2007]

Negative streamer in strong field

Branching streamer:

Electrons **Space charge** **|Field|**





Paper, comment and reply in Phys Rev Lett 2002:

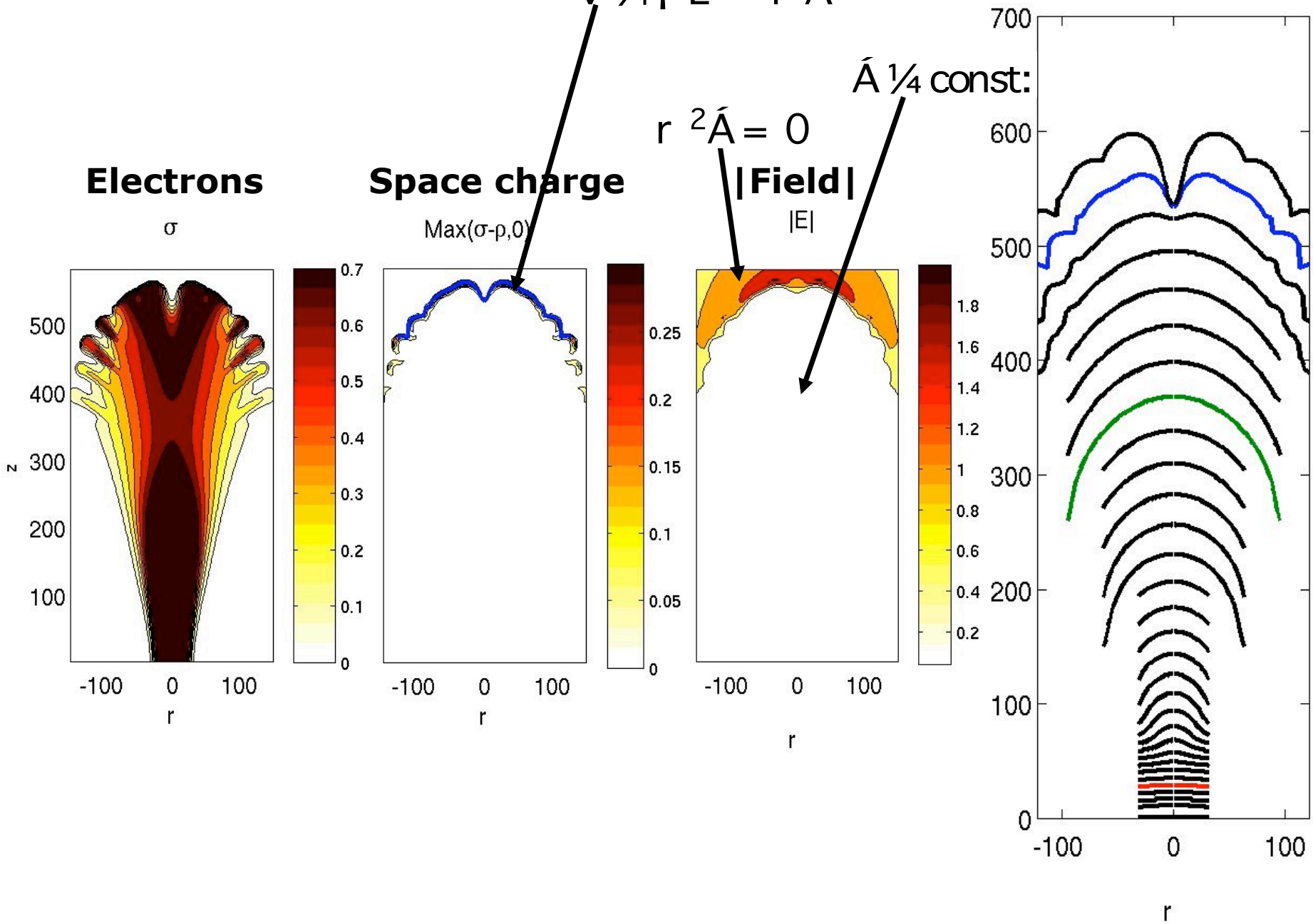
Is branching physical or a numerical artifact?

Thin layer with velocity

v ¼ ¡ E = r Á

Á ¼ const:

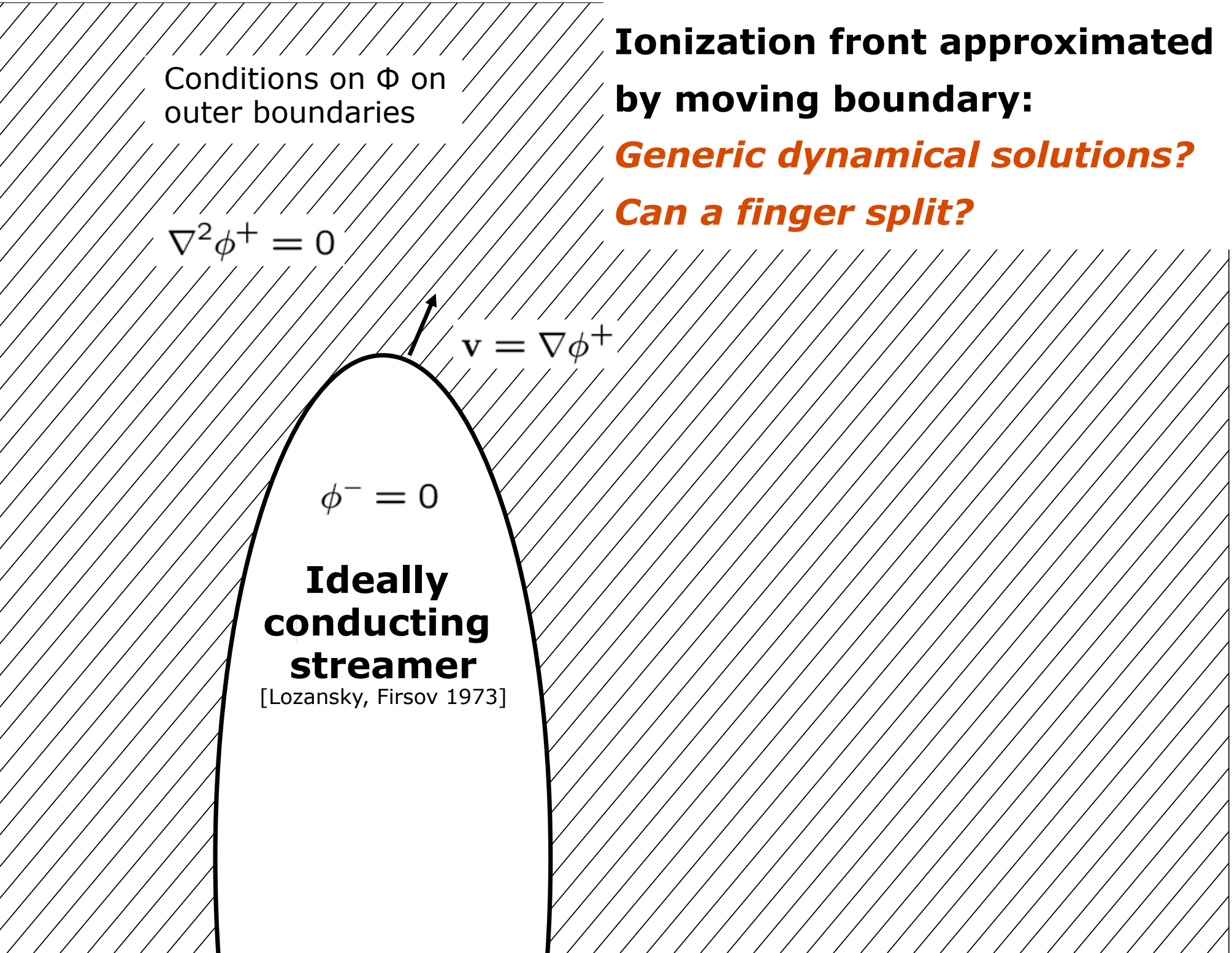
r ²Á = 0



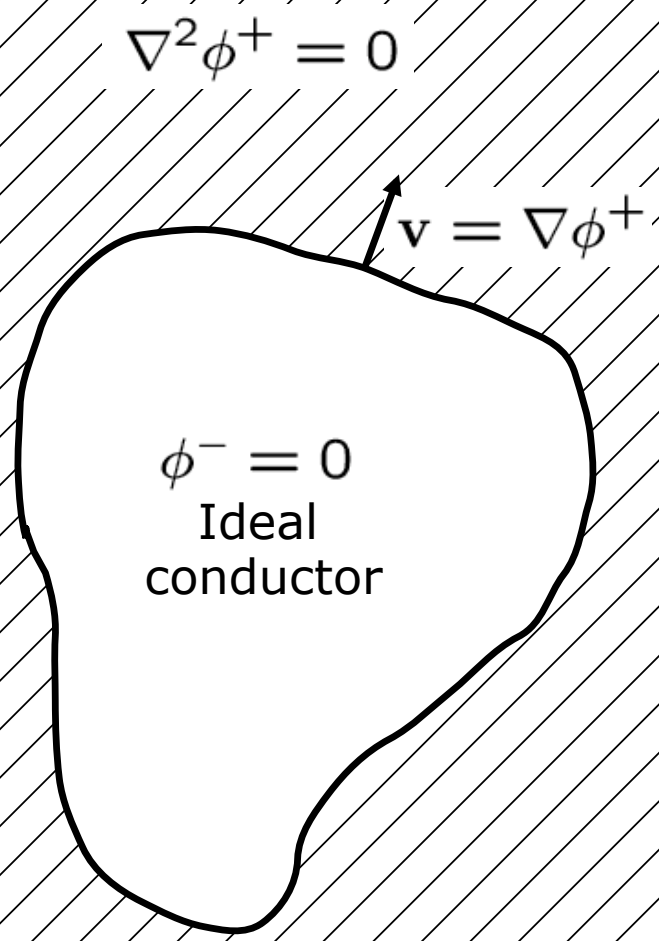
Ionization front approximated by moving boundary:

Generic dynamical solutions?

Can a finger split?



Moving Ionization Boundaries



Boundary conditions:

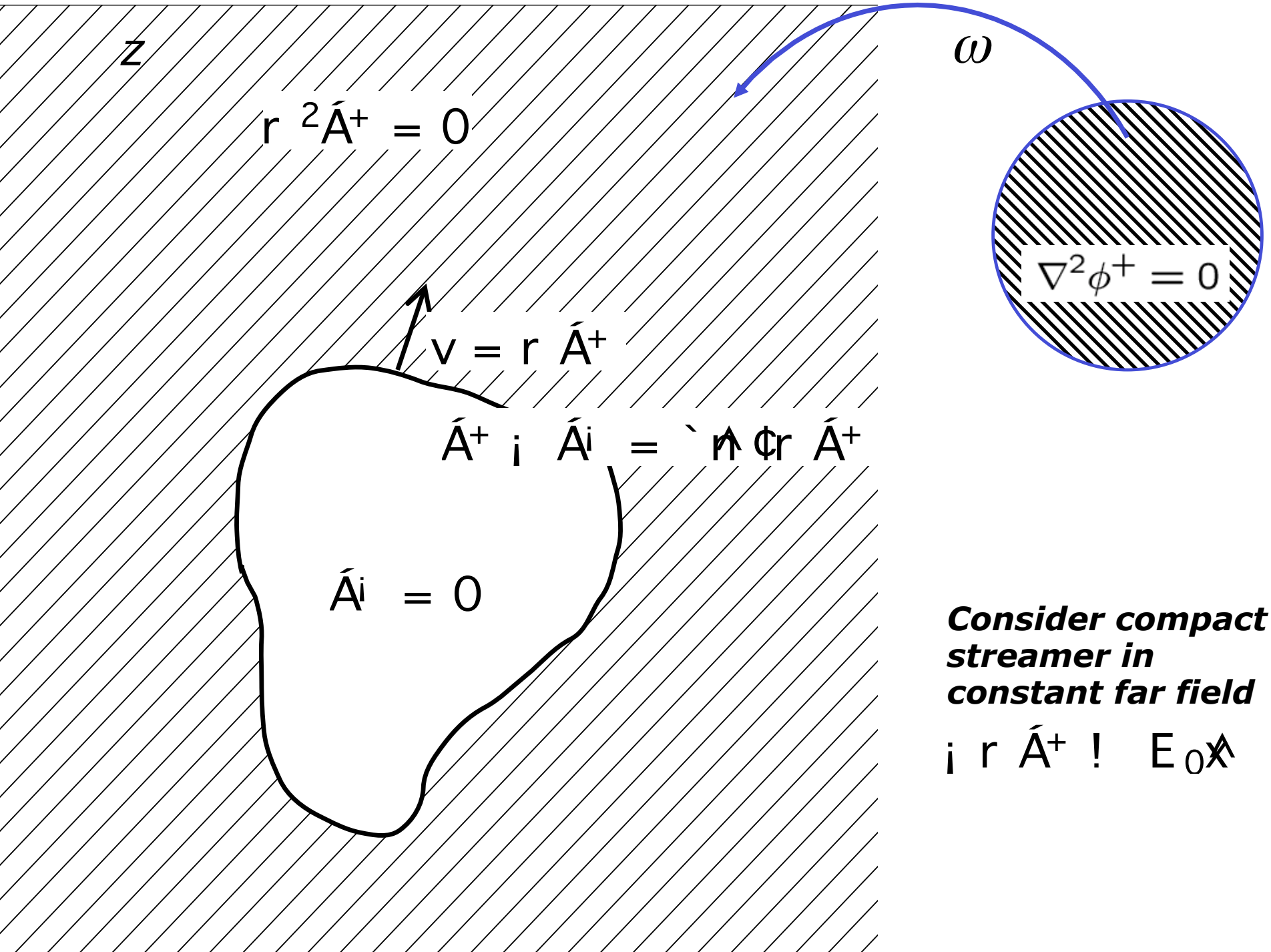
1) simple:
$\phi^+ - \phi^- = 0$
nicely solvable, but ill posed

2) Hele-Shaw:
$\phi^+ - \phi^- = \tau\kappa$
regularized motion

3) **planar ionization fronts (D=0):**
$\phi^+ - \phi^- = \hat{\mathbf{n}} \cdot \nabla\phi^+$
regularizing?
("kinetic undercooling")

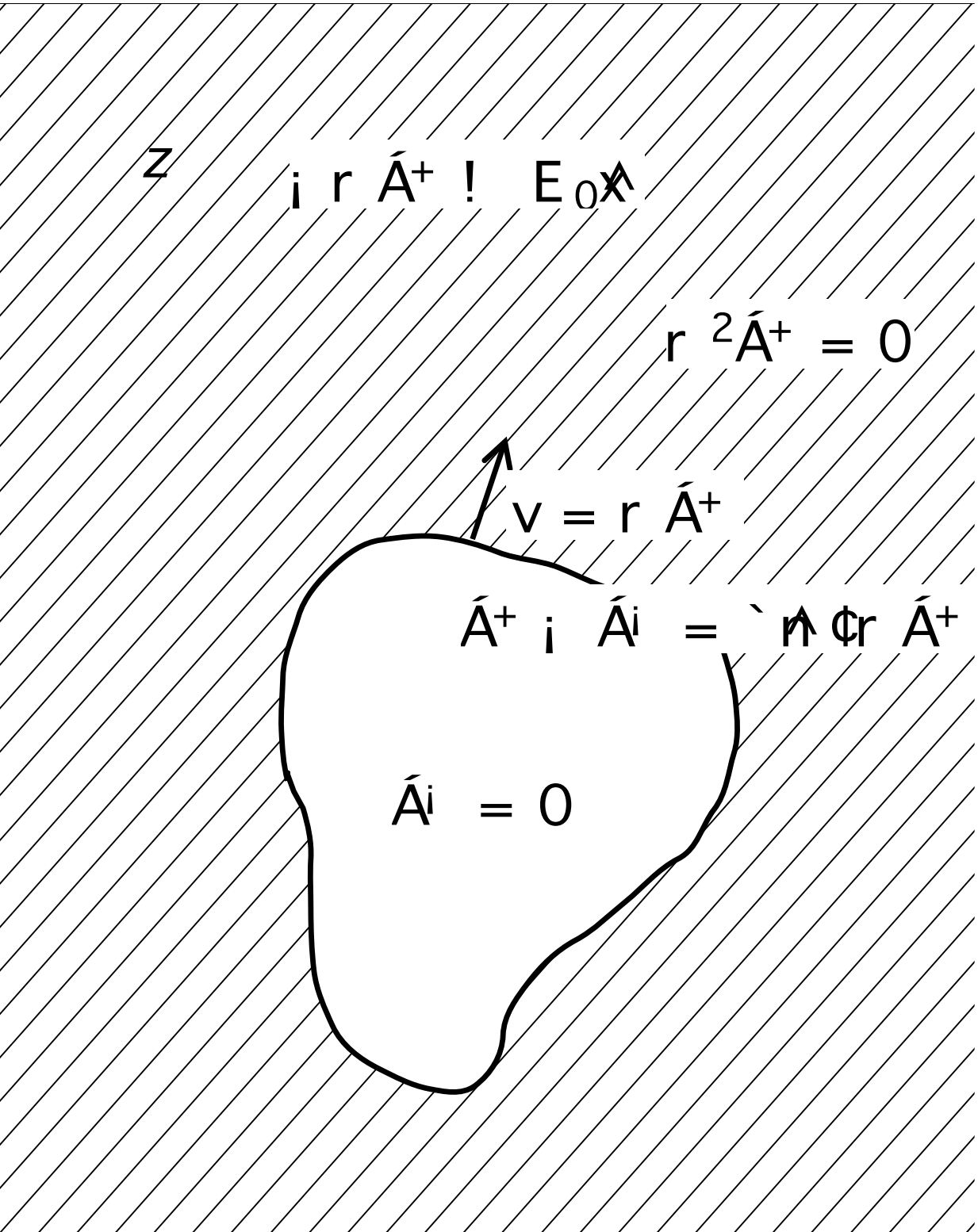
Model derived and tested in
[Brau et al., PRE 2008]

2D - Conformal Mapping



Consider compact streamer in constant far field

¡ r Á+ ! E0x



Traveling waves/ uniformly translating solutions:

For $` = 0$: All ellipses, two parameter family.

For $` > 0$: All circles, one parameter family: radius R.

Solutions characterized by

$$^2 = `=R;$$

E_0 can be scaled out.

ε=1 unphysical, but special

Investigate **linear stability** of these solutions:

$$f(\omega;\xi) = \frac{1}{\omega}\left(\mu + \xi + \varepsilon\bar{\varphi}(\omega;\xi)\right),$$

$$\hat{c}(\omega;\xi) = \left(\frac{1}{\omega} - \frac{1-\mu^2}{1+\mu^2}\,\omega\right) + \varepsilon\hat{\beta}(\omega;\xi).$$

Impose and use that β and φ are analytical.

φ can be completely eliminated, β solves

$$L_2\bar{\varphi} = 0,$$

$$L_2 = \frac{\mu^2}{2}\partial_\omega\,\omega\,(\omega^2-1)\partial_\omega + \mu^2\,\partial_\omega\,\omega\,\partial_\xi + \partial_\xi - \partial_\omega.$$

For ε=1, the initial perturbation ¯(!,0) defines the function

$$G(!) = (2 + ! @_!)\,{}^{-}(!\,;0);$$

and the general solution is

$$^{-}(!\,;¿) = \int_0^{!} \frac{x\,dx}{!^2}\, G\left(\frac{x+T}{1+Tx}\right); \qquad T = \tanh\frac{¿}{2}:$$

For ¿→1, the solution follows the universal slow manifold

$$^{-}(!\,;¿) \quad ¿!\,!^{1} \quad \frac{G(1)}{2} + G^{0}(1)\,F(!)\,e^{i\,¿} + O\left(e^{i\,2¿}\right);$$

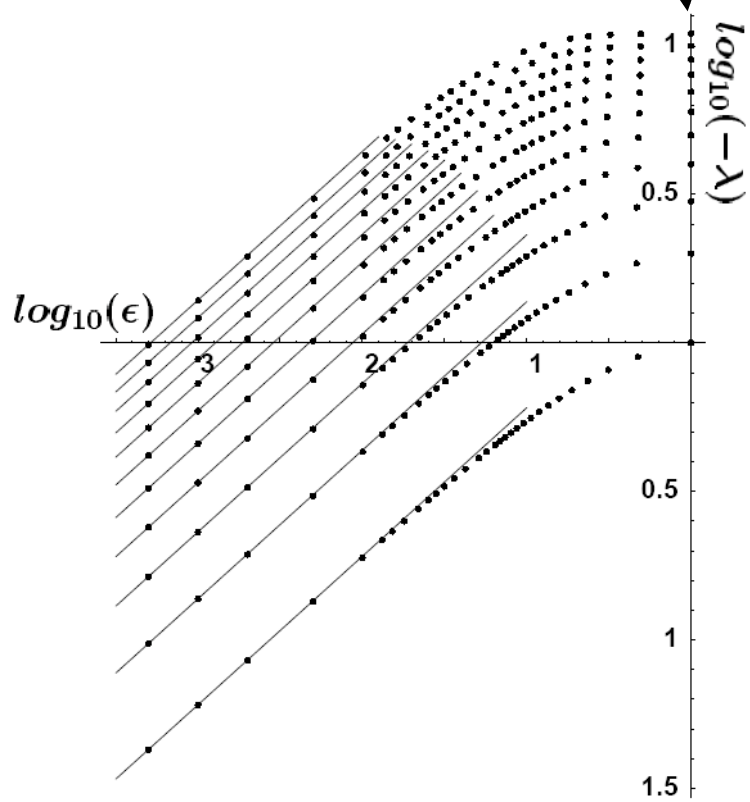
$$F(!) = 1 ¡ \frac{4}{!} + \frac{4\ln(1+!)}{!^2}:$$

[Meulenbroek, Ebert, Schäfer, Phys Rev Lett 2005, SIAP 2007]

Explicit eigenfunctions for ε=1:

$$\bar{}(!\,;¿) \,/\, e^{¿}\,\bar{}_{¸}(!\,); \qquad \bar{}_{¸}(!\,) = \int_0^{!} \frac{x\,dx}{!^{2}} \left(\frac{x ¡ 1}{x+1}\right)^{¶ ¡ ¸}$$

analytical except for !=-1 (on the back of the circle)
for the discrete spectrum: λ = 0, -1, -2, -3, …



The behavior is generic for ε>0:

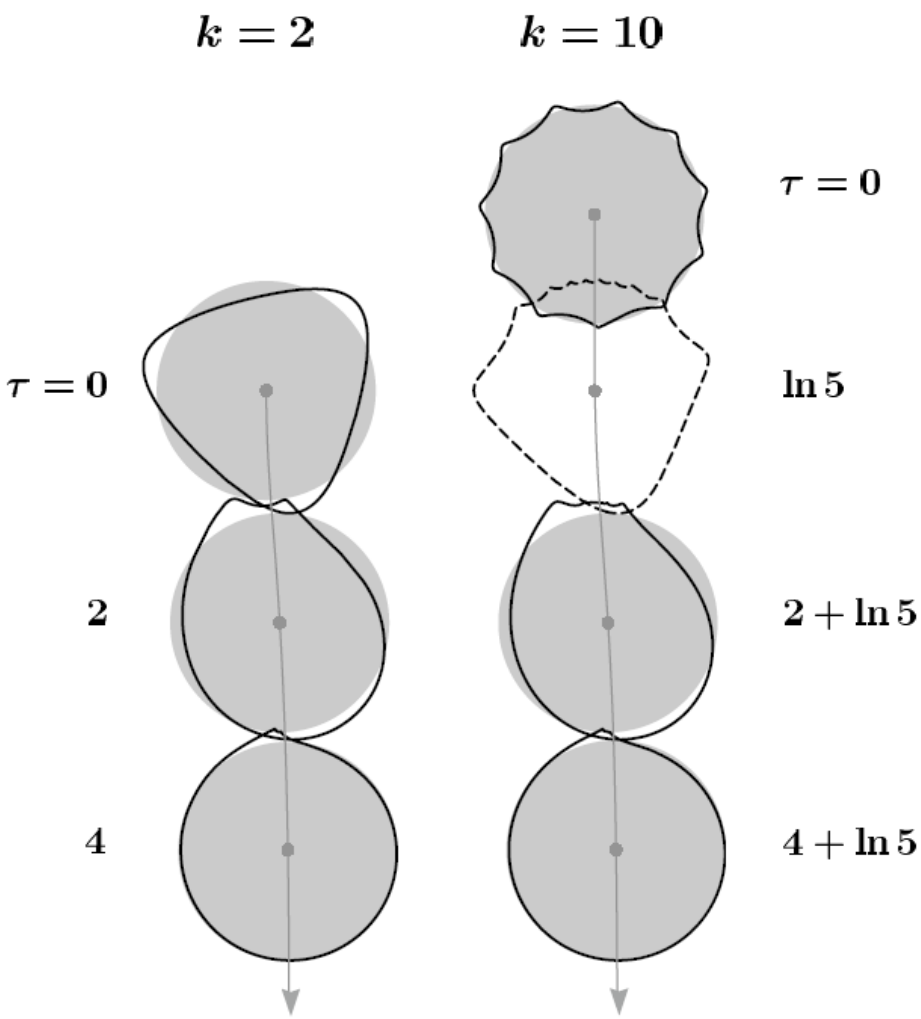
Discrete negative eigenvalues

[Tanveer, Schäfer, Brau, Ebert, Physica D 2009].

Temporal evolution of linear perturbations:

Analytical solutions (ε=1):

Convection, universality for ¿→1



[Ebert, Meulenbroek, Schäfer, SIAP 2007]

Temporal evolution of linear perturbations:

Analytical solutions (ε=1):

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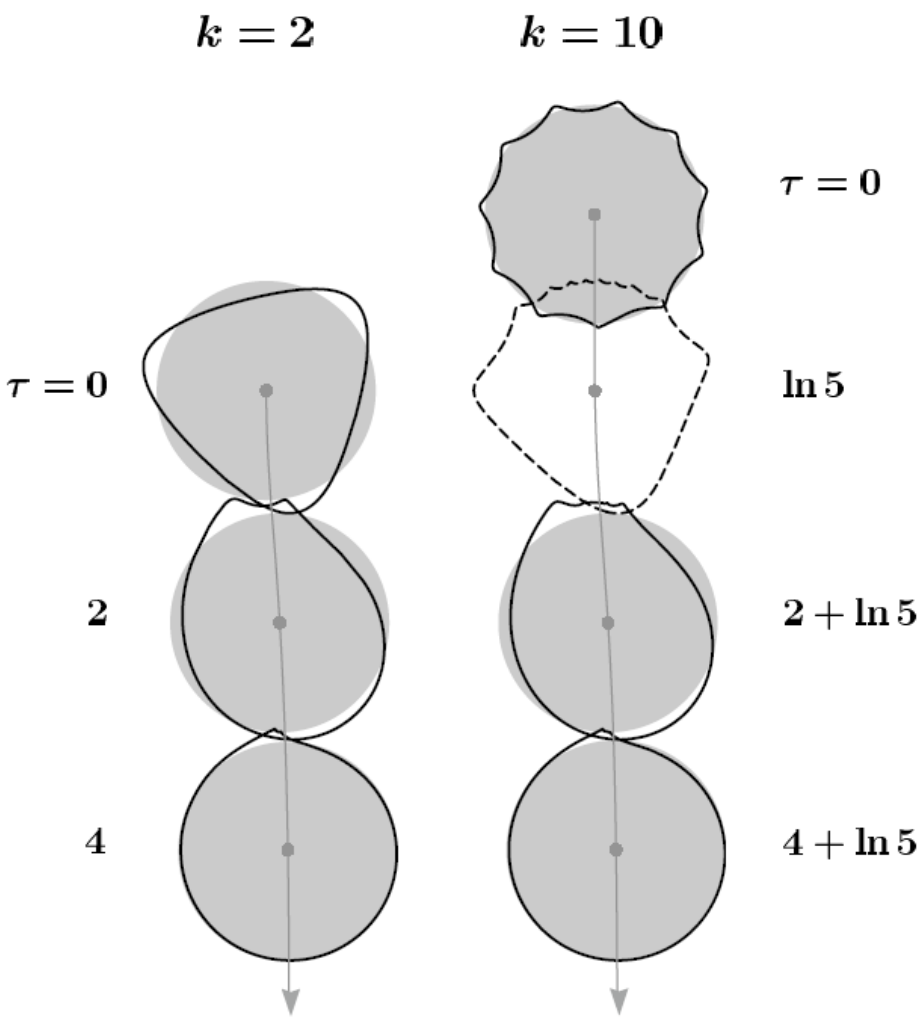
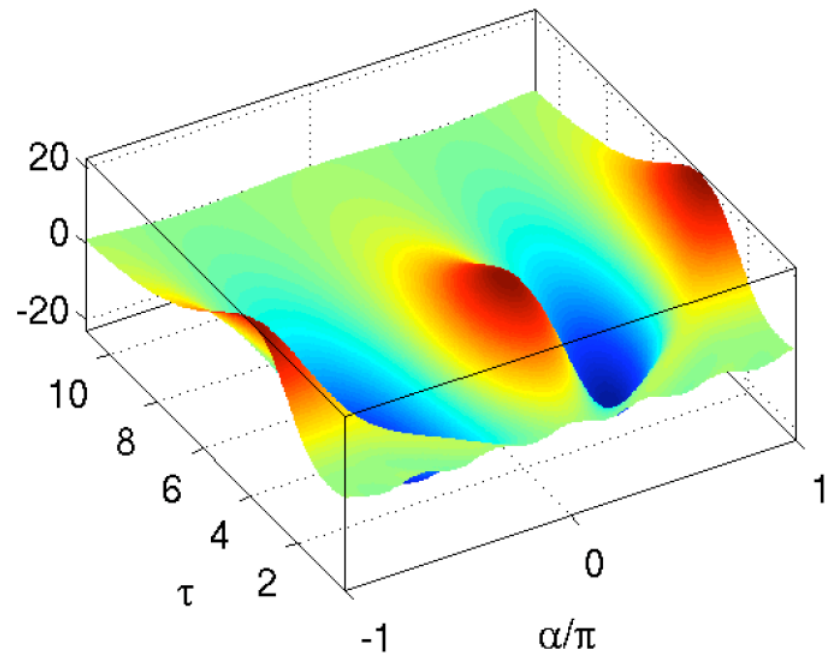


[Ebert, Meulenbroek, Schäfer, SIAP 2007]

Numerical solutions (ε=1/10):

Growth on front half and convection

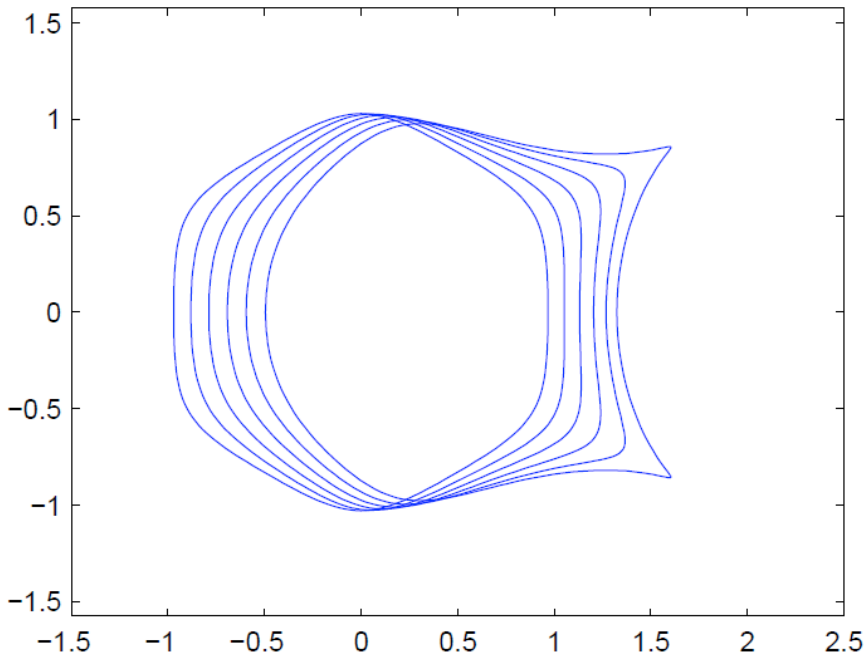


[Kao, Brau, Ebert, Schäfer, Tanveer]

Finite perturbations

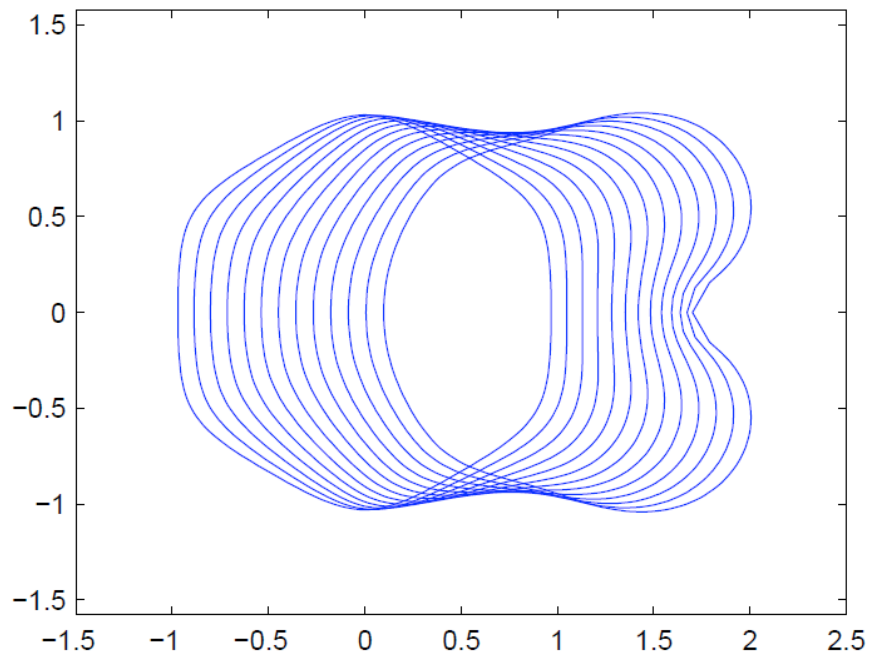
ε=0

Cusps



ε=1/10

Branching

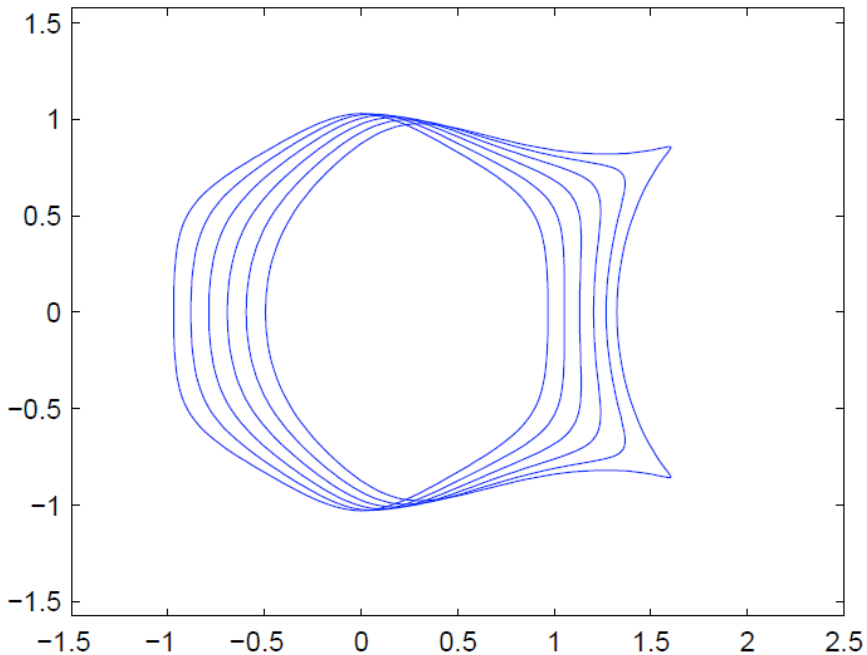


[Kao, Brau, Ebert, Schäfer, Tanveer, subm to Physica D]

Finite perturbations

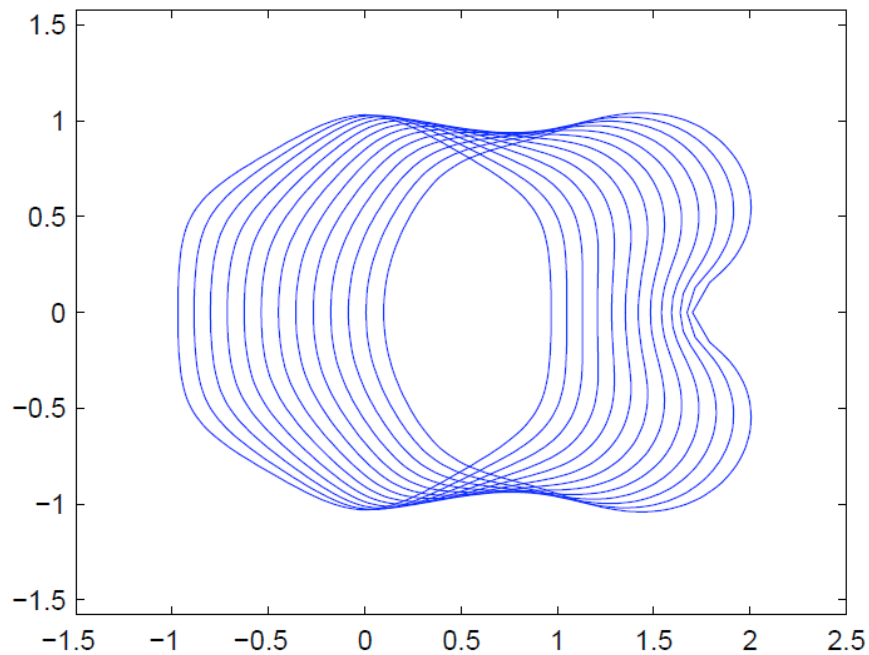
ε=0

Cusps



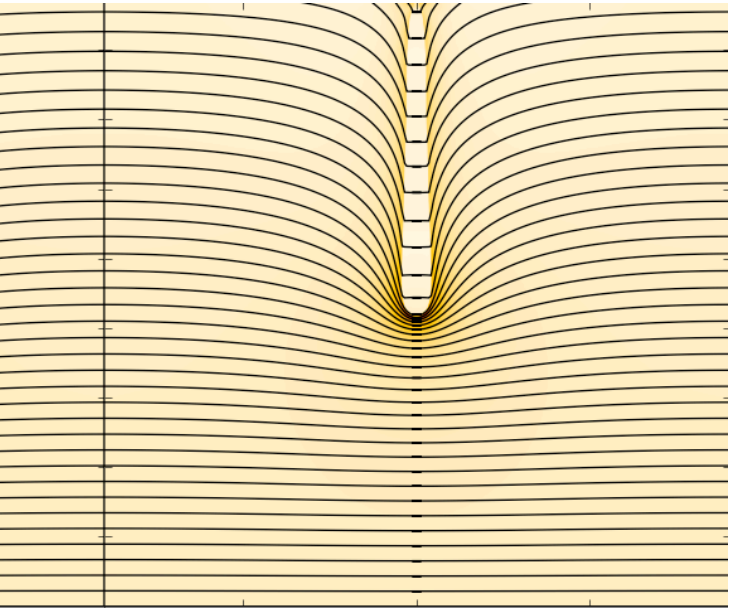
ε=1/10

Branching

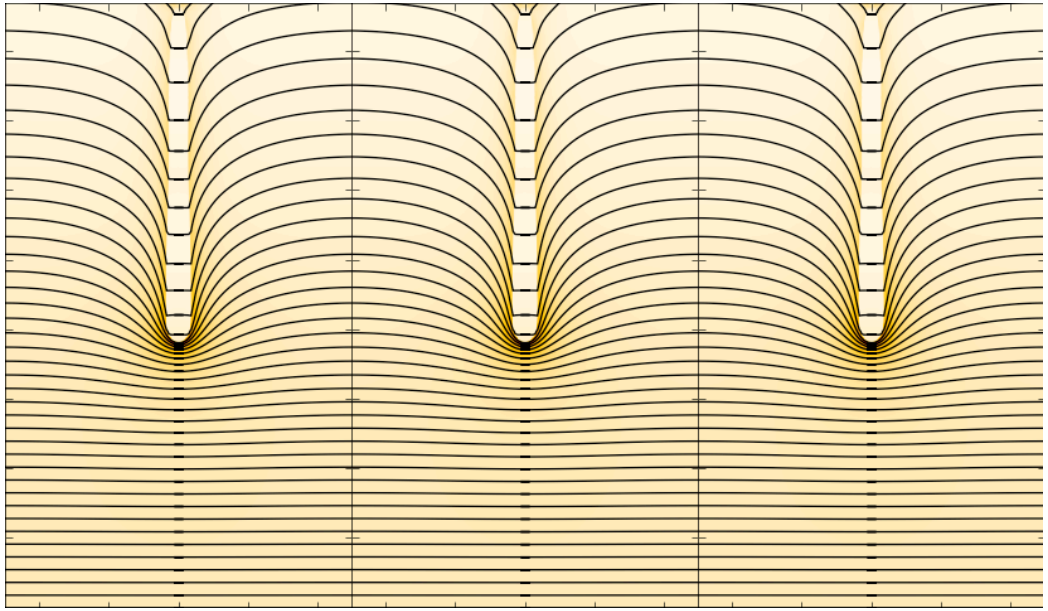


Due to convection,
expect similar dynamics
at the tip of a finger.

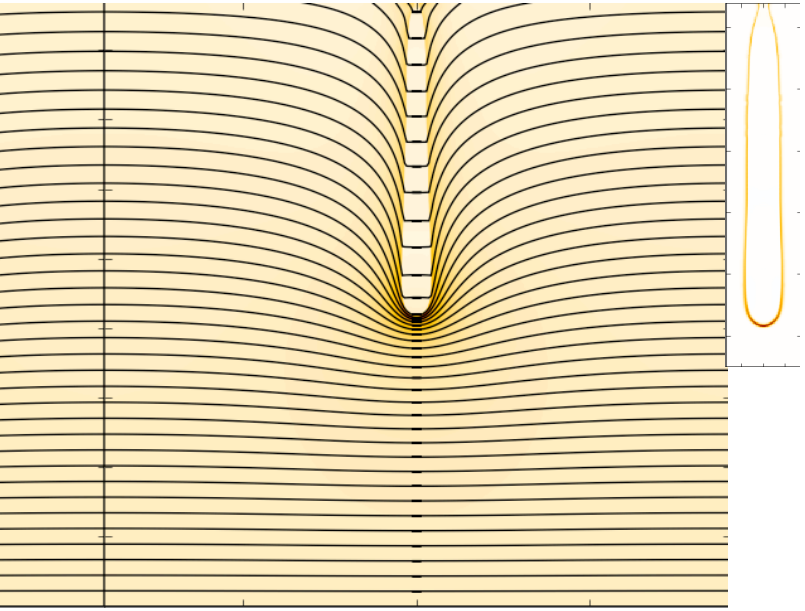
Single streamer



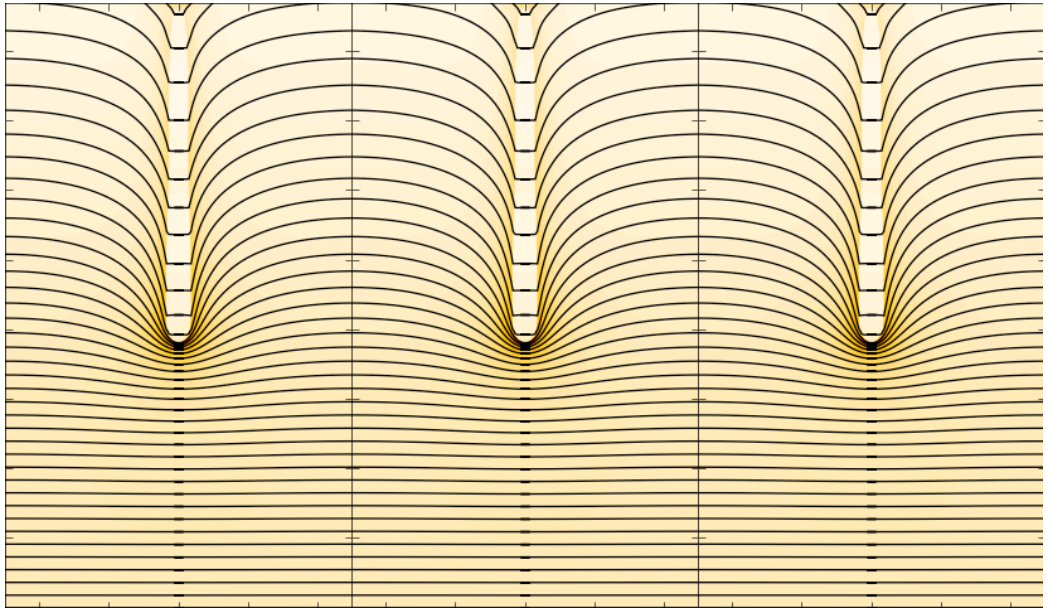
Streamers with electrostatic interaction



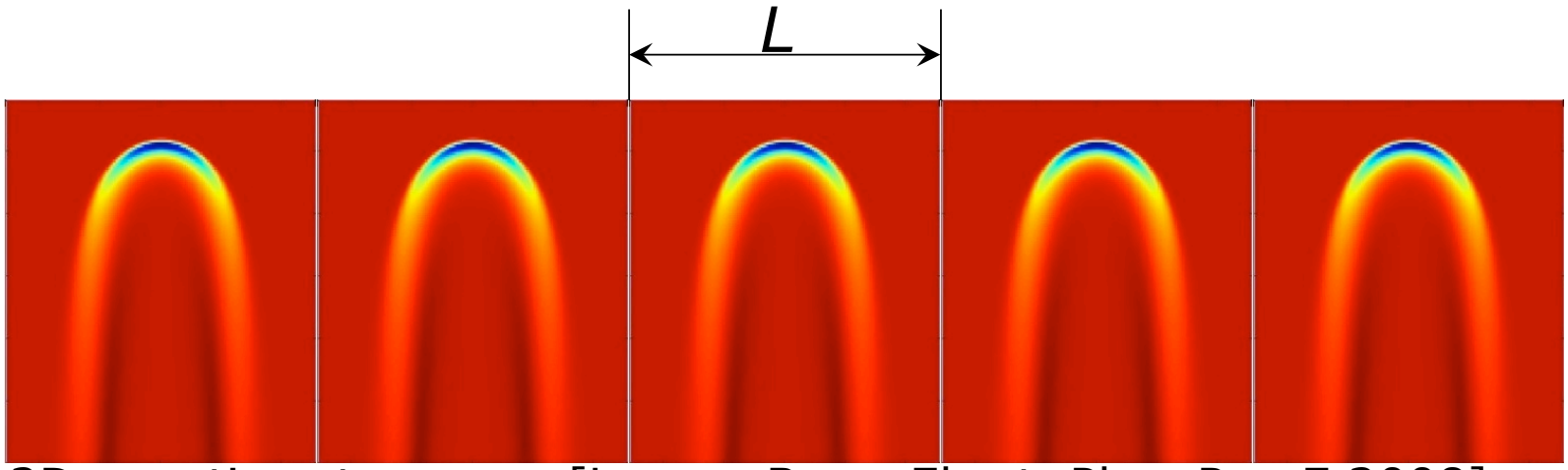
Single streamer



Streamers with electrostatic interaction

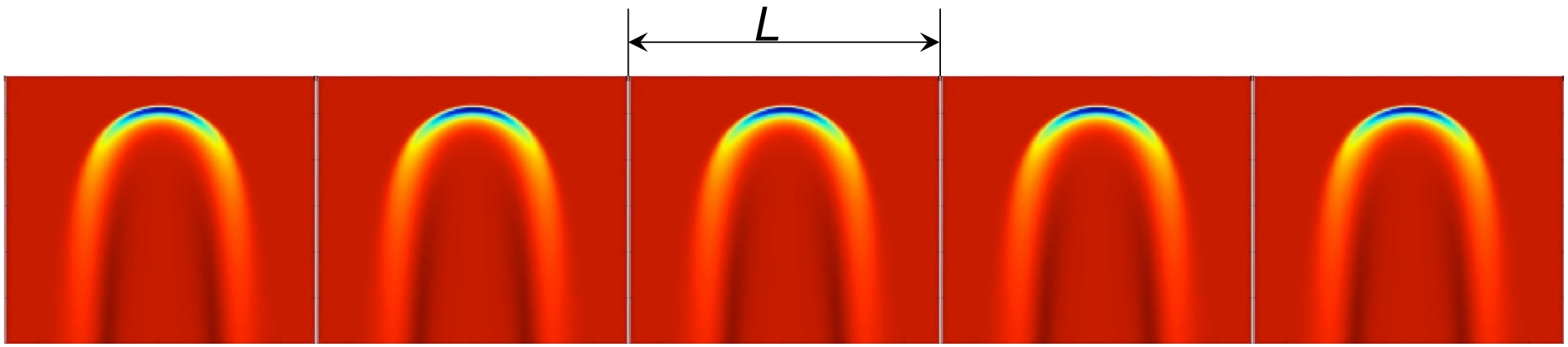


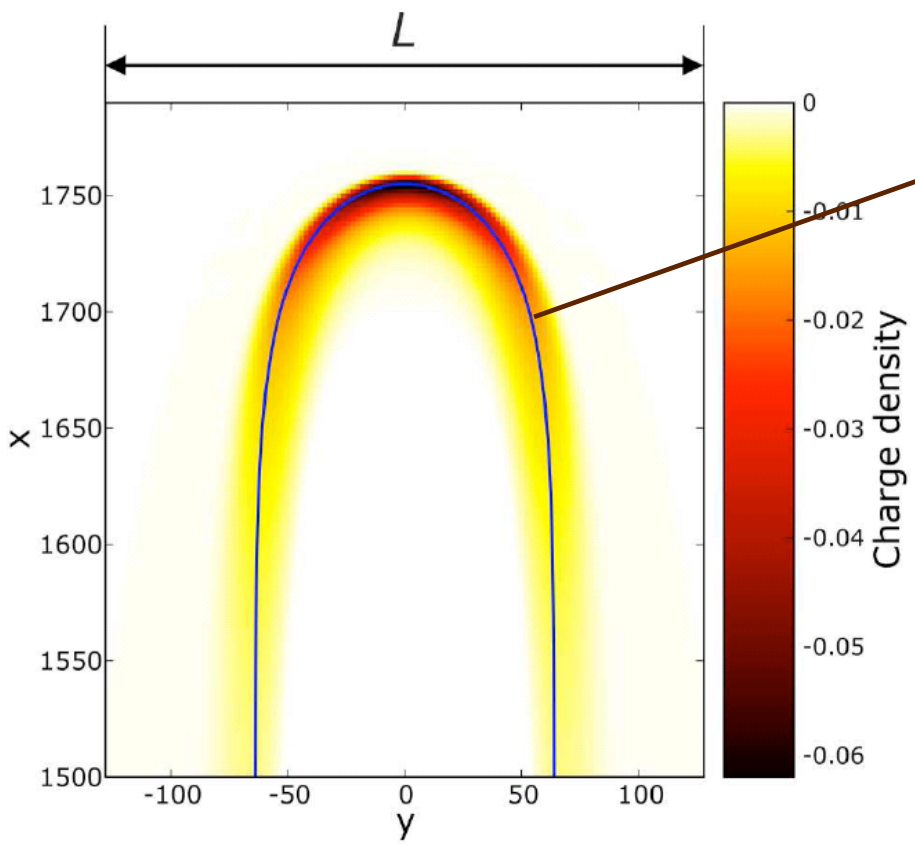
Extreme case: complete screening behind the group of heads



2D negative streamers [Luque, Brau, Ebert, Phys Rev E 2008]

An array of streamers (2D, PDE-model):





$$x = \frac{L(1-\lambda)}{2\pi} \ln\left[\frac{1}{2}\left(1+\cos\left(\frac{2\pi y}{\lambda L}\right)\right)\right]$$

Saffman-Taylor finger with λ=½!

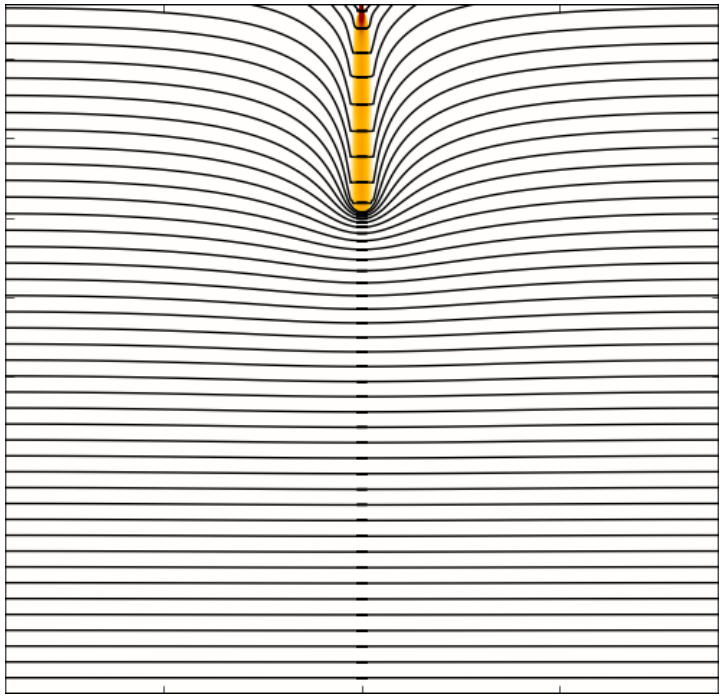
Mathematics of selection?

Prediction: field at streamer tip is enhanced by factor 2 in 2D!

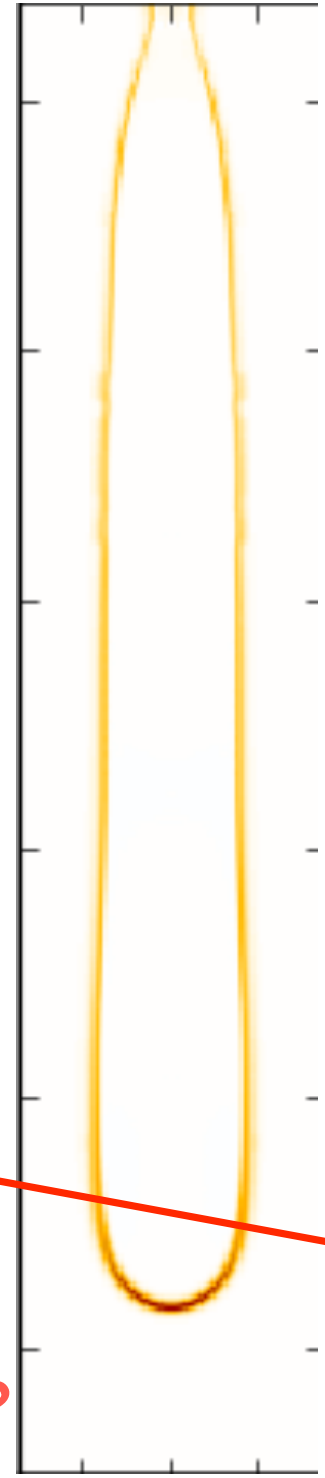
3D analogue?

[Luque, Brau et al., Phys Rev E 2008]

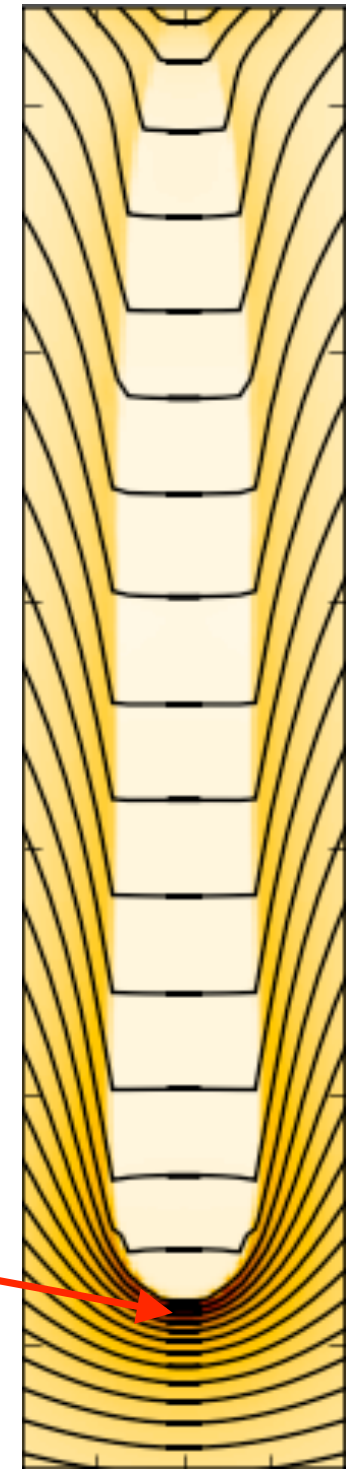
Positive streamer in air



n_+-n_e



|**E**|,
equi-φ



Electron acceleration in enhanced field

-> X-rays and terrestrial gamma-rays?

The multiple scales of lightning and a moving boundary problem

Ute Ebert[1,2], www.cwi.nl/~ebert

Theory:
[1] CWI Amsterdam (0.8)

Willem Hundsdorfer (staff)
Manuel Arrayas
Fabian Brau
Wouter Brok
Chao Li
Alejandro Luque
Bernard Meulenbroek
Carolynne Montijn
Margreet Nool
Valeria Ratushnaya
Andrea Rocco
Gideon Wormeester

Experiment:
[2] Eindhoven Univ. Tech. (0.2)

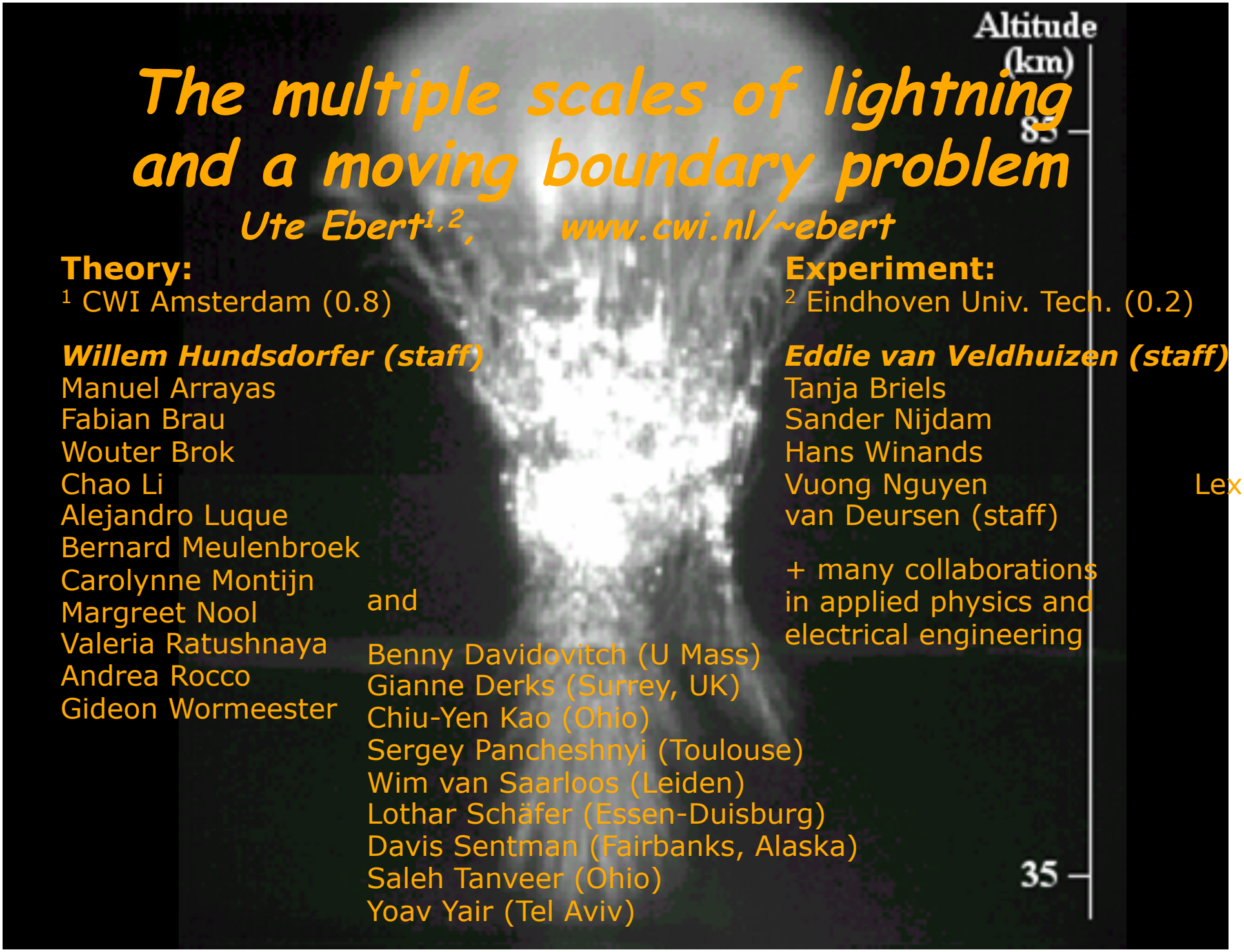
Eddie van Veldhuizen (staff)
Tanja Briels
Sander Nijdam
Hans Winands
Vuong Nguyen
van Deursen (staff)

Lex

+ many collaborations in applied physics and electrical engineering

and

Benny Davidovitch (U Mass)
Gianne Derks (Surrey, UK)
Chiu-Yen Kao (Ohio)
Sergey Pancheshnyi (Toulouse)
Wim van Saarloos (Leiden)
Lothar Schäfer (Essen-Duisburg)
Davis Sentman (Fairbanks, Alaska)
Saleh Tanveer (Ohio)
Yoav Yair (Tel Aviv)



TexPoint fonts used in EMF.