Network Topology:

Sensors & Systems

Robert Ghrist

University of Pennsylvania

Departments of Mathematics &

Electrical & Systems Engineering

Sensors in the news

Motivation

ws

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News, Reviews and Opinions from the marvelous world of chips that are changing the

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PALIT Folding@Home Challenge 2008

It's official: Digital sensor overtakes the human eye(s)

Posted by: **theovalich** | December 20, 2008

With the popularization of digital cameras, we witnessed numerous discussions about the precision of human eye e.g. how many "megapixels" a human eye has? There are several answers on-line, but general consensus is that a single human eye has between 105 and 126 million rods and cones, e.g. "126 MPixels". This number was extremely impressive at the time when most digital cameras had 1.3 MPixels, and still is impressive, with the world's consumer cameras passing the 10 million barrier.

But, RED is the company that intends to do things differently. A while ago, I wrote a small piece announcing the revolution of digital cinema and digital photography in general - RED introducing their "double o" (Obsolescence Obsolete) concept. At the top of the future line-up stands RED 617, powered by single CMOS sensor in "beyond huge" category (9744mm2). What didn't made the news then was the fact that this single CMOS sensor actually features a higher resolution than human eye... both of them. According to Jim Jannard, founder of RED, RED 617 Mysterium Monstro features 261,352,000 pixels, while both human eyes feature 252 million rods/cones. In short - RED 617 has 135 million pixels more than a single human eyes.

Motivation

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News, Reviews and Opinions from the marvelous world of chips that are changing the

California Institute of Technology

Caltech336

Revolution in a radar chip

Imagine driving down a twisty mountain road on a foggy night. Visibility is near zero, yet you still can see clearly. Not through your windshield, but via an image on a screen in front of you.

Such a built-in radar system in our cars has long been in the domain of science fiction, as well as wishful thinking on the part of commuters. But such gadgets could become available in the near future, thanks to Caltech's High Speed Integrated Circuits group.

The group is directed by Ali Hajimiri, an associate professor of electrical engineering. Hajimiri and his team have used revolutionary techniques to build the world's first radar implemented a novel

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Sniper rifle software launched for iPod touch

New BulletFlight program could be a 'killer' app for Apple

Last Updated: 3:19PM GMT 20 Jan 2009

BulletFlight is a new application has been launched for the iPod touch to help gun users line up a clean shot at their target

A new application has been launched for the iPod touch to help gun users line up a clean shot at their target.

The BulletFlight app, which costs

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By Charlie Sorrel May 10, 2007 | 5:07:42 AM Categories: Innovations

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http://blog.wired.com/wiredscience/2008/12/ingestible-chip.html

Edible Electronics Monitor Drugs in Your Body

By Priya Ganapati

December 04, 2008 | 2:50:23 PM Categories: Personalized Medicine

Over the next few years popping a pill

A biomedi

EE Times:

DNA-based sensor chip detects metals in real-time

R. Colin Johnson R. Colin Johnson
EE Times
(12/28/2000 4:13 PM EST)

CHAMPAIGN, Ill. — An inexpensive, real-time sensor technology harnesses living DNA to detect dangerous metals such as lead, mercury and cadmium.

Developed by researchers at the University of Illinois, the DNA sensors immediately react to the presence of specific metals by emitting light into an inexpensive fiber-optic lens. Traditional methods require lengthy batch testing or expensive instrumentation. Genetic algorithms were used to discover the specific required DNA strands required to detect specific metals from within a population of trillions of random DNA sequences.

Engineers could benefit from this method by designing their own DNA strands that would test in real-time for specific metallic substances.

"We have created a new class of simple and environmentally safe sensors — the world's first example of a catalytic DNA-based biosensor with highly sensitive fluorescence detection for metals," said professor Yi Lu. Lu was assisted by graduate student Jing Li, co-author of the recent paper in the Journal of the American Chemical S

process patent is pending.

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Science News

Portable Cocaine Sensor Developed At UC Santa Barbara

ScienceDaily (Feb. 28, 2006) — A real-time sensor for detecting cocaine -- made with inexpensive, off-the-shelf electronics -- has been developed by a team of researchers at the University of California, Santa Barbara. Two local high school students and a Nobel laureate participated in the discovery. The potential applications of the sensor are far-reaching and include bioterrorism detection and important medical uses.

See also:

Matter & Energy
- Detectors
- Chemistry
- Organic Chemistry
- Sports Science
- Medical Technology
- Inorganic Chemistry

Reference
- Macromolecule
- White gold
- Alkaloid
- Catalytic converter

The high school students made their own sensors and collected data shown in a graph in the scientific article they co-authored describing the work. In the article, published in the Feb. 18 issue of the Journal of the American Chemical Society (JACS), the authors state, "Cocaine serves as an ideal and representative target for testing new analytical techniques due to pressing needs for its rapid detection in law enforcement and clinical settings." The sensor can be housed in supporting electronics that are the size of small hand-held device

Co-author and Nobel laureate Alan Heeg
developed a method of detecting

EE Times:

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One fish, two fish: New sensor improves fish counts

Anne Trafton, News Office
February 2, 2006

Researchers at MIT have found a new way of looking beneath the ocean surface that could help definitively determine whether fish populations are shrinking.

A remote sensor system developed by Associate Professor Nicholas Makris of mechanical engineering, along with others at MIT, Northeast University and the Naval Research Laboratory, allows scientists to tra enormous fish populations, or shoals, as well as small schools, ove 10,000-square-kilometer area -- a vast improvement over conventio technology that can survey only about 100 square meters at a time

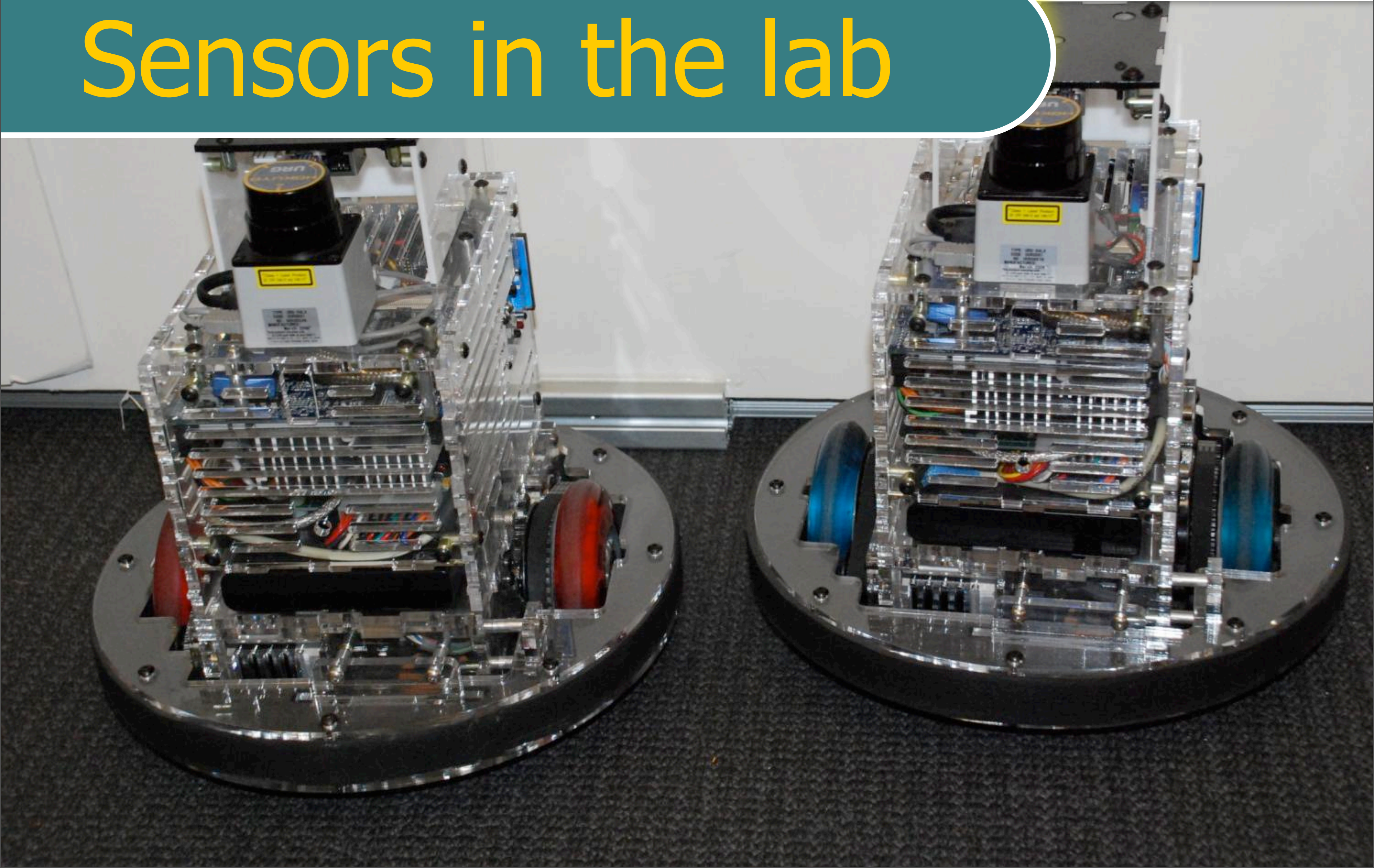
"We're able to see for the first time what a large group of fish look said Makris, who compared the dramatic improvement to the diff between seeing everything on a television screen and seeing o pixel.

The new sensor system, described in the Feb. 3 issue of Sci allow government agencies to figure out what's really happe populations, which many environmentalists and scientists rapid decline.

"The world's fish stocks are being depleted at a horrible kris, who attributed declining populations to overfishi abetted by inaccurate fish counts. "One the darkness in the ocean. Y

Sensors in the lab

Sensors in the lab



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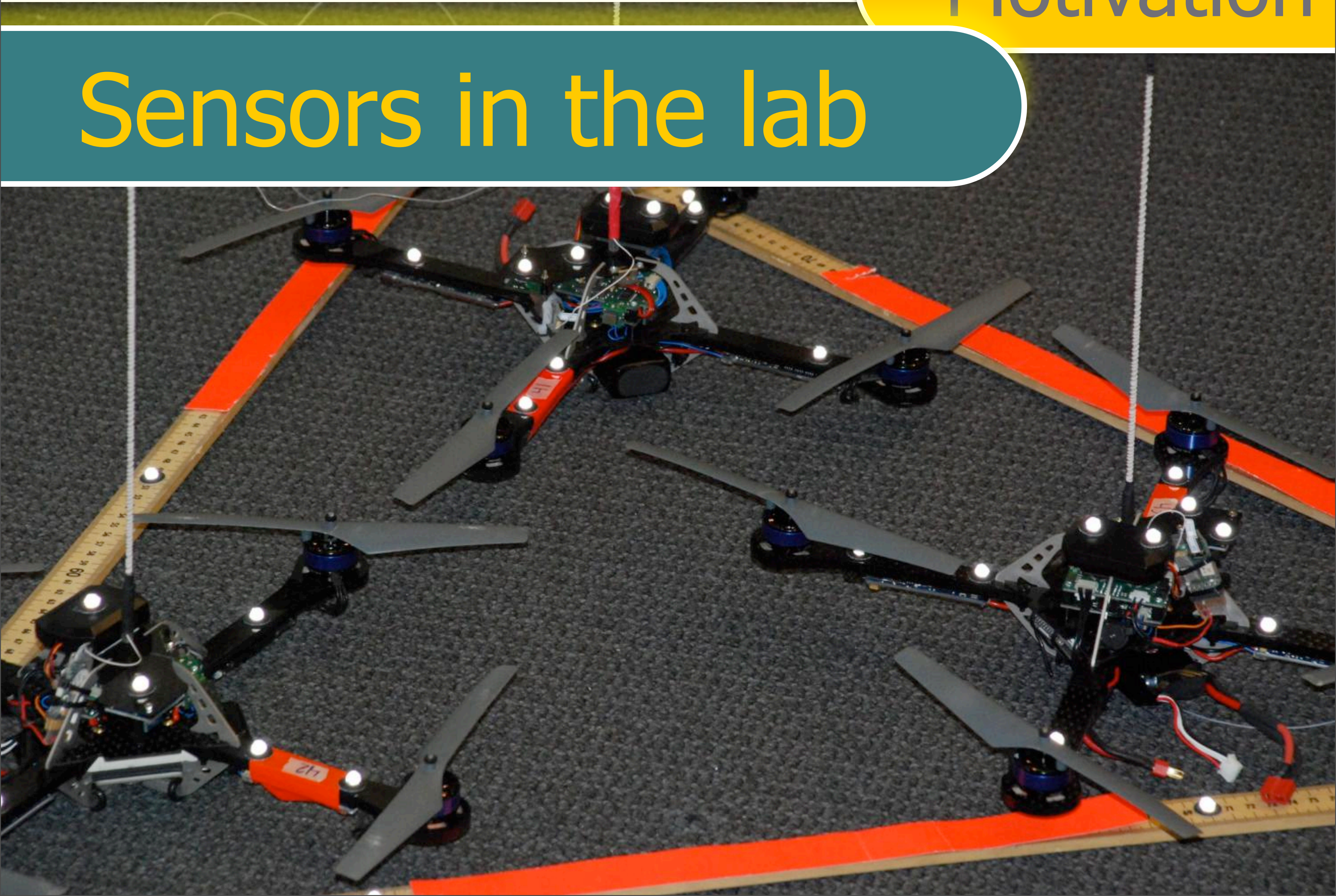

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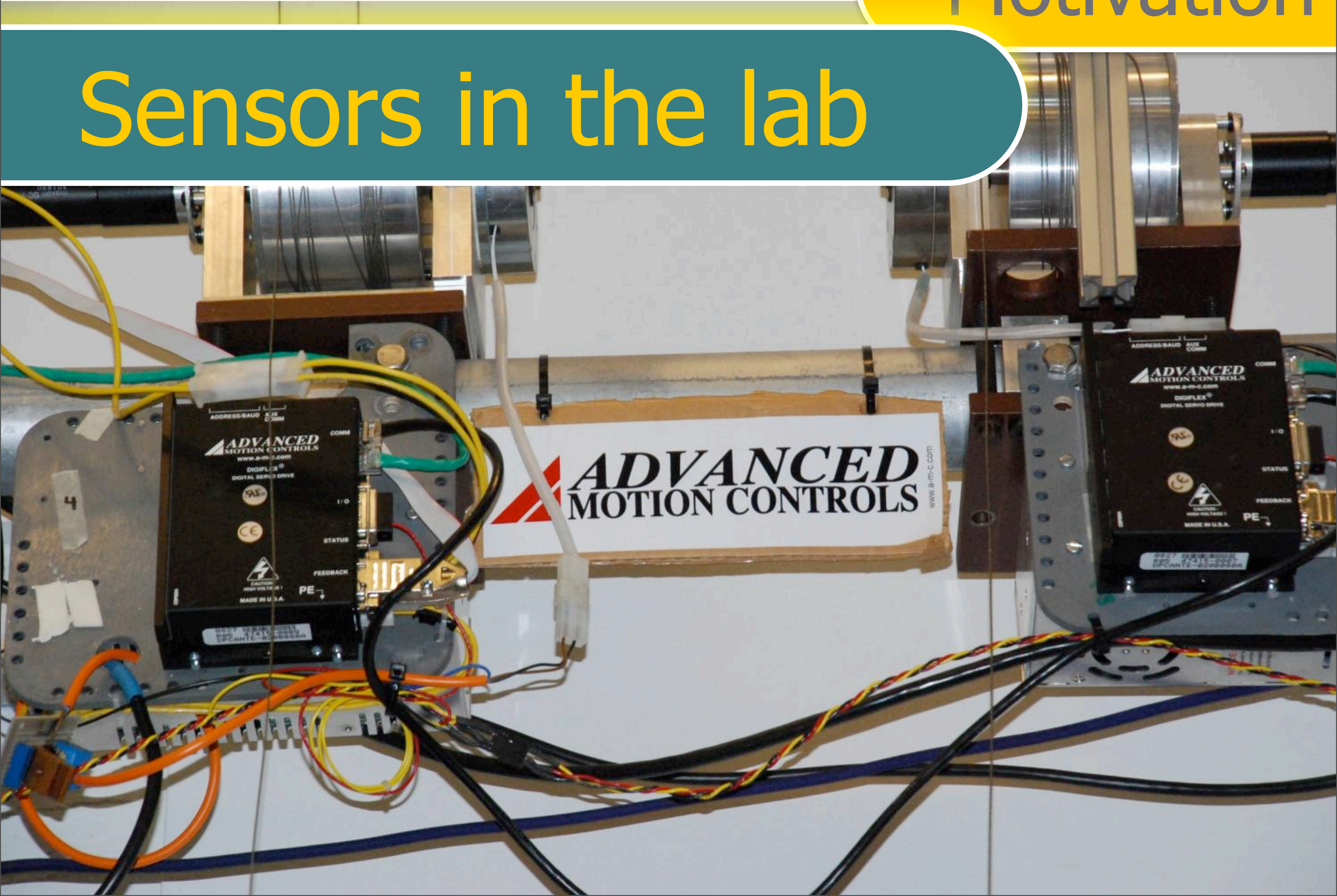

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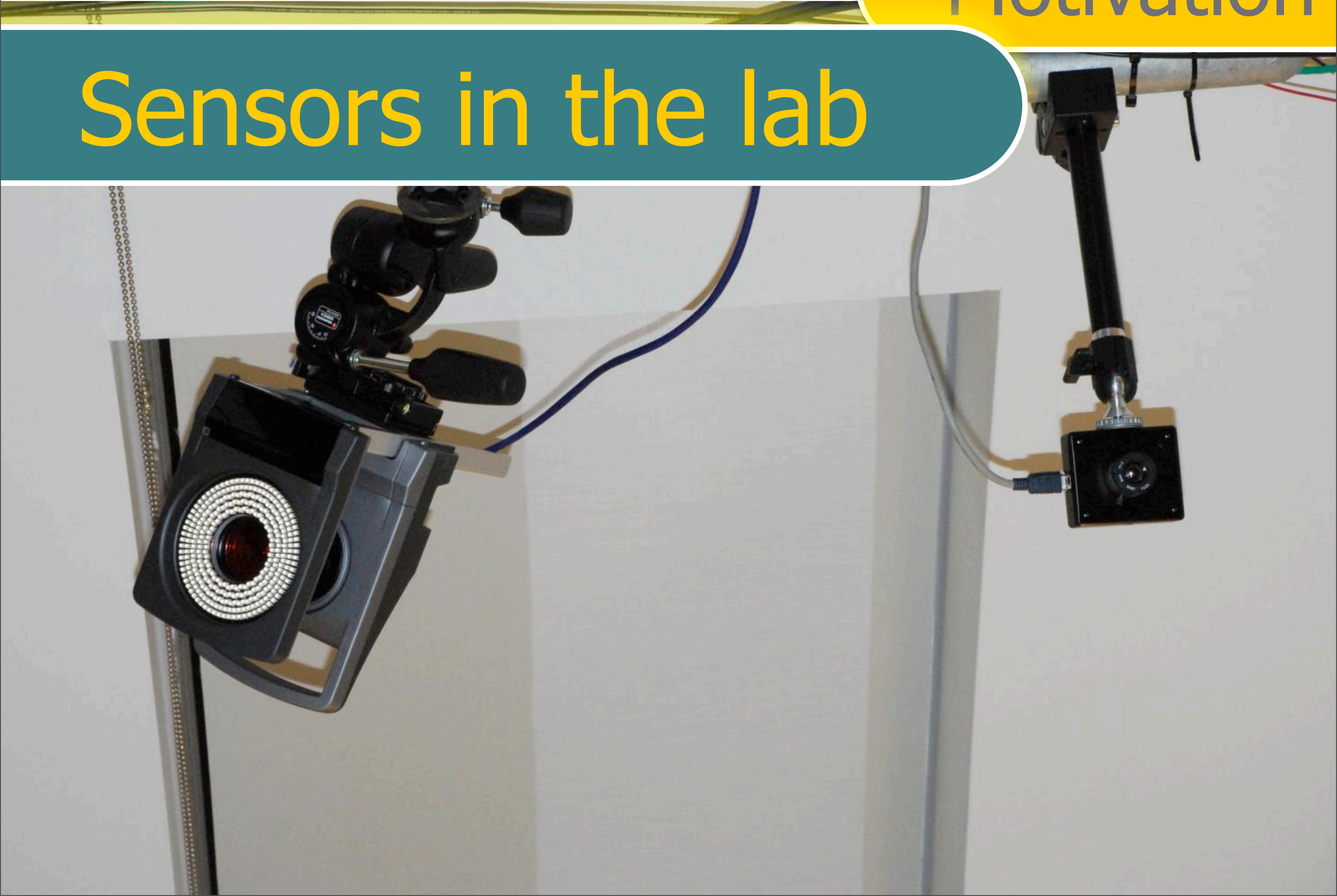

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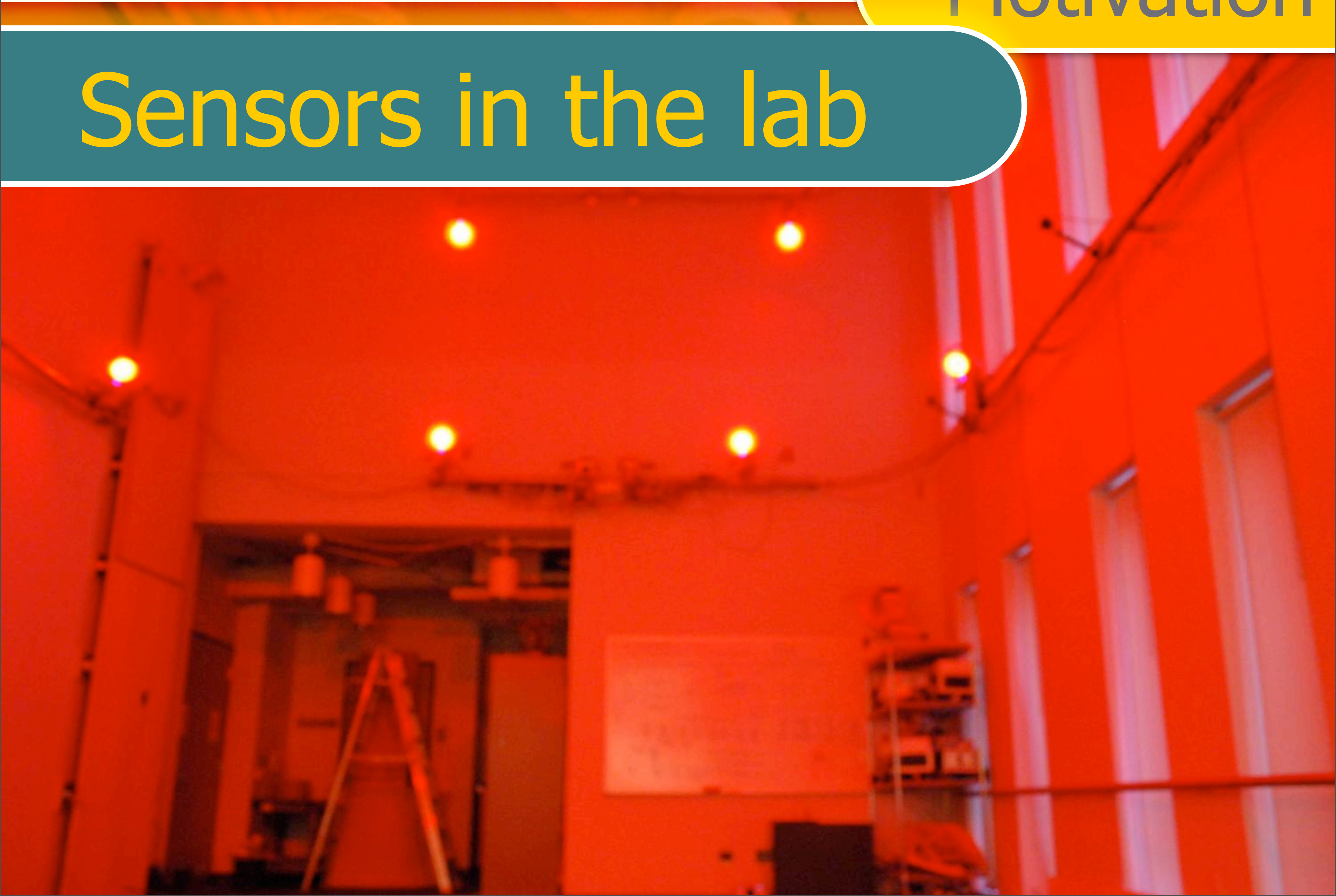

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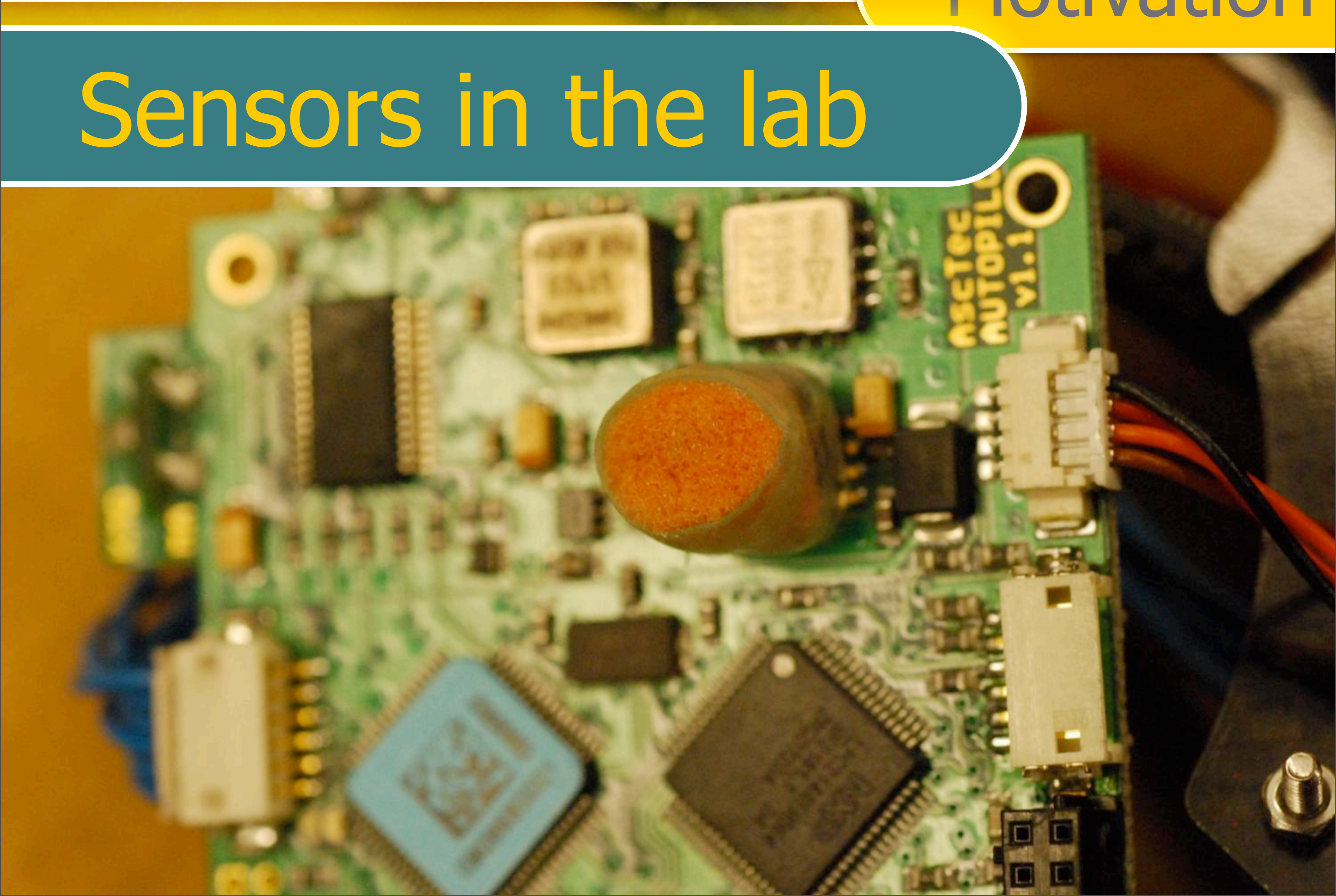

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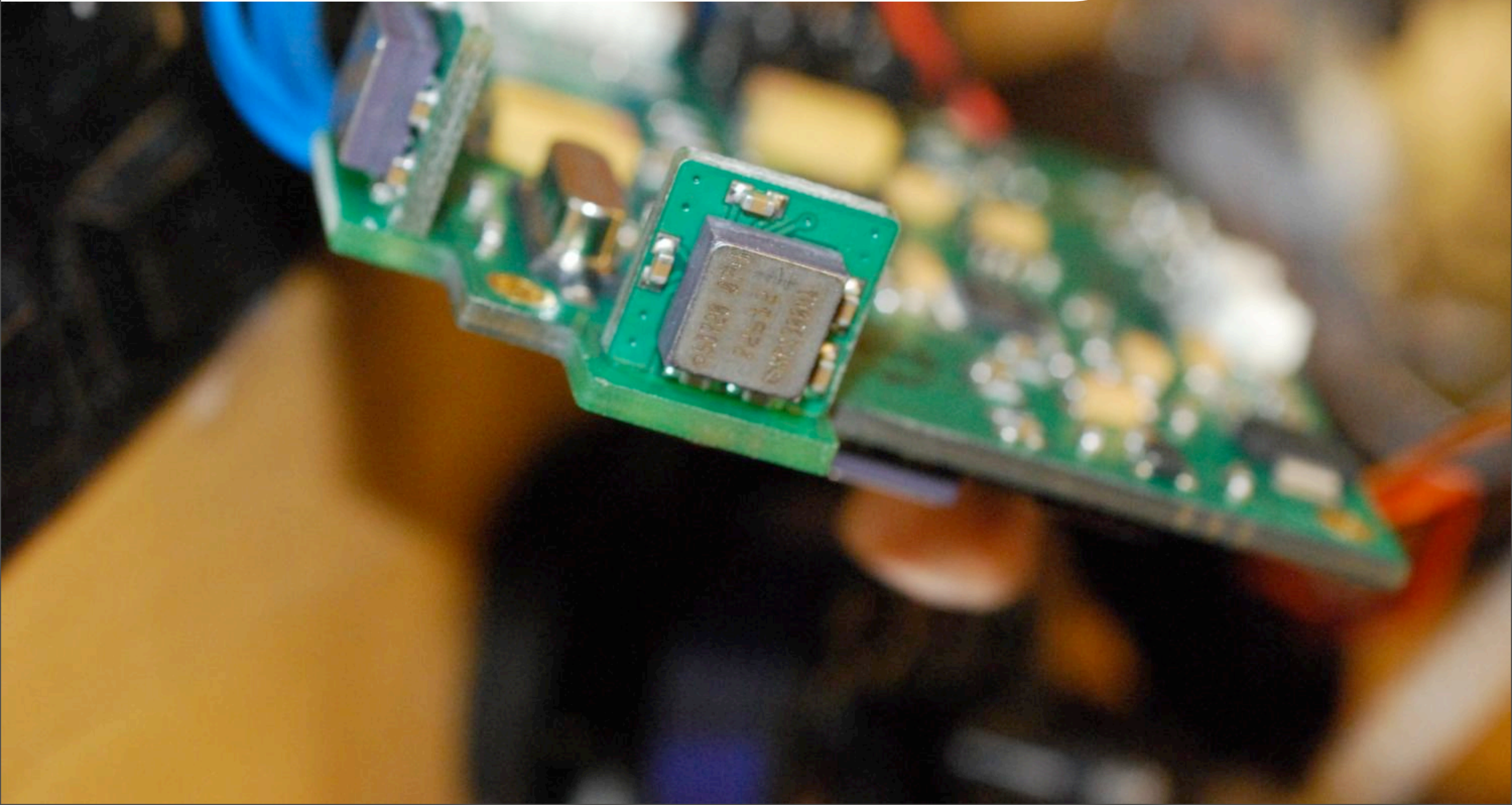

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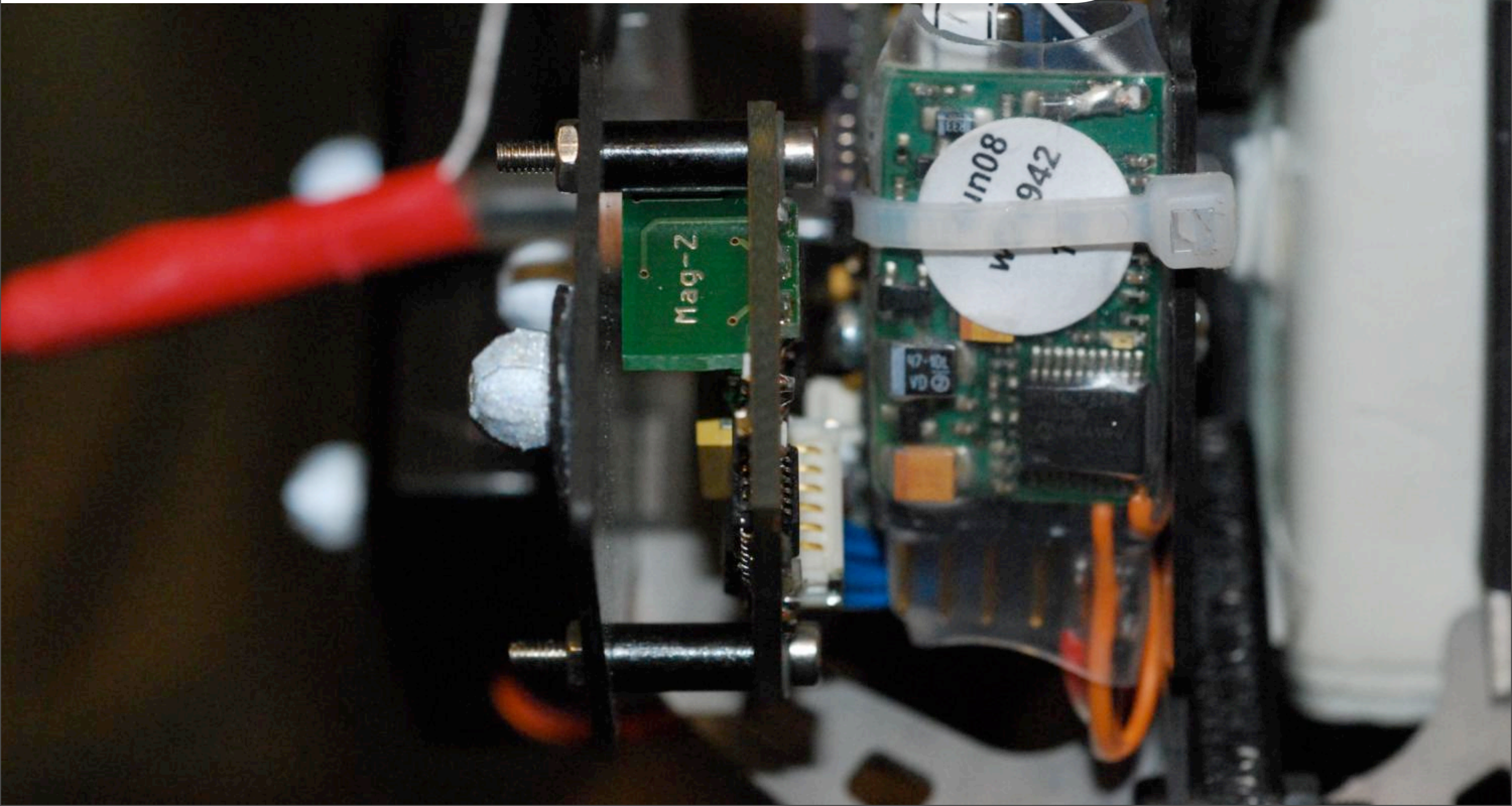

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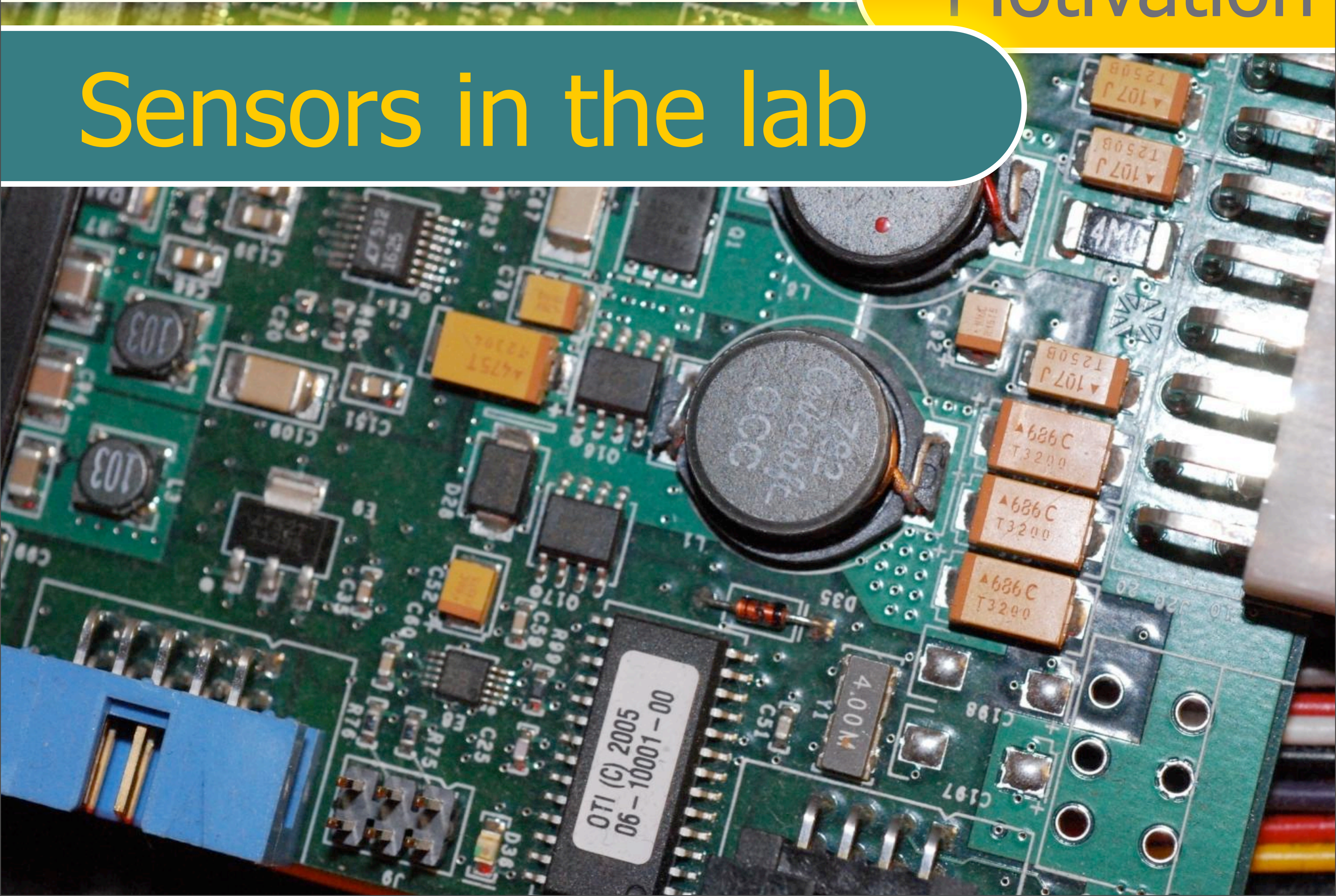

Sensors in the lab

Sensors in the lab

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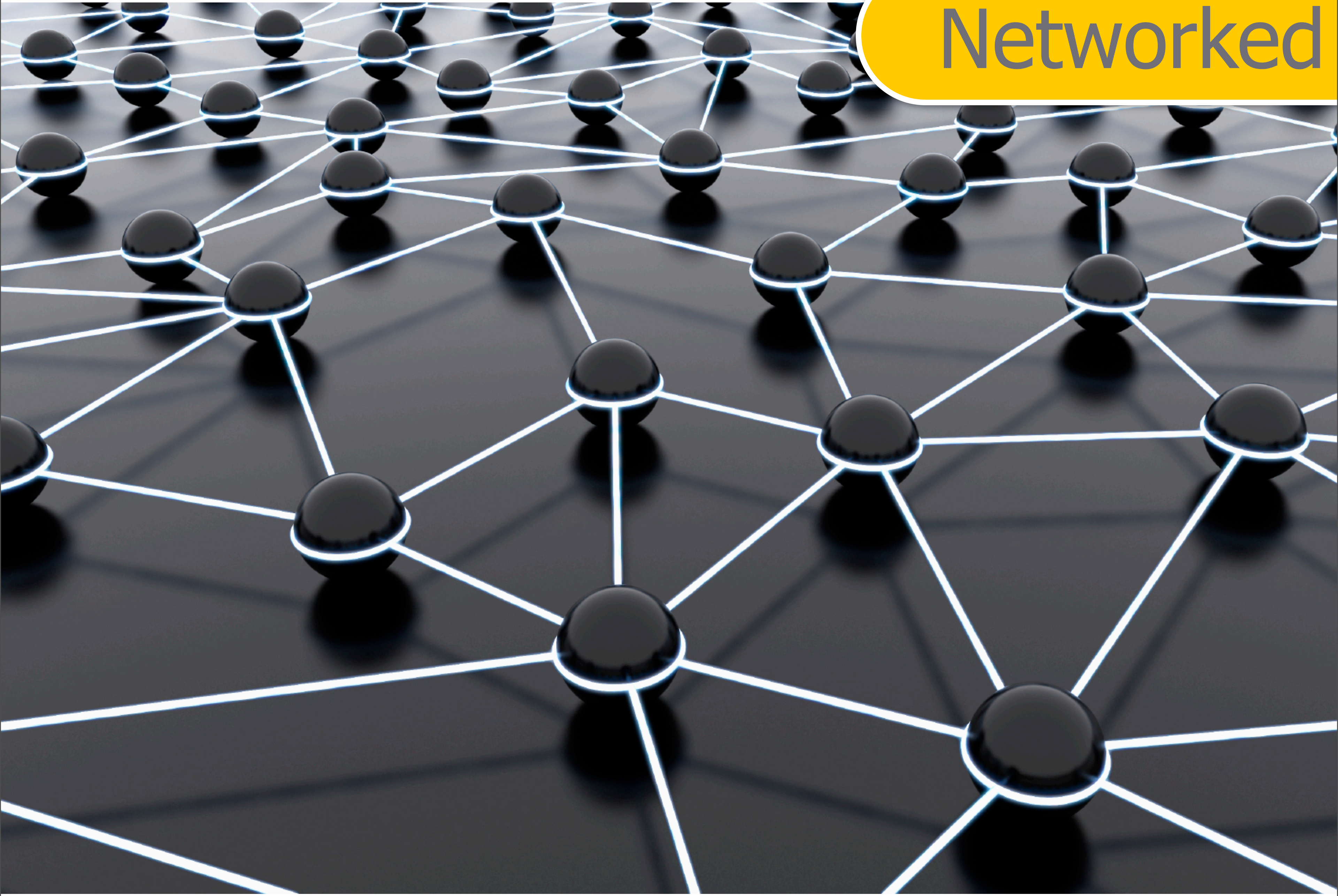

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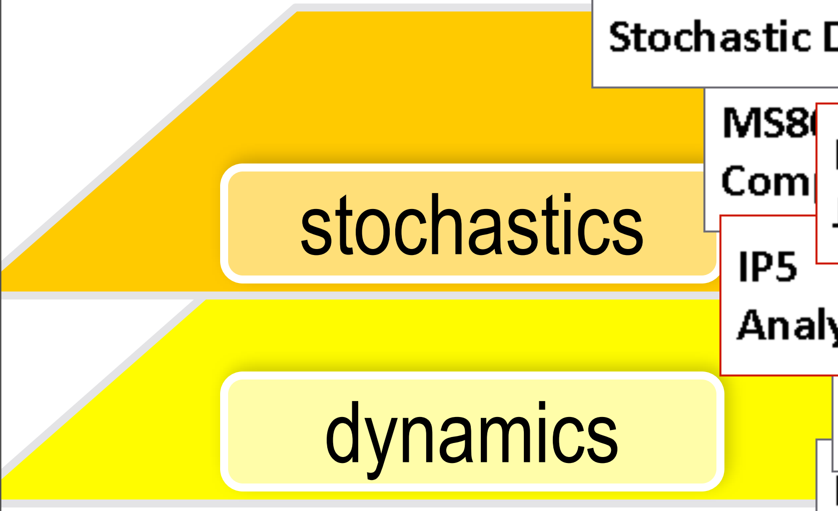

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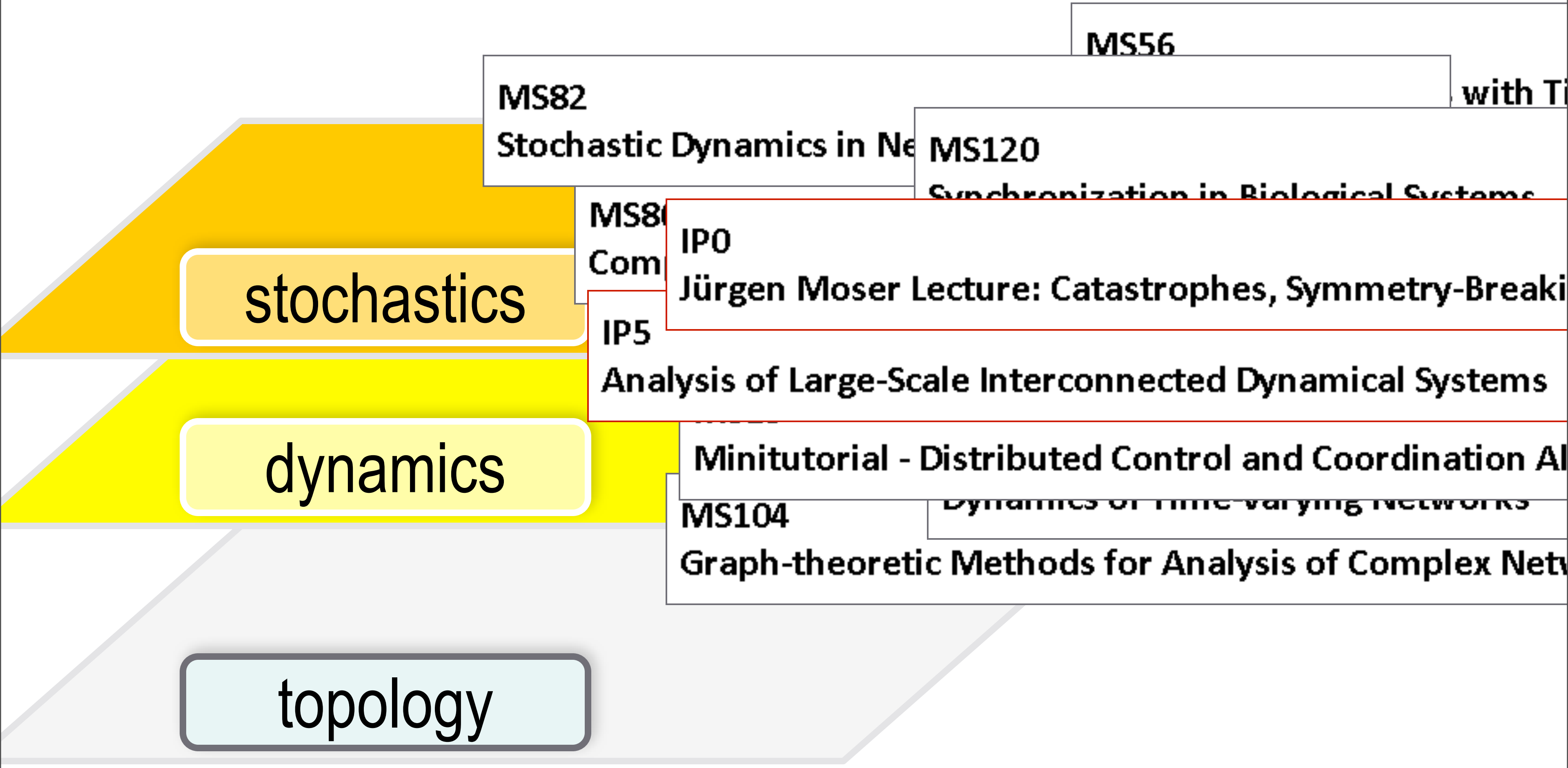

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Topological Networks

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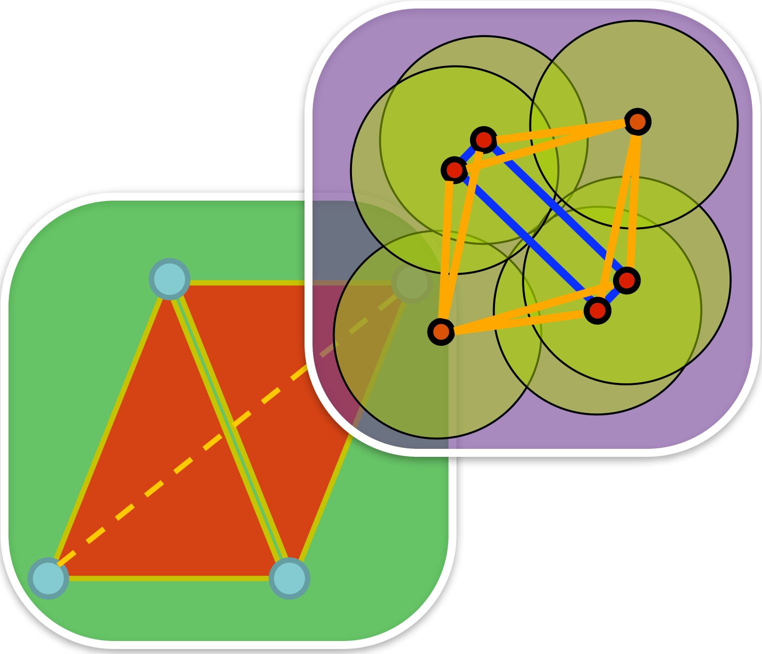
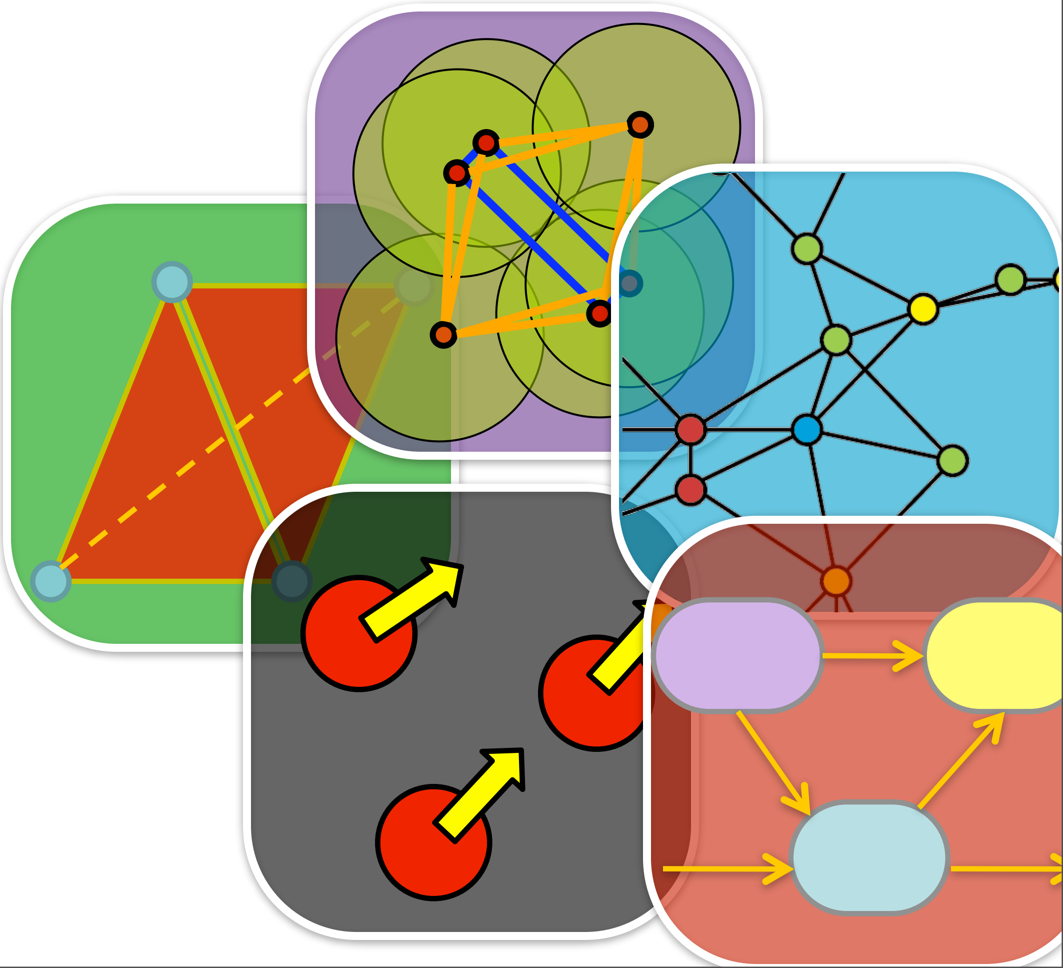


Outline

Topological Networks



Homology

Index Theory

Precursor

Index Theory

Homology has two ingredients: counting and cancelling

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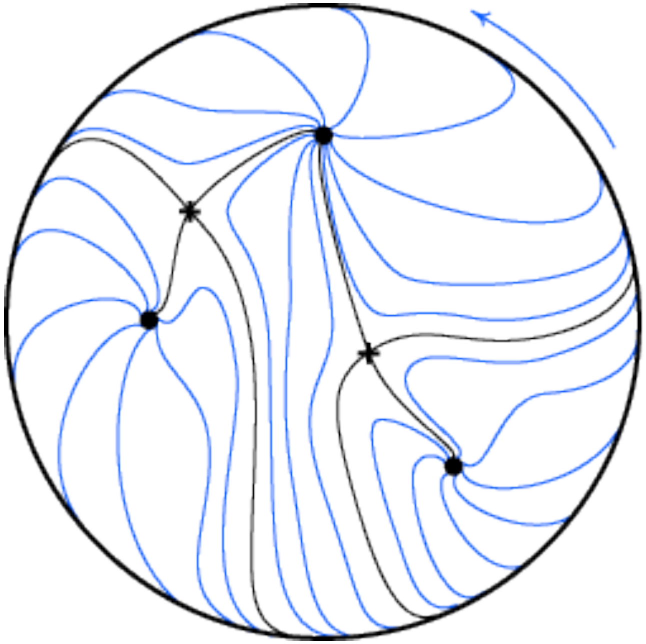

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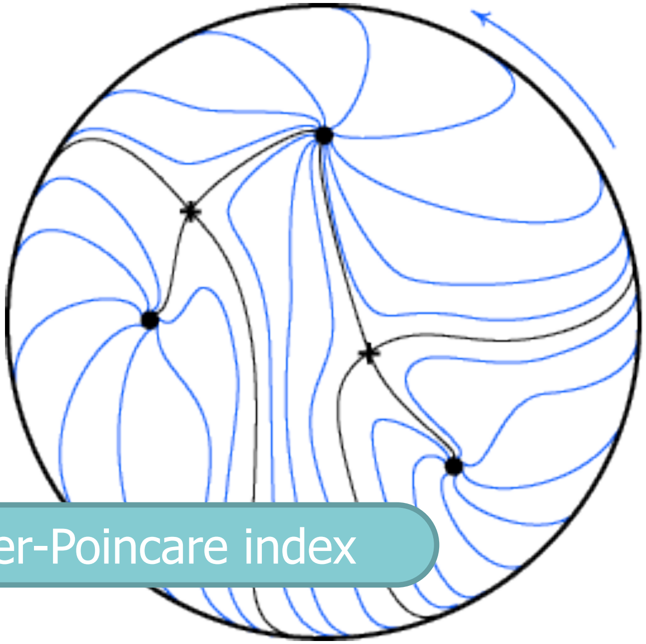

Euler-Poincare index

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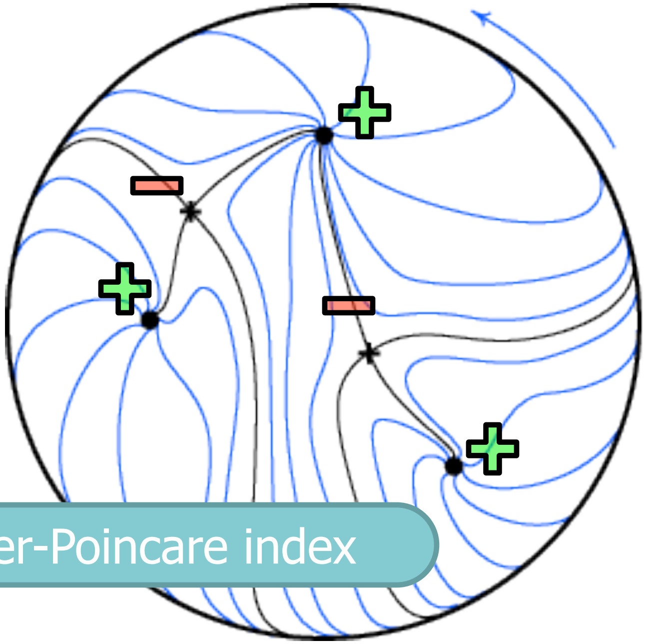

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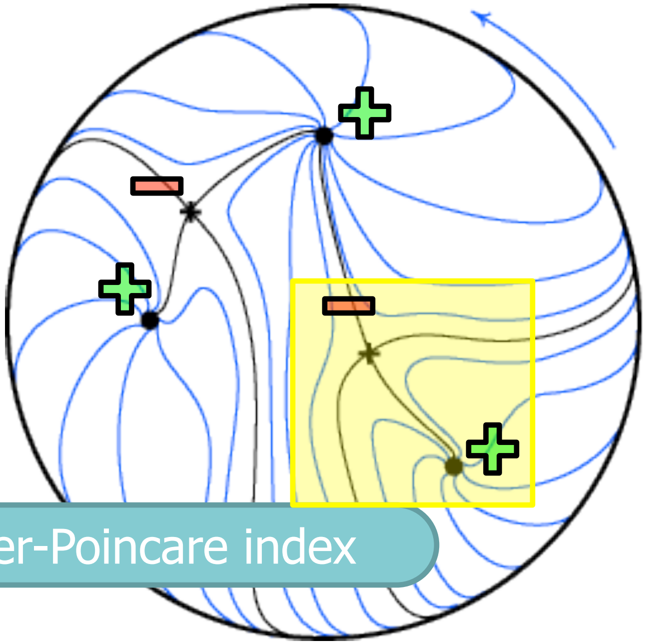

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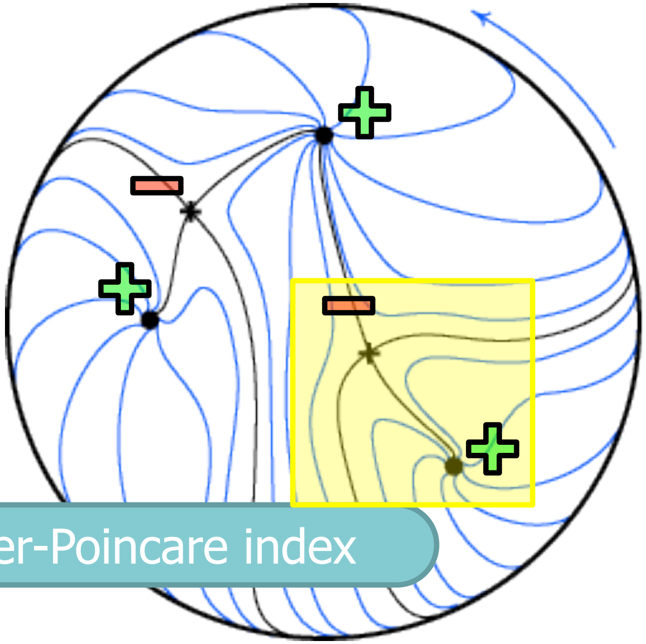

Euler-Poincare index

Invariant under deformation

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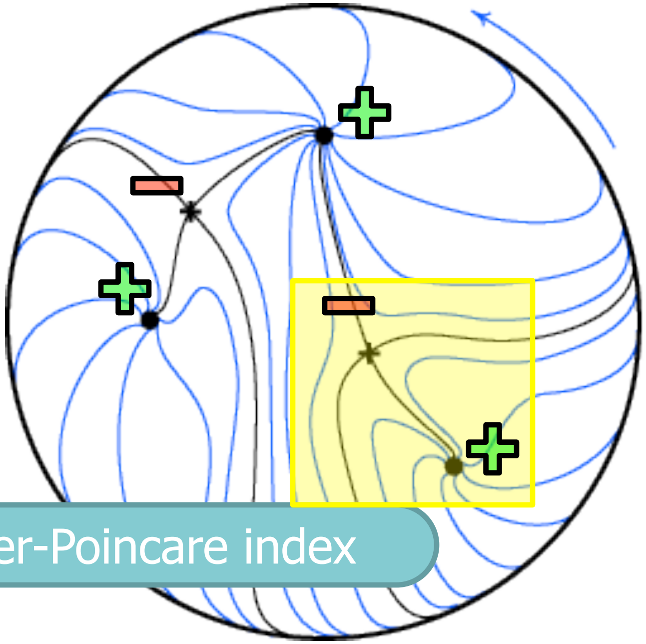

Euler-Poincare index

Invariant under deformation

Local computation

Precursor

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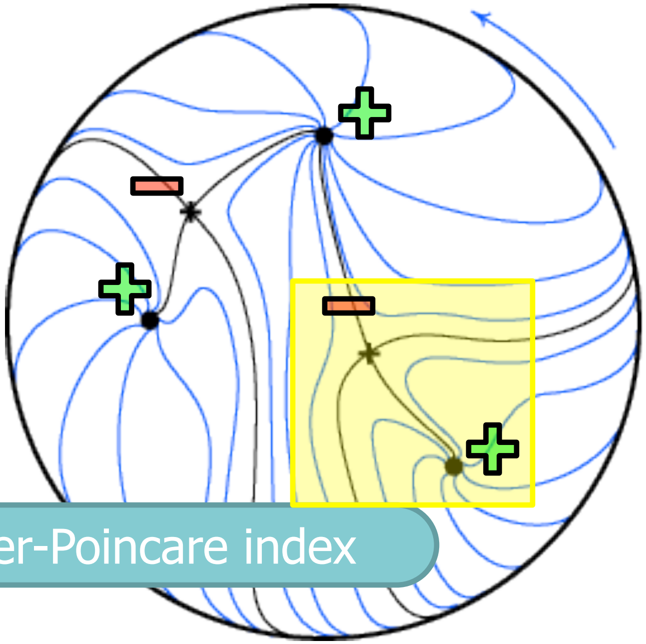

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Invariant under deformation

Local computation

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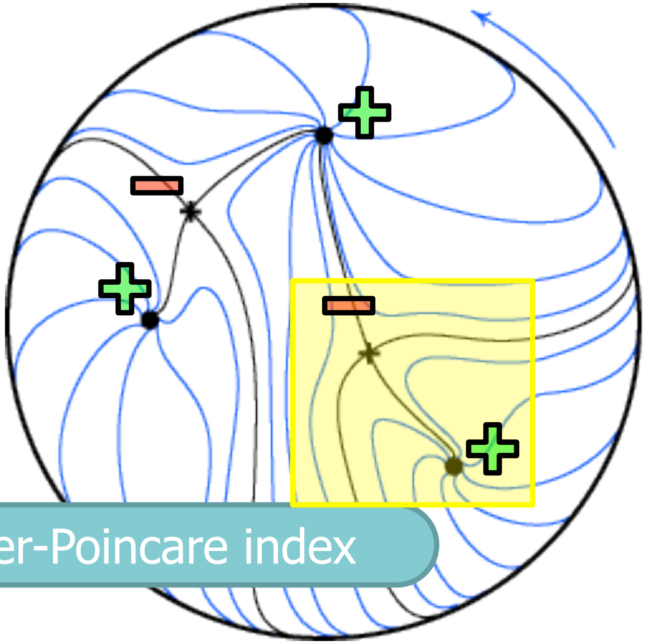

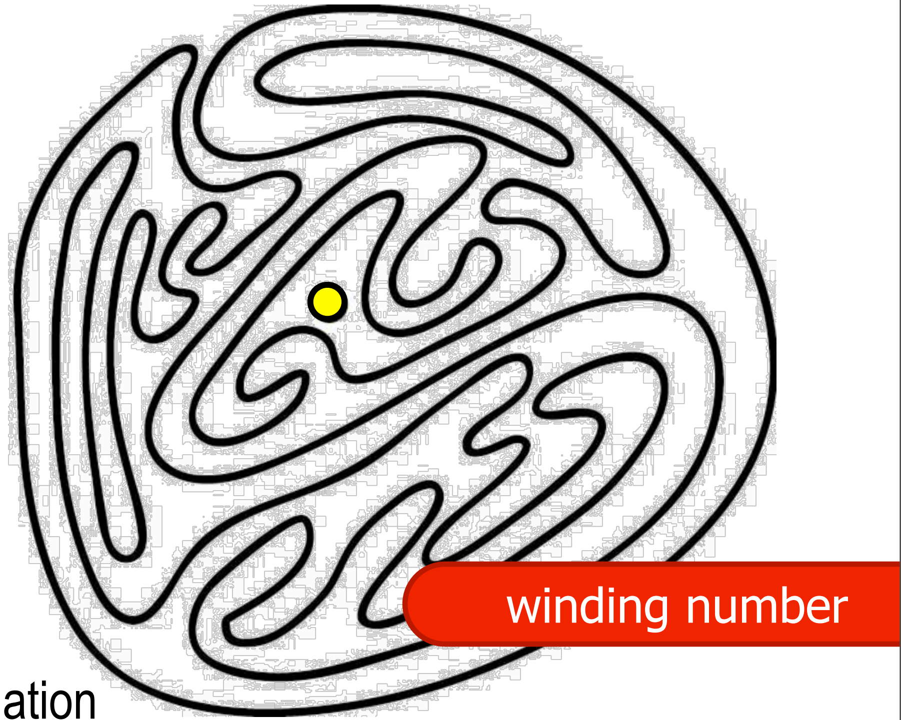

Invariant under deformation

Local computation

Index Theory

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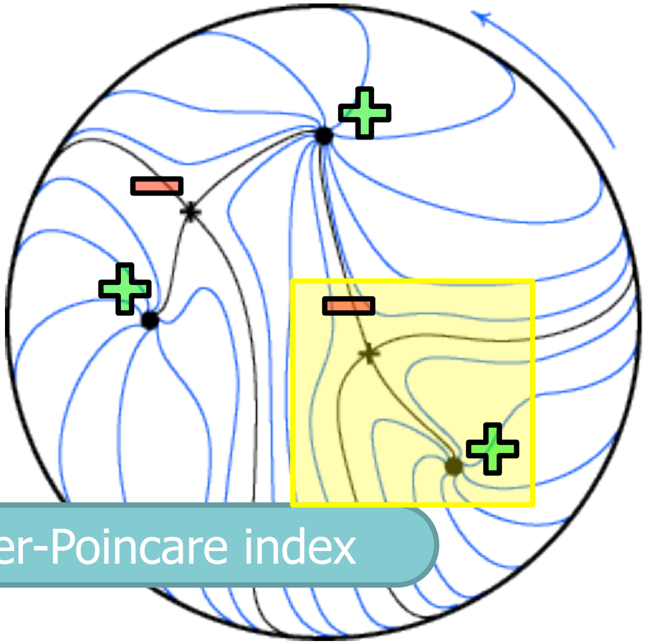
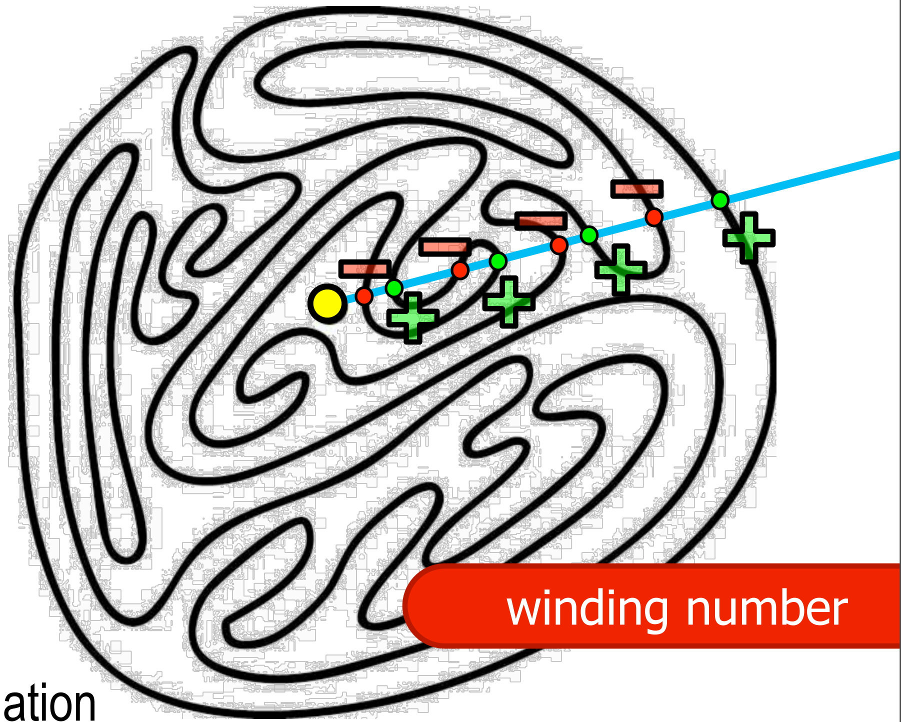




Invariant under deformation

Local computation

18th c.

Euler characteristic

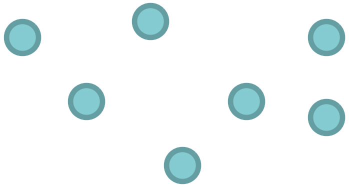
18th c.

Euler characteristic

Let’s count a collection of…

Euler characteristic

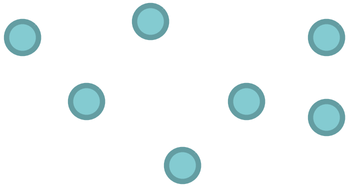
Let’s count a collection of…



Euler characteristic

Let’s count a collection of…

vertices



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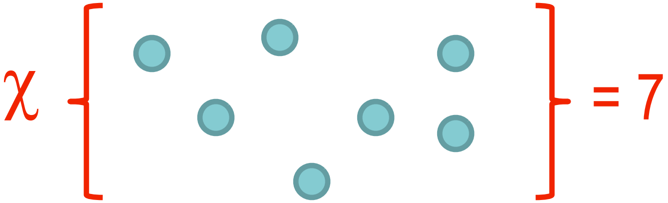
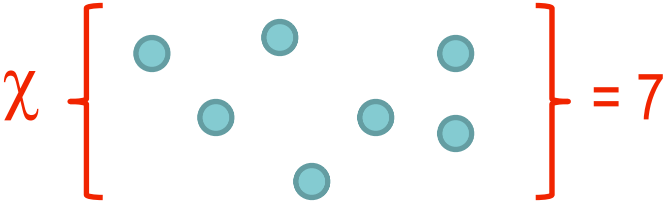


Euler characteristic

Let’s count a collection of…

vertices & edges



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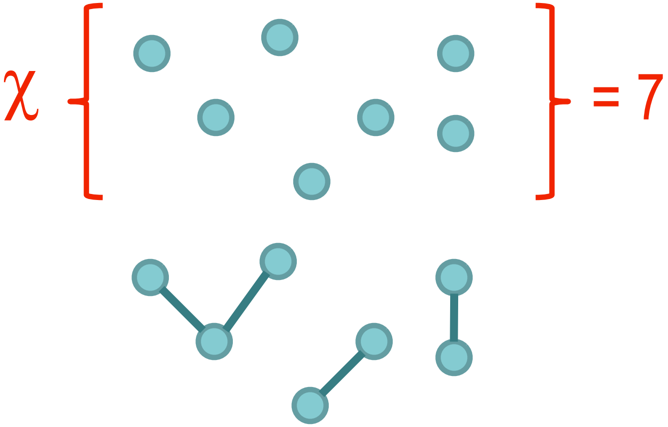

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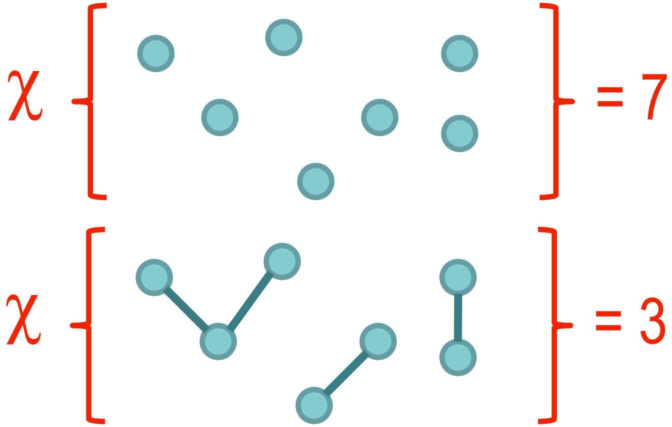

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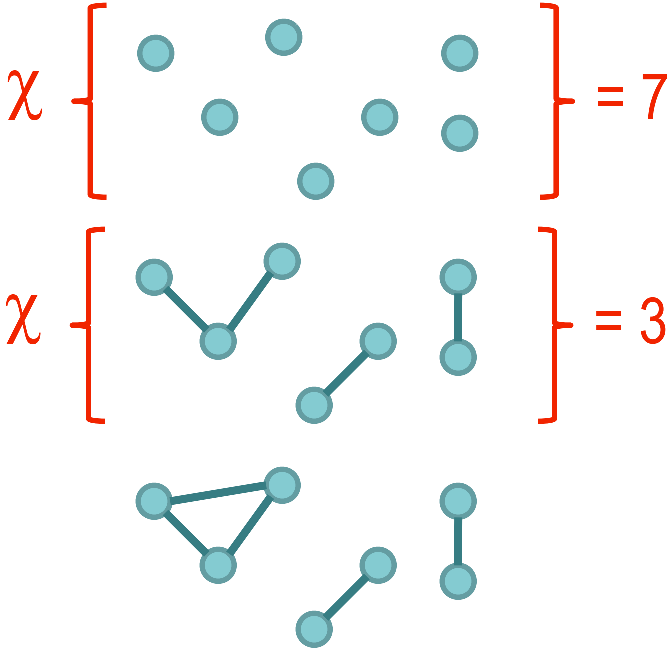

18th c.

Euler characteristic

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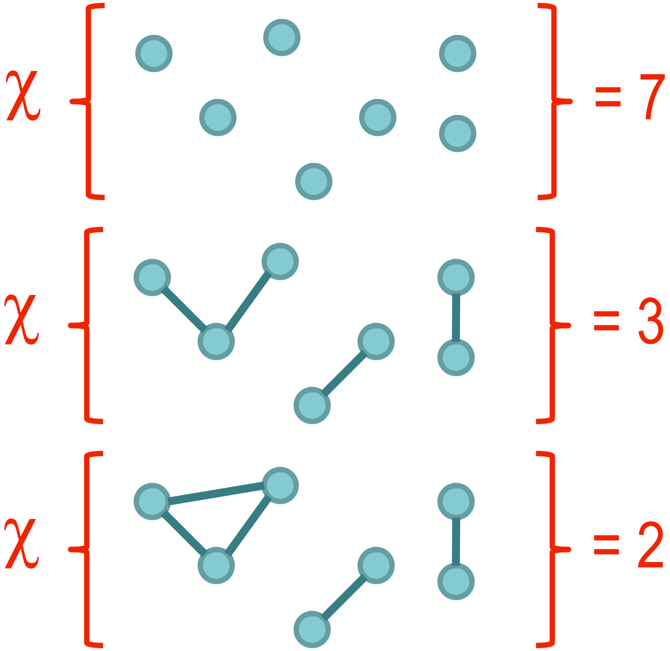

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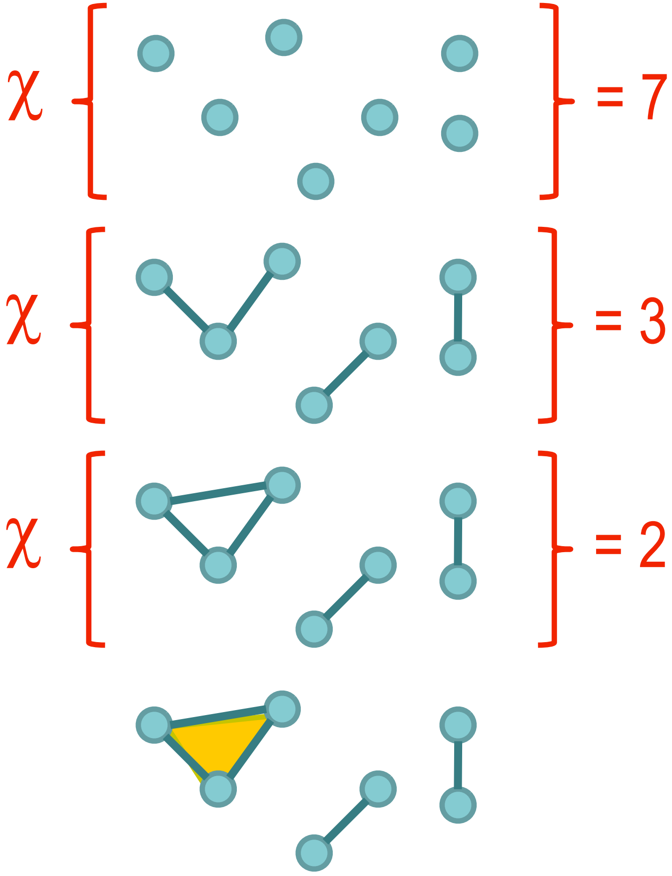
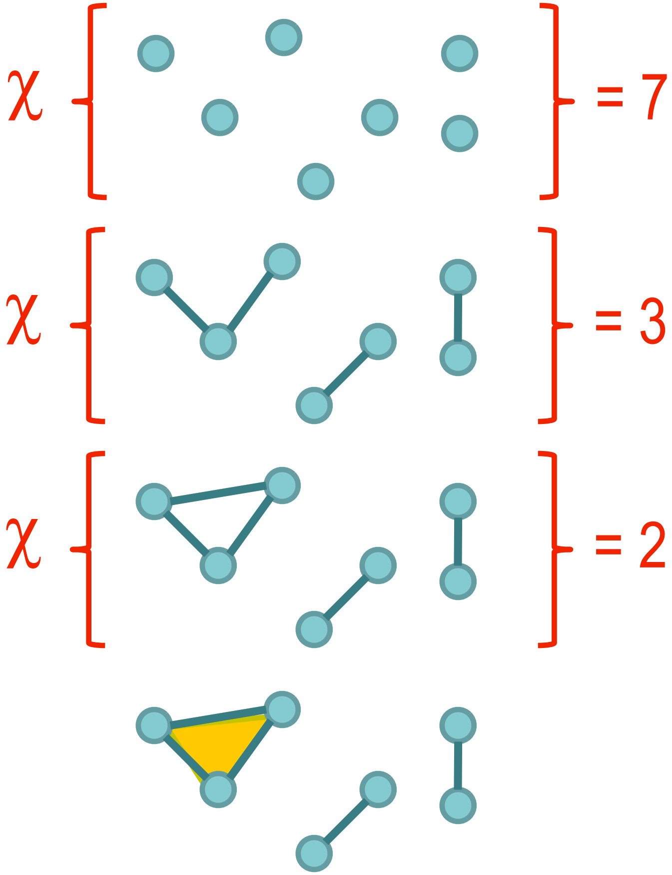


Euler characteristic

Let’s count a collection of…

vertices & edges & faces



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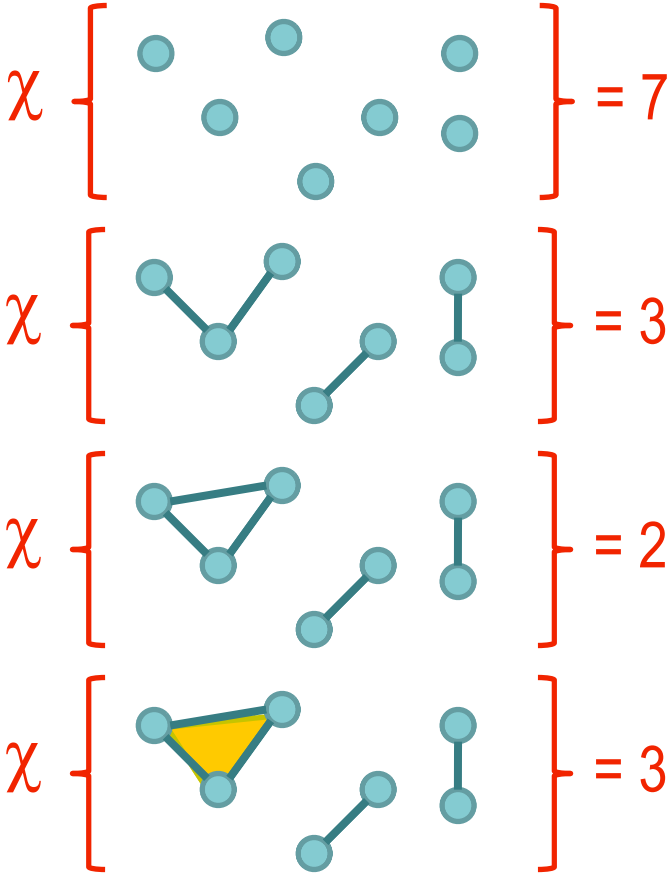

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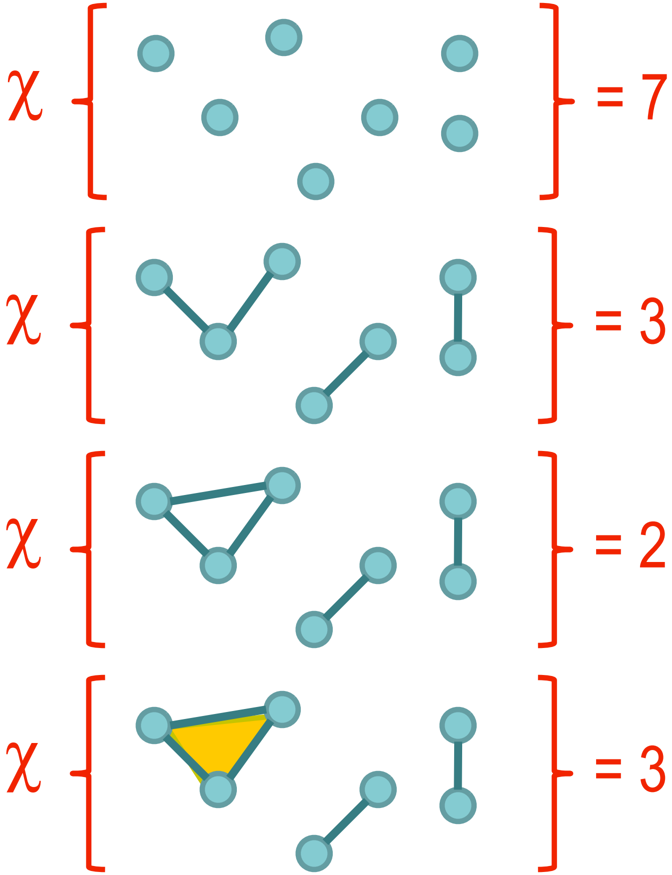

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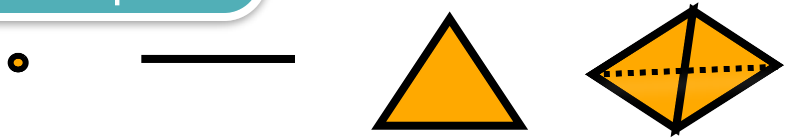
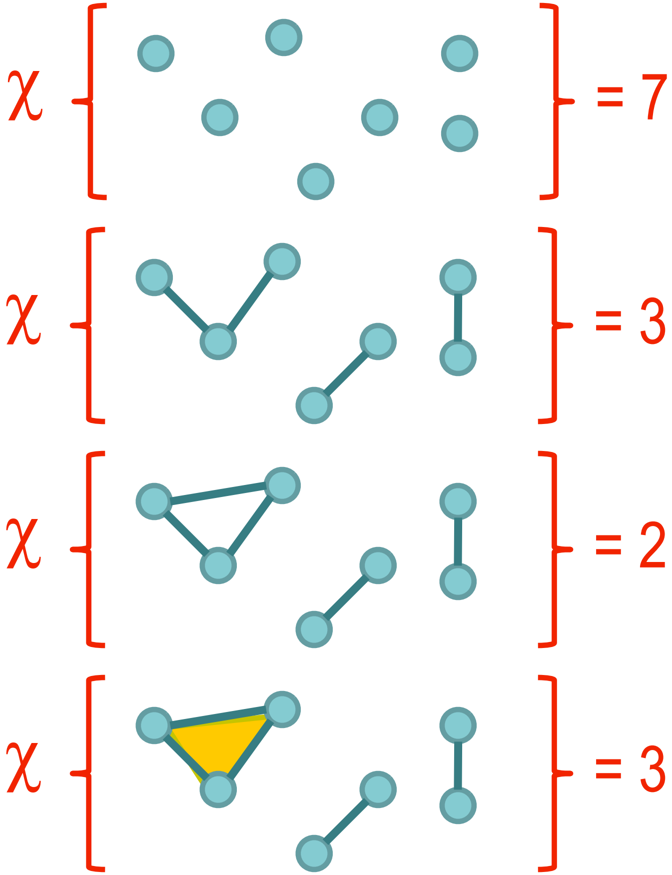
vertices & edges & faces & …

simplicial complex





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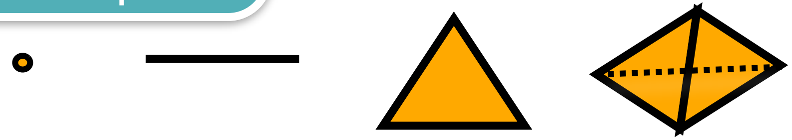
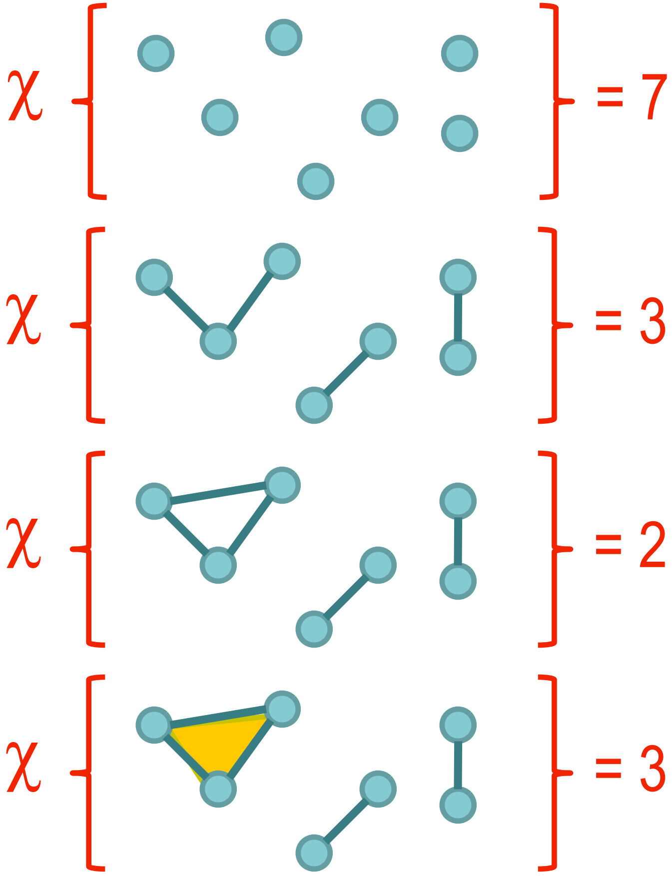


$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$

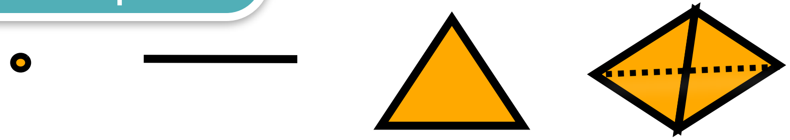


Euler characteristic

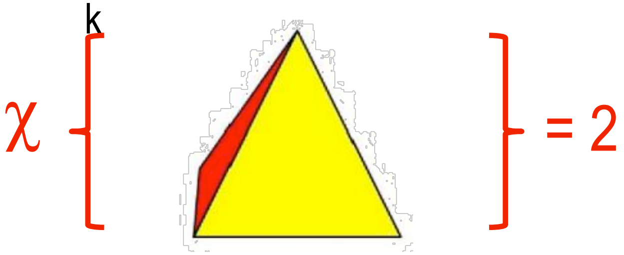
Let’s count a collection of…

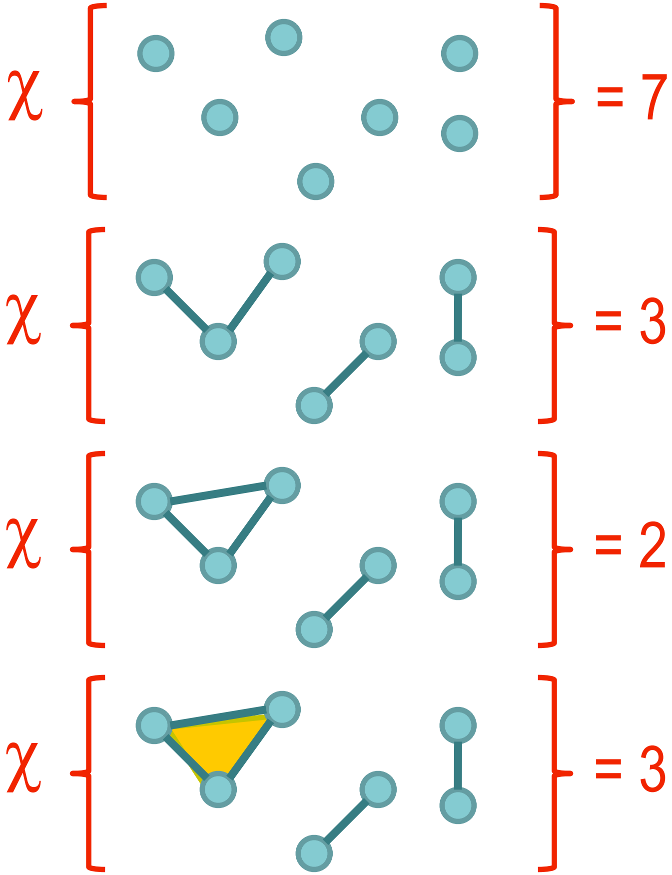
vertices & edges & faces & …

simplicial complex



$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$





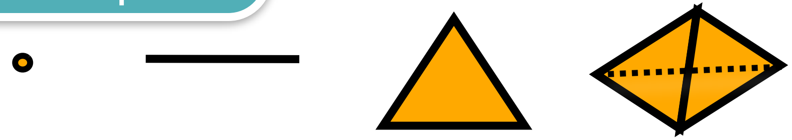
18th c.

Euler characteristic

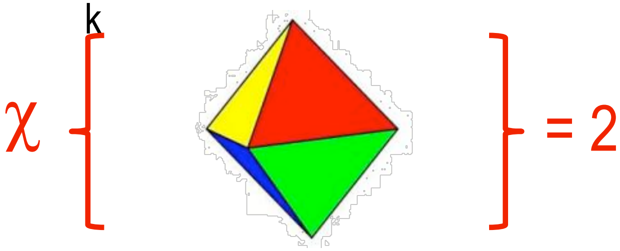
Let's count a collection of…

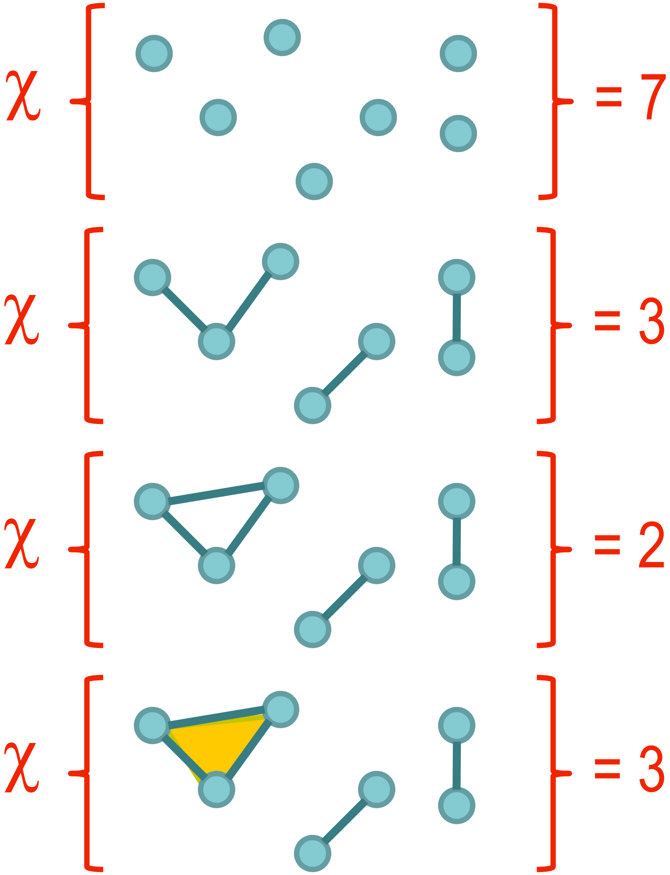
vertices & edges & faces & …

simplicial complex



$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$





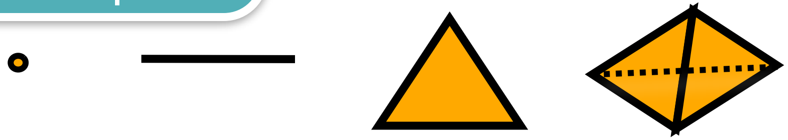
18th c.

Euler characteristic

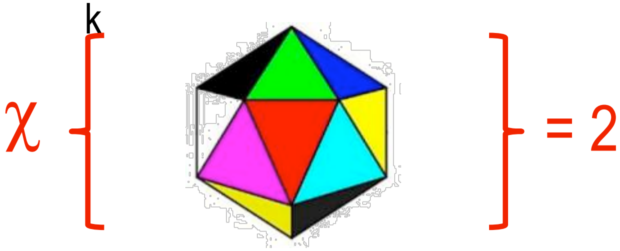
Let's count a collection of…

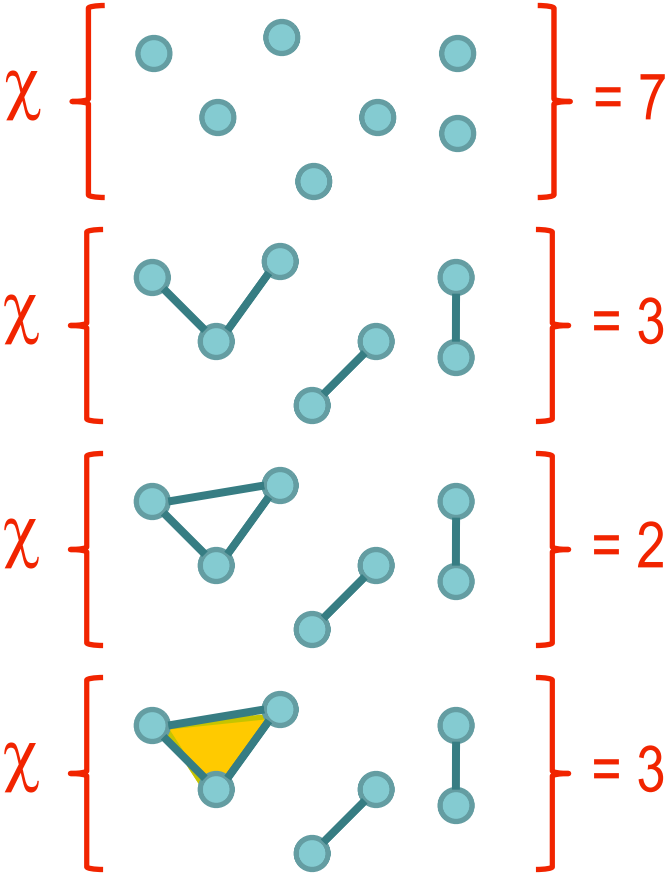
vertices & edges & faces & …

simplicial complex



$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$





18th c.

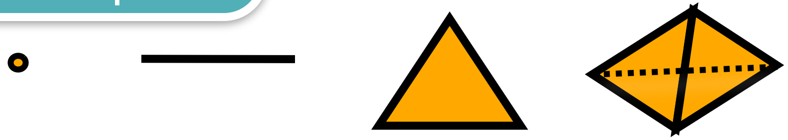
Homology

Euler characteristic

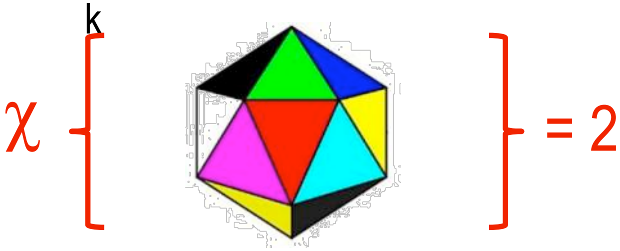
Let's count a collection of…

vertices & edges & faces & …

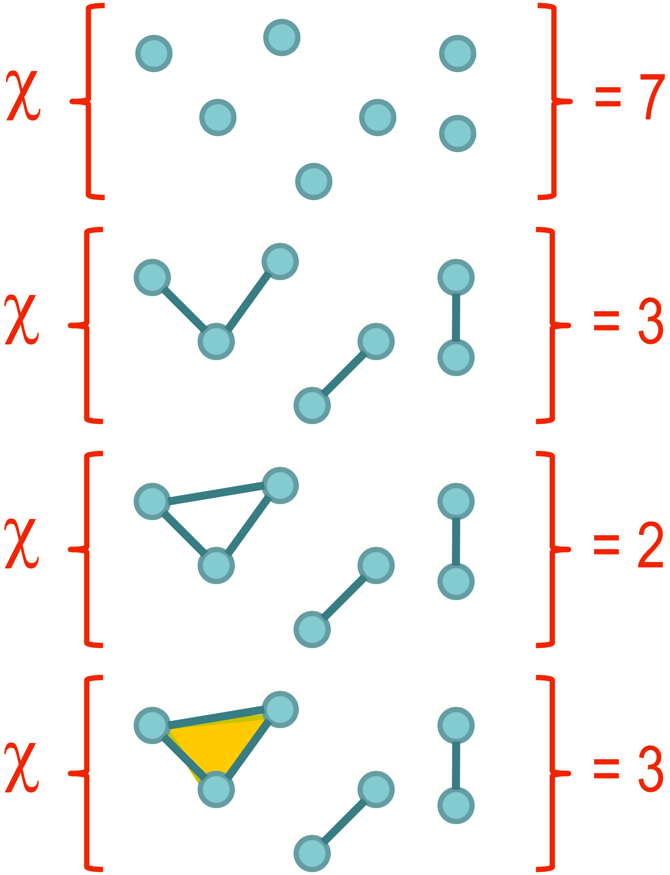
simplicial complex



$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$



Invariant under decomposition



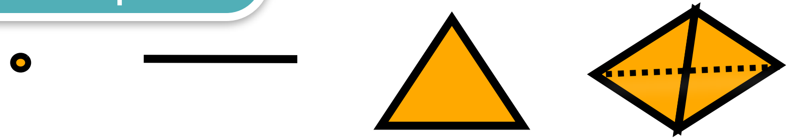
18th c.

Euler characteristic

Let’s count a collection of…

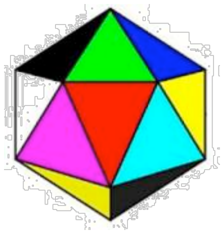
vertices & edges & faces & …

simplicial complex



$$\chi = \sum_k (-1)^k \# \{\text{k-dimensional pieces}\}$$

χ [

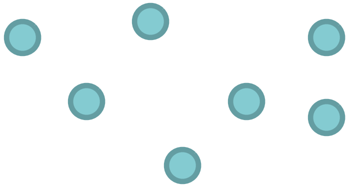


] = 2

Invariant under decomposition

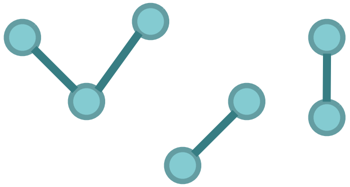
Local computation

χ [



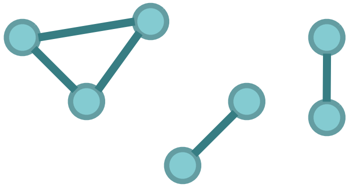
] = 7

χ [



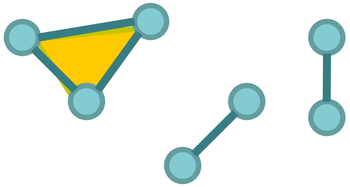
] = 3

χ [



] = 2

χ [



] = 3

20th c.

definition

Algebraic Homology

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

20th c.

definition

Algebraic Homology

count

objects

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linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

$0 \quad C_0 \quad C_1 \quad C_2 \quad C_3 \quad \ldots$

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

$0 \quad C_0 \quad C_1 \quad C_2 \quad C_3 \quad \ldots$

boundary maps

20th c.

definition

Algebraic Homology

count

objects

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linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$

boundary maps

20th c.

definition

Algebraic Homology

count

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chain complex

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boundary maps

$$\partial\partial = 0$$

20th c.

definition

Algebraic Homology

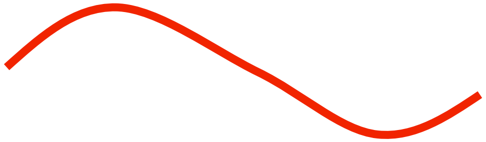
count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects



chain complex

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$

boundary maps

$$\partial\partial = 0$$

20th c.

definition

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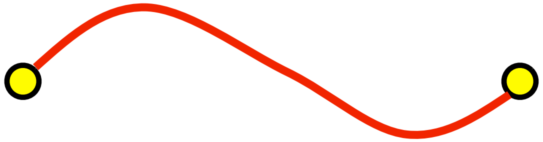
boundary maps

$$\partial\partial = 0$$



20th c.

definition

Algebraic Homology

count

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chain complex

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boundary maps

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20th c.

definition

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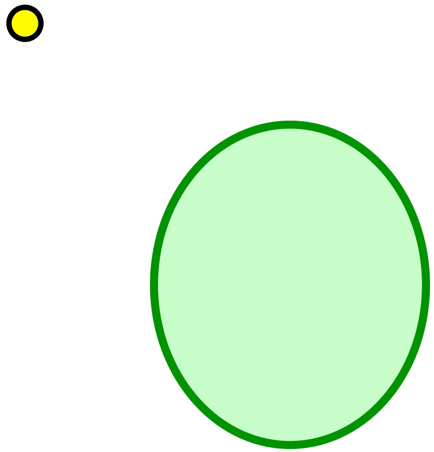
boundary maps

$$\partial\partial = 0$$



20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

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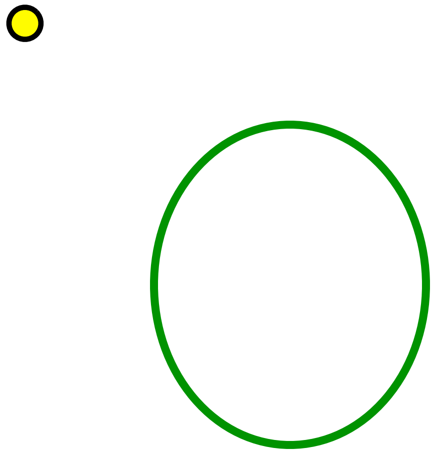
boundary maps

$$\partial\partial = 0$$



20th c.

definition

Algebraic Homology

count

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linear maps

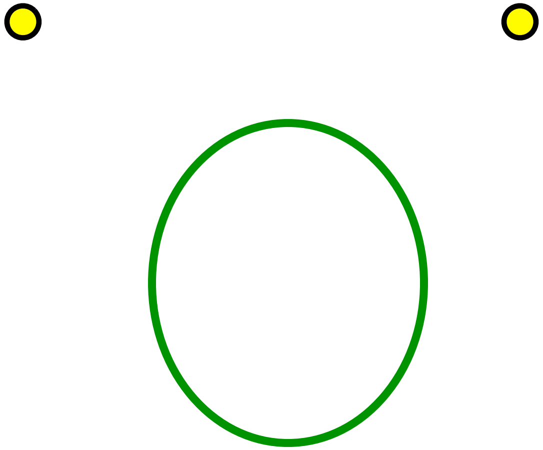
Homology is the algebraic means of counting and cancelling objects

chain complex

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$

boundary maps

$$\partial\partial = 0$$



curl(grad(f)) = 0

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

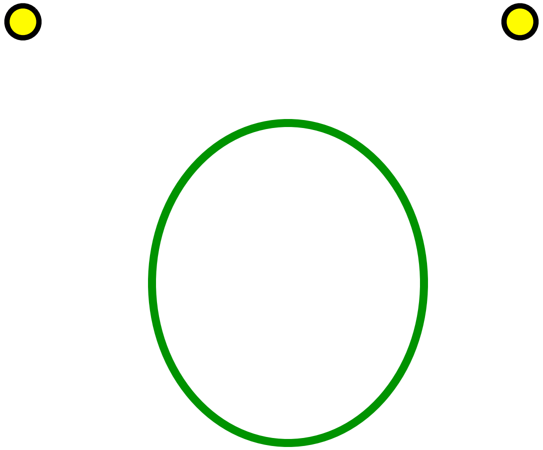
Homology is the algebraic means of counting and cancelling objects

chain complex

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$

boundary maps

$$\partial\partial = 0$$



curl(grad(f)) = 0

div(curl(V)) = 0

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

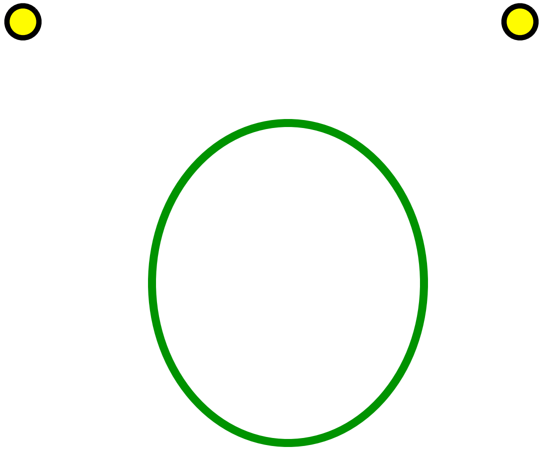
$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$

boundary maps

$$\partial\partial = 0$$

definintion

$$H_k = \ker \partial \,/\, \operatorname{im} \partial$$



curl(grad(f)) = 0

div(curl(V)) = 0

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$

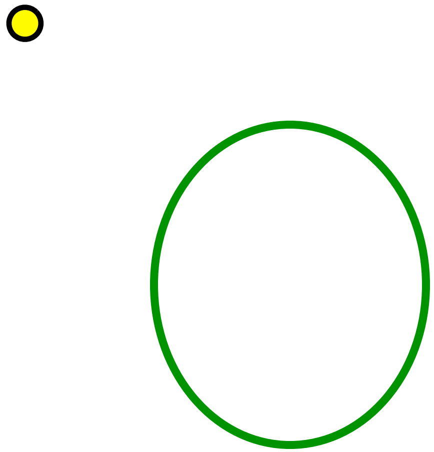
boundary maps

$$\partial\partial = 0$$

cycles

definintion

$$H_k = \ker \partial \, / \, \operatorname{im} \partial$$



curl(grad(f)) = 0

div(curl(V)) = 0

20th c.

definition

Algebraic Homology

count

objects

cancel

linear maps

Homology is the algebraic means of counting and cancelling objects

chain complex

$$\begin{tikzcd} 0 & C_0 \arrow[l, "\partial"'] & C_1 \arrow[l, "\partial"'] & C_2 \arrow[l, "\partial"'] & C_3 \arrow[l, "\partial"'] & \dots \arrow[l, "\partial"'] \end{tikzcd}$$

boundary maps

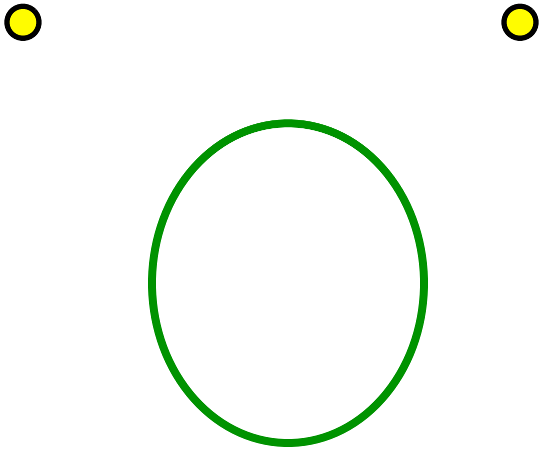
$$\partial\partial = 0$$

cycles

definintion

$$H_k = \ker \partial \, / \, \mathrm{im}\, \partial$$

boundaries



curl(grad(f)) = 0

div(curl(V)) = 0

mid 20th c.

intuition

Homology

count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$

$$H_k = \ker \partial / \operatorname{im} \partial$$

mid 20th c.

intuition

Homology

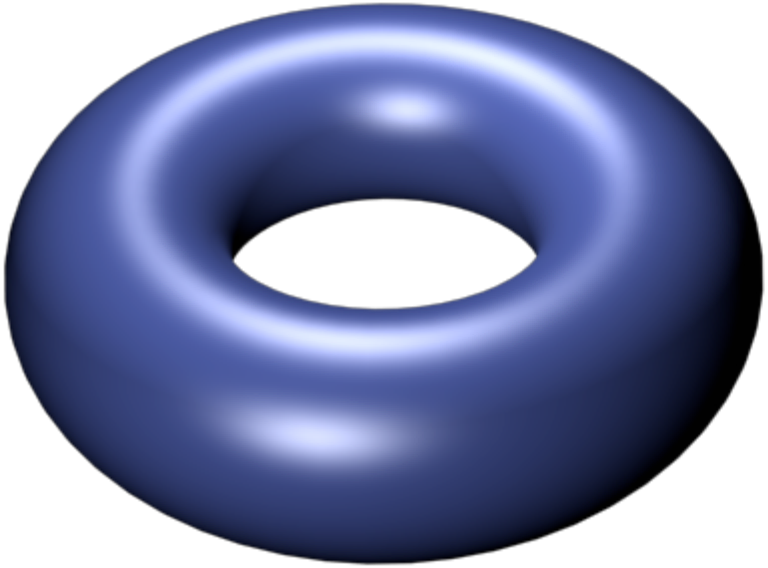
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



$H_k = \ker \partial \,/\, \mathrm{im}\, \partial$

mid 20th c.

intuition

Homology

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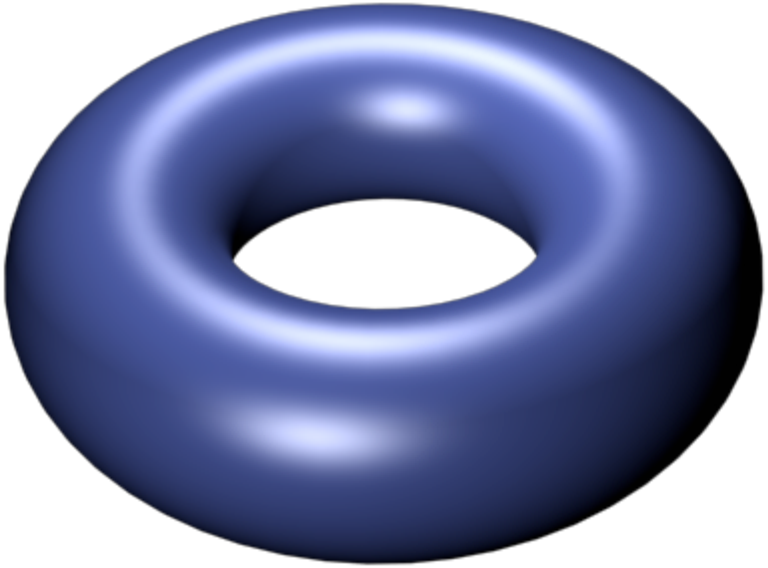
$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



$H_k = \ker \partial \,/\, \mathrm{im}\, \partial$

mid 20th c.

intuition

Homology

count

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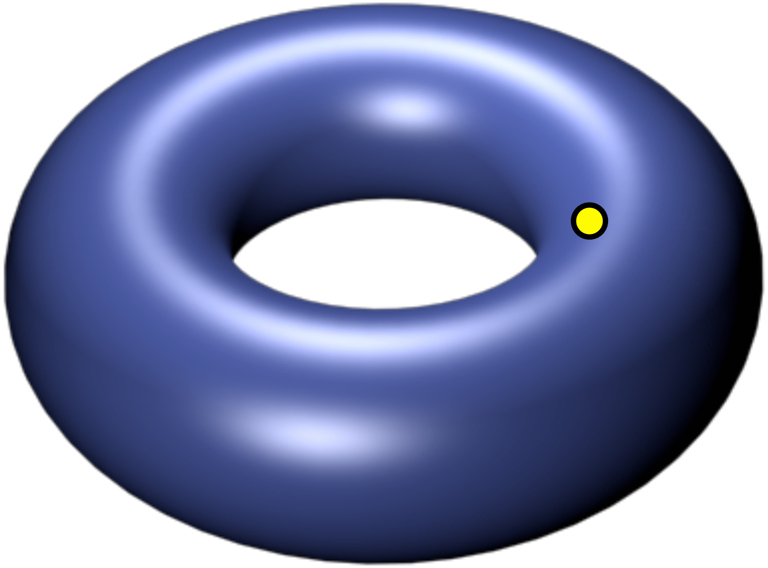
$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



$$H_k = \ker \partial \,/\, \operatorname{im} \partial$$

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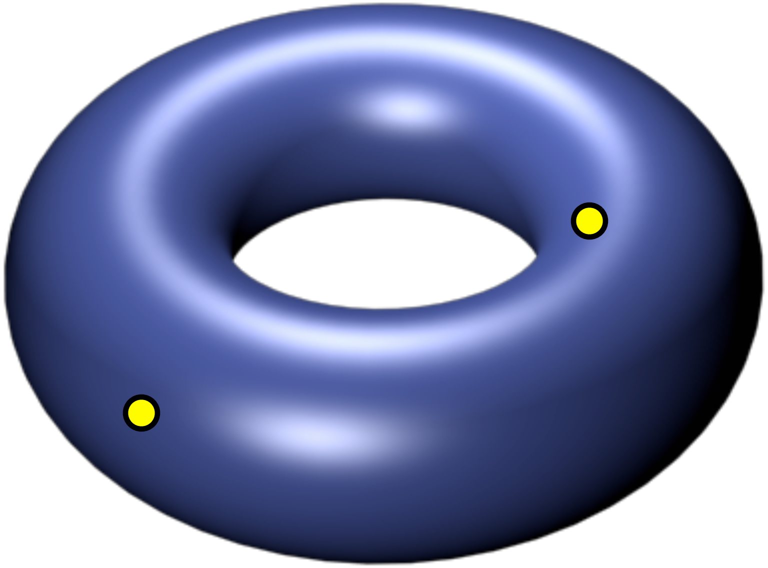

$H_k = \ker \partial \,/\, \operatorname{im} \partial$

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intuition

Homology

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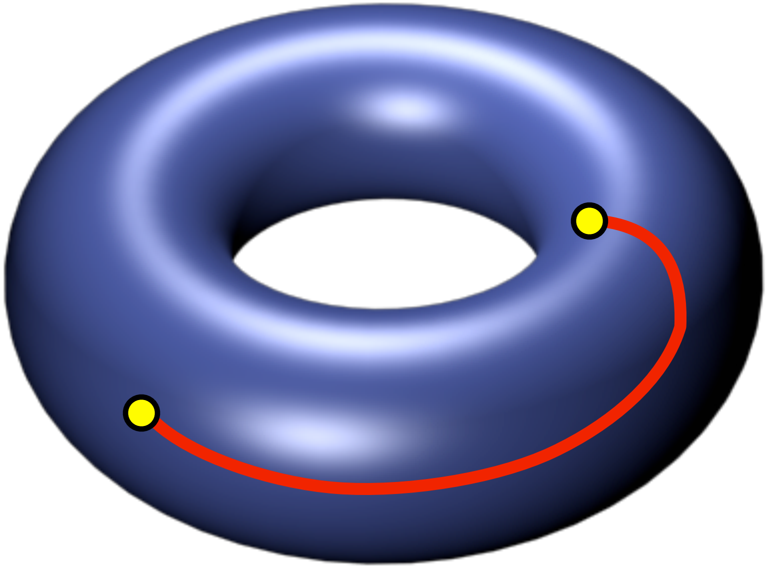


$H_k = \ker \partial \,/\, \operatorname{im} \partial$

mid 20th c.

intuition

Homology

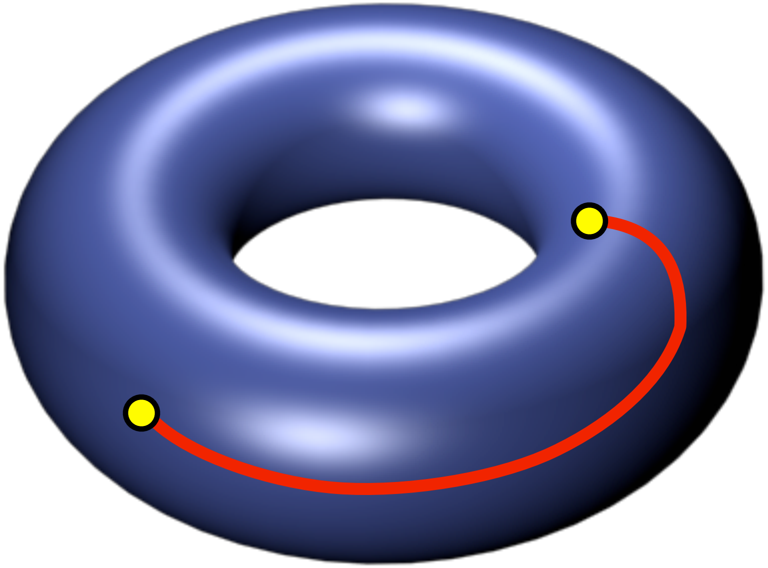
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



dim $H_0 = 1$

$H_k = \ker \partial \,/\, \operatorname{im} \partial$

mid 20th c.

intuition

Homology

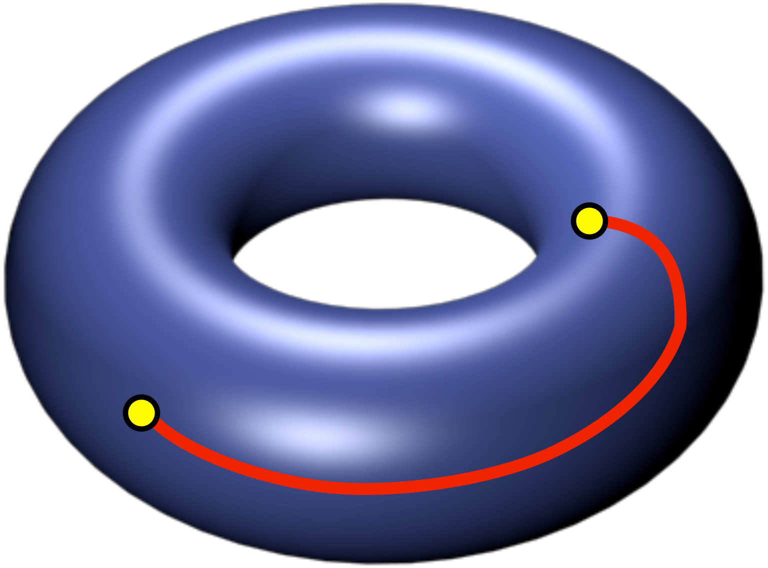
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} \boxed{C_0} \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



$\dim H_0 = 1$

connectivity

$H_k = \ker \partial \,/\, \operatorname{im} \partial$

mid 20th c.

intuition

Homology

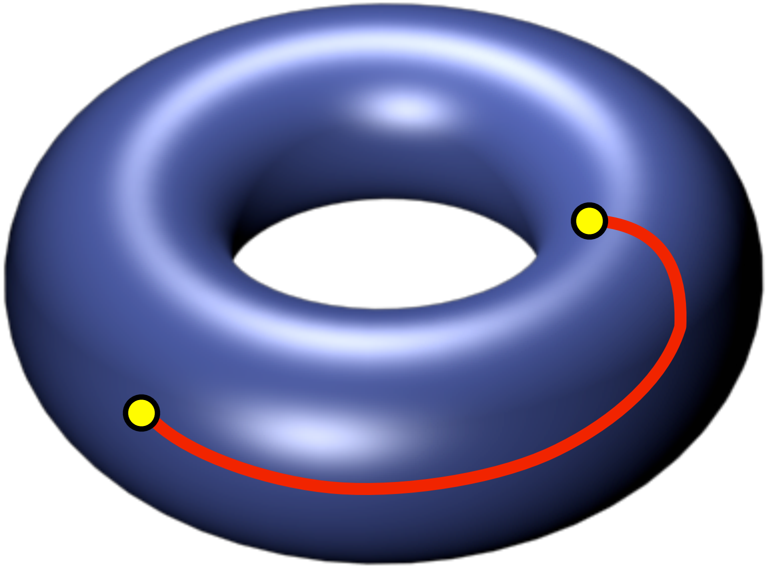
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} \boxed{C_1} \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



$\dim H_0 = 1$

connectivity

$H_k = \ker \partial / \operatorname{im} \partial$

mid 20th c.

intuition

Homology

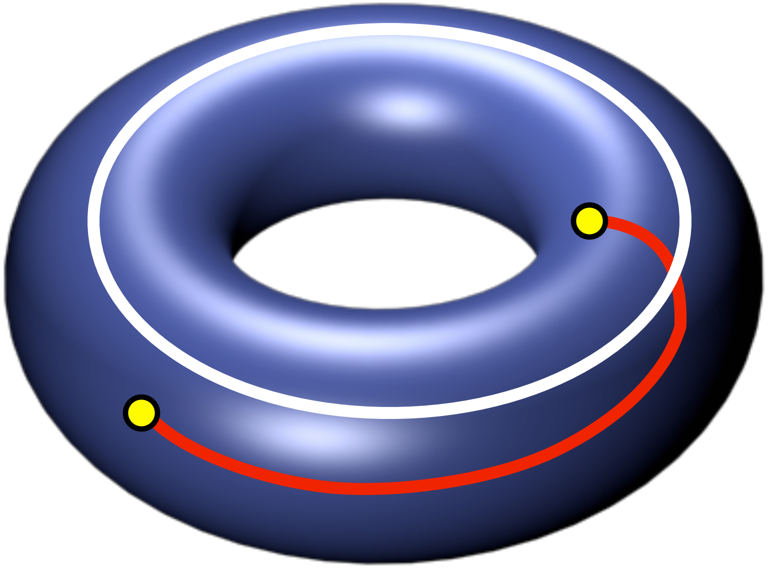
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



dim $H_0 = 1$

connectivity

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mid 20th c.

intuition

Homology

count

points, loops, …

cancel

boundaries

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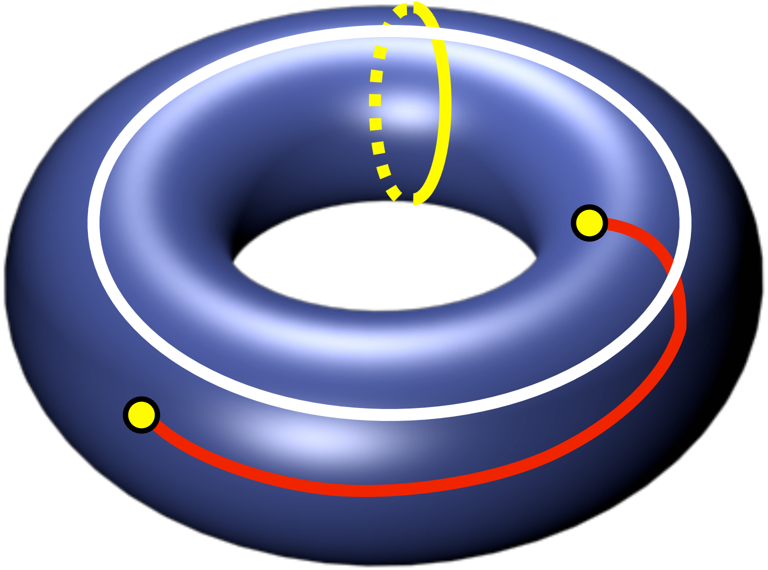

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connectivity

$H_k = \ker \partial \,/\, \operatorname{im} \partial$

mid 20th c.

intuition

Homology

count

points, loops, …

cancel

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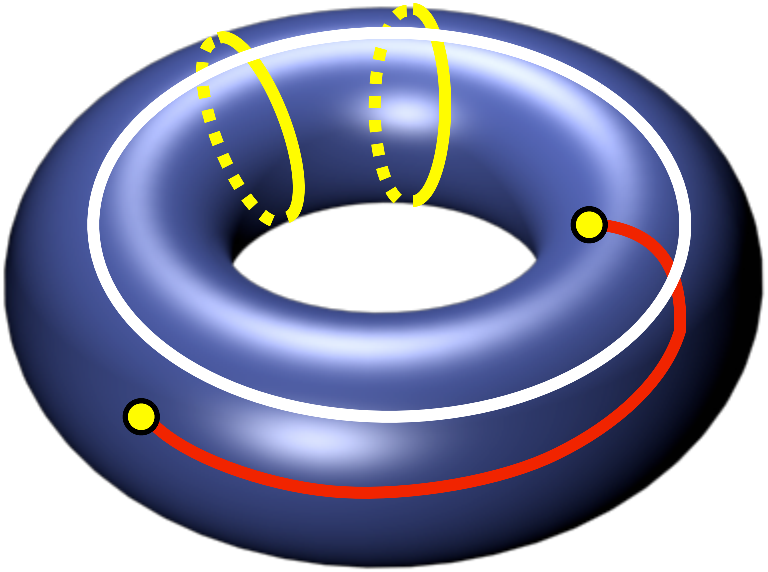
$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



$\dim H_0 = 1$

connectivity

$$H_k = \ker \partial \,/\, \operatorname{im} \partial$$

mid 20th c.

intuition

Homology

count

points, loops, …

cancel

boundaries

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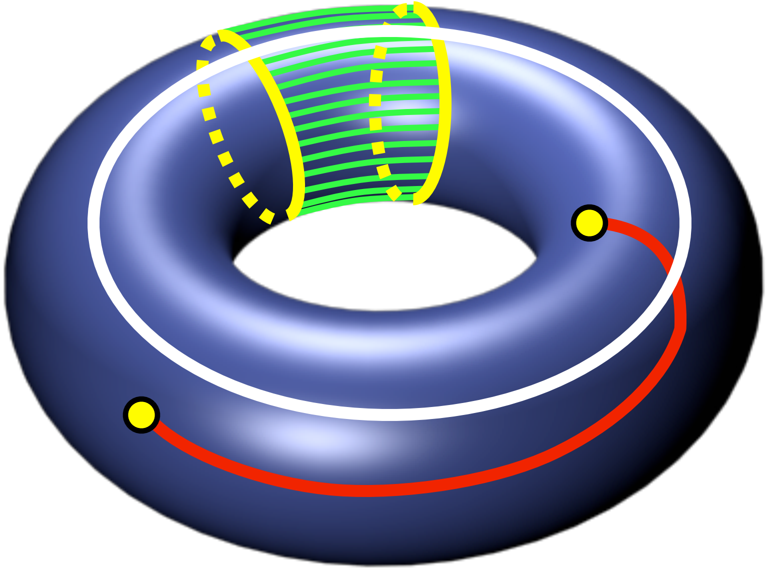


$\dim H_0 = 1$

connectivity

$H_k = \ker \partial \, / \, \mathrm{im} \, \partial$

mid 20th c.

intuition

Homology

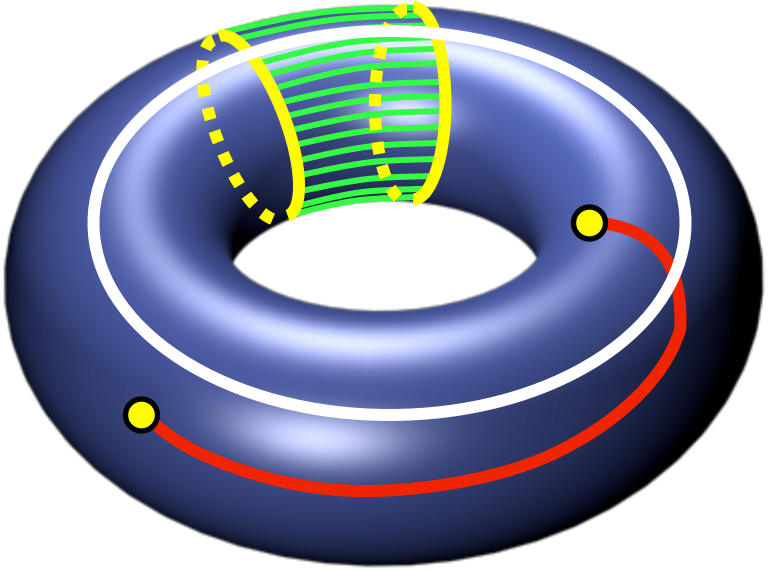
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



$\dim H_0 = 1$

$\dim H_1 = 2$

connectivity

$$H_k = \ker \partial / \operatorname{im} \partial$$

mid 20th c.

intuition

Homology

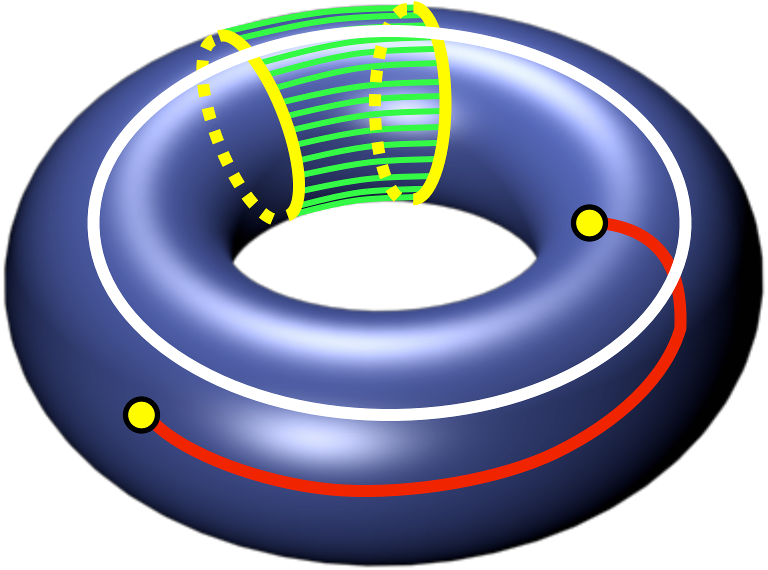
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



$\dim H_0 = 1$ connectivity

$\dim H_1 = 2$ loops

$$H_k = \ker \partial \,/\, \operatorname{im} \partial$$

mid 20th c.

intuition

Homology

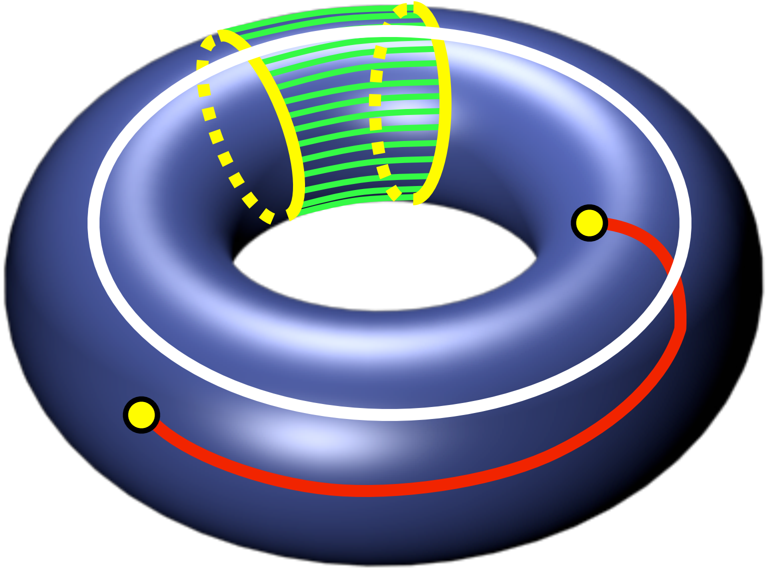
count

points, loops, …

cancel

boundaries

$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$



$\dim H_0 = 1$

$\dim H_1 = 2$

$\dim H_2 = 1$

connectivity

loops

$H_k = \ker \partial / \operatorname{im} \partial$

mid 20th c.

intuition

Homology

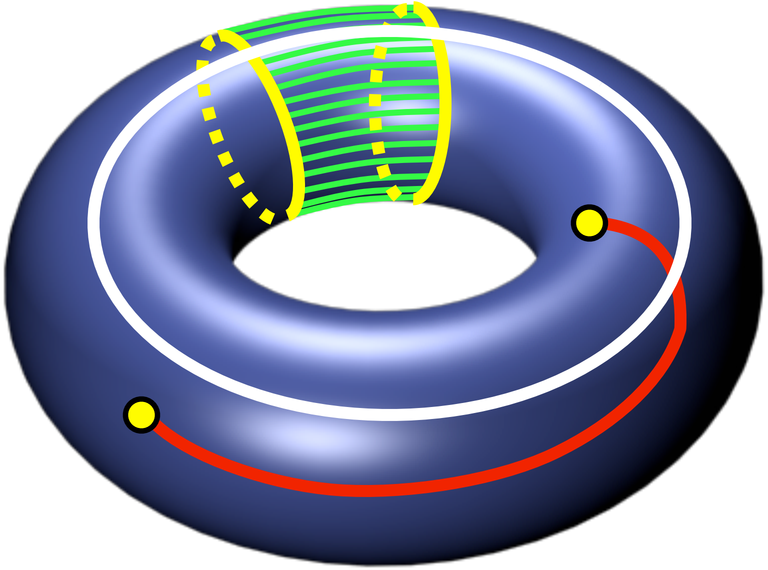
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$



$\dim H_0 = 1$	connectivity
$\dim H_1 = 2$	loops
$\dim H_2 = 1$	surface

$$H_k = \ker \partial \, / \, \operatorname{im} \partial$$

mid 20th c.

intuition

Homology

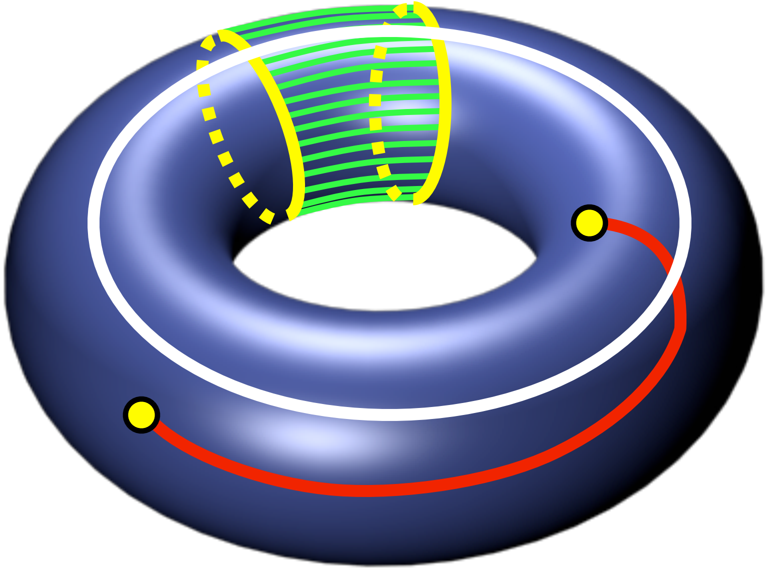
count

points, loops, …

cancel

boundaries

$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} \boxed{C_1} \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \ldots$$



$\dim H_0 = 1$ connectivity

$\dim H_1 = 2$ loops

$\dim H_2 = 1$ surface

$H_{k>2} = 0$

$H_k = \ker \partial \, / \, \operatorname{im} \partial$

20th c.

example

Cellular Homology

count

cancel

20th c.

example

Cellular Homology

count

cells of dimension k

cancel

20th c.

example

Cellular Homology

count

cells of dimension k

cancel

boundaries

20th c.

example

Cellular Homology

count

cells of dimension k

cancel

boundaries

simplicial complex

20th c.

example

Cellular Homology

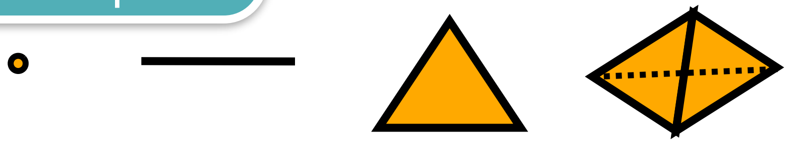
count

cells of dimension k

cancel

boundaries

simplicial complex



20th c.

example

Cellular Homology

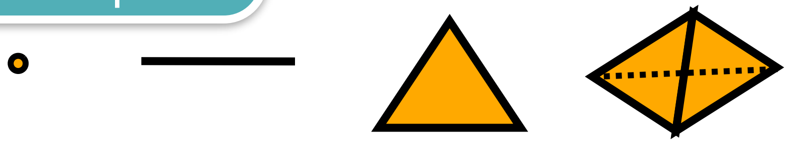
count

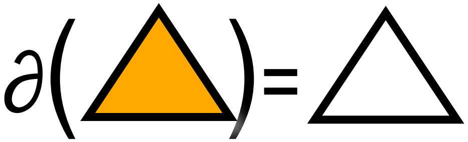
cells of dimension k

cancel

boundaries

simplicial complex





20th c.

example

Cellular Homology

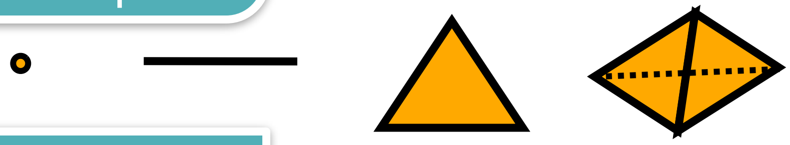
count

cells of dimension k

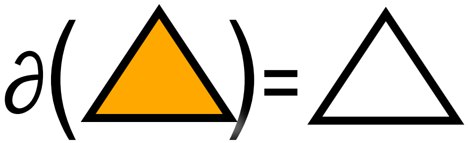
cancel

boundaries

simplicial complex



cubical complex



$\partial(▲)=△$

20th c.

example

Cellular Homology

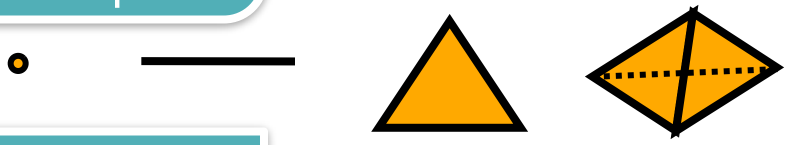
count

cells of dimension k

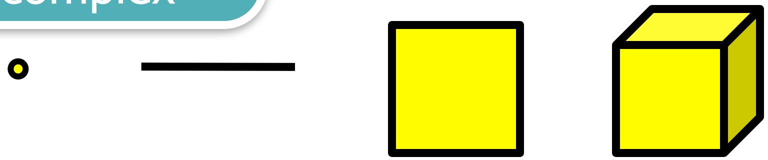
cancel

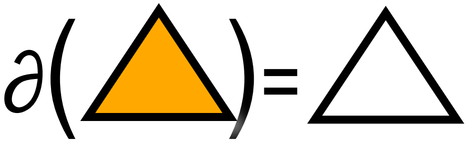
boundaries

simplicial complex



cubical complex





Cellular Homology

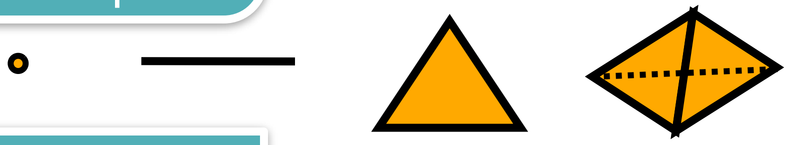
count

cells of dimension k

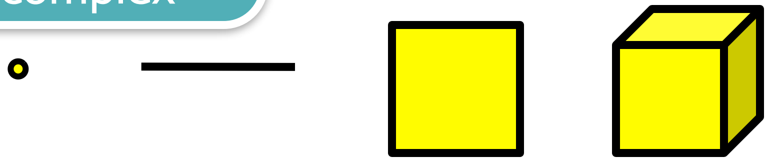
cancel

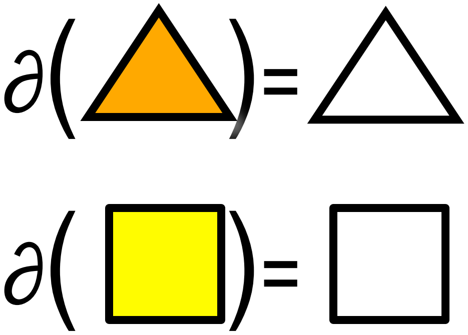
boundaries

simplicial complex



cubical complex





20th c.

example

Cellular Homology

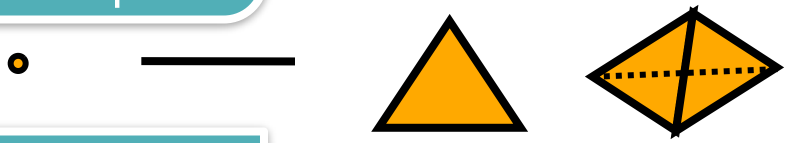
count

cells of dimension k

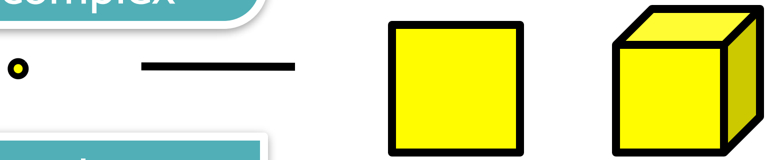
cancel

boundaries

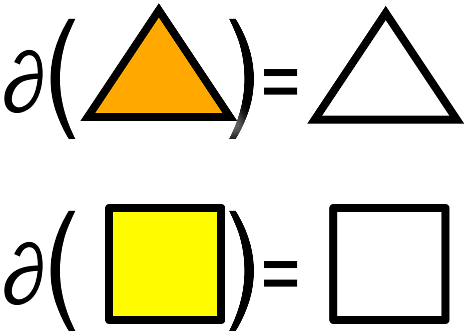
simplicial complex



cubical complex



cell complex



20th c.

example

Cellular Homology

count

cells of dimension k

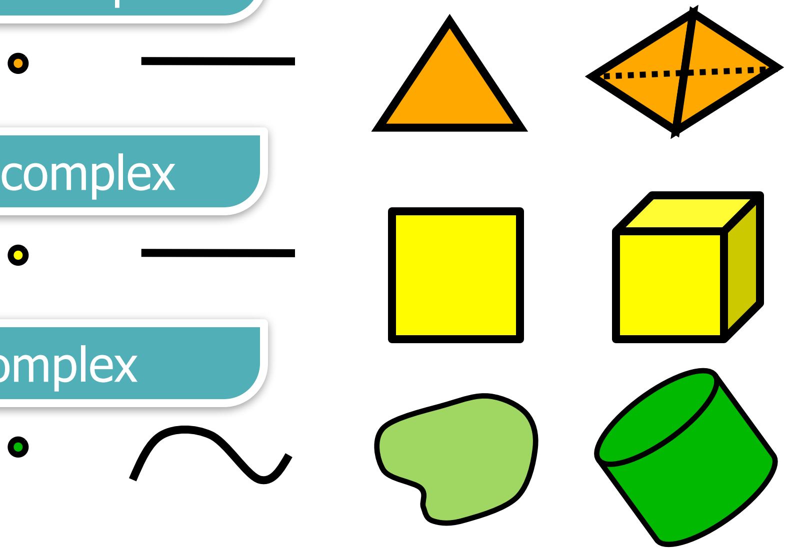
cancel

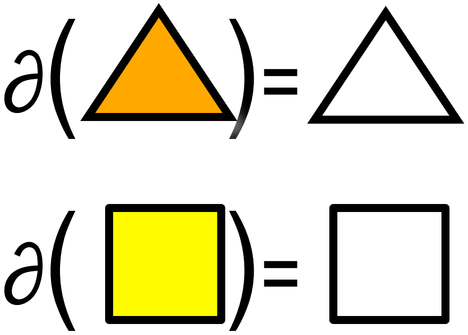
boundaries

simplicial complex

cubical complex

cell complex





20th c.

example

Cellular Homology

count

cells of dimension k

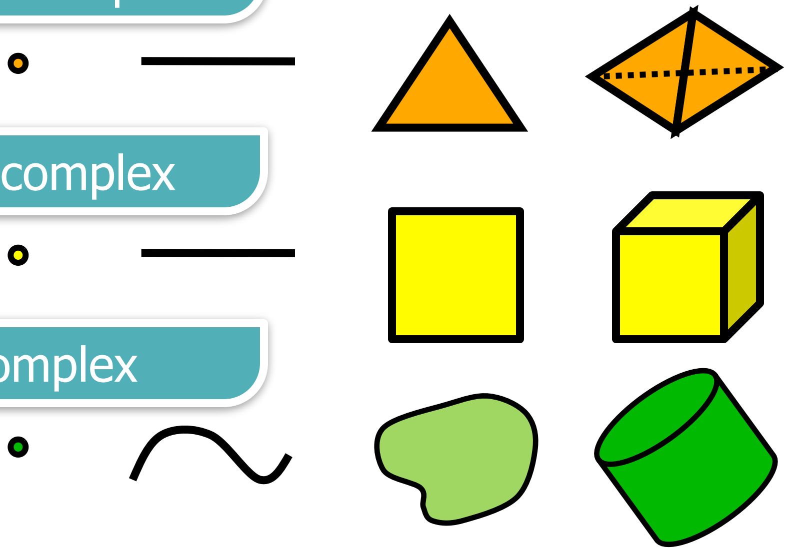
cancel

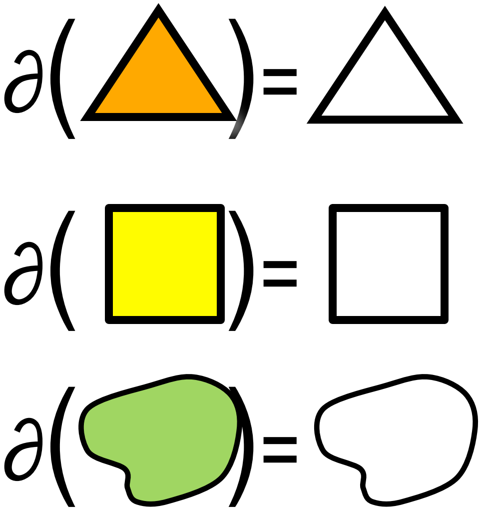
boundaries

simplicial complex

cubical complex

cell complex





20th c.

example

Cellular Homology

count

cells of dimension k

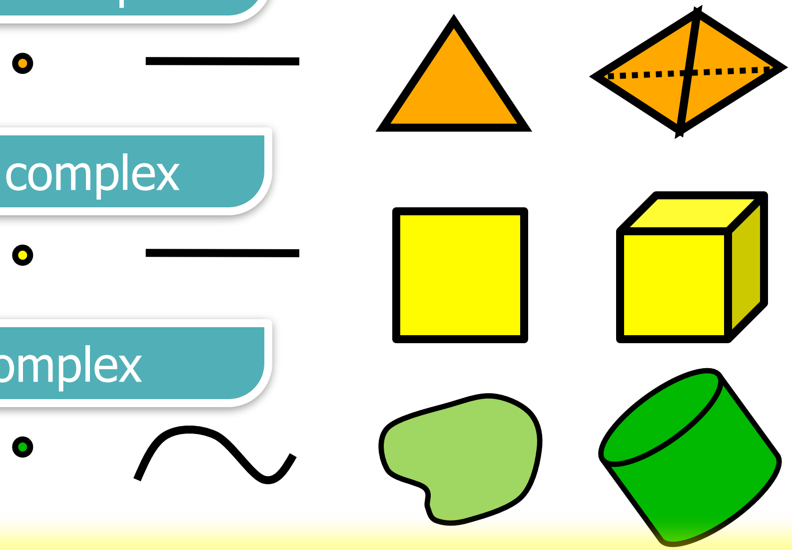
cancel

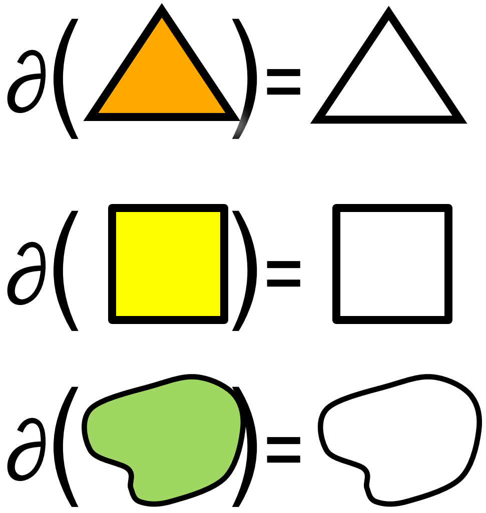
boundaries

simplicial complex

cubical complex

cell complex





theorem

cellular homology independent of cells

20th c.

example

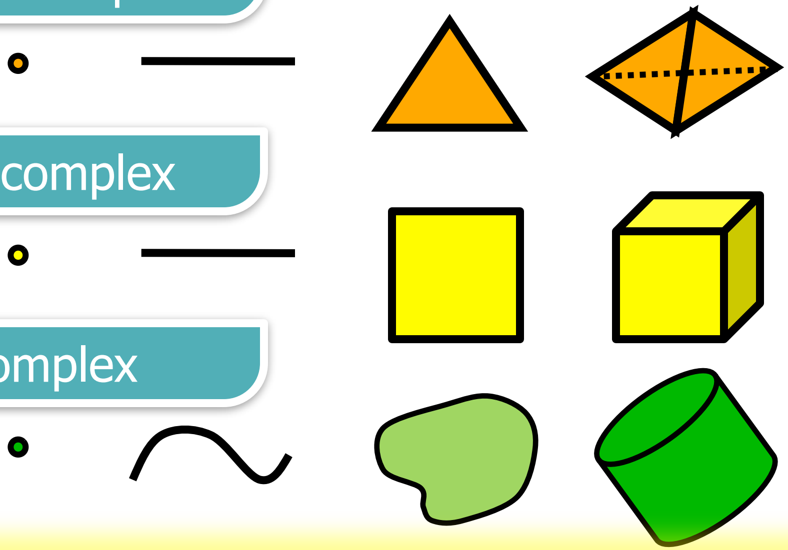
Cellular Homology

count

cells of dimension k

cancel

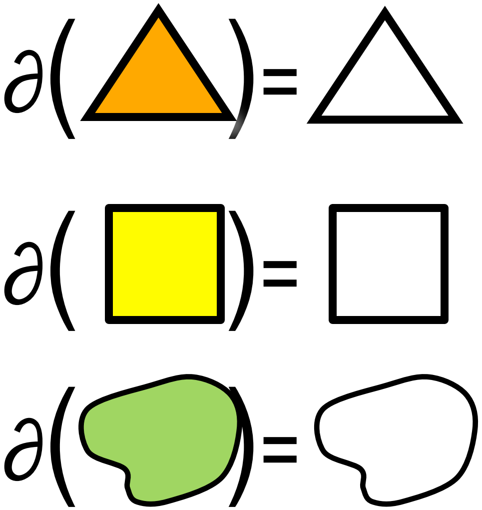
boundaries



simplicial complex

cubical complex

cell complex



$$\chi = \sum_k (-1)^k \dim C_k$$

theorem

cellular homology independent of cells

20th c.

example

Cellular Homology

count

cells of dimension k

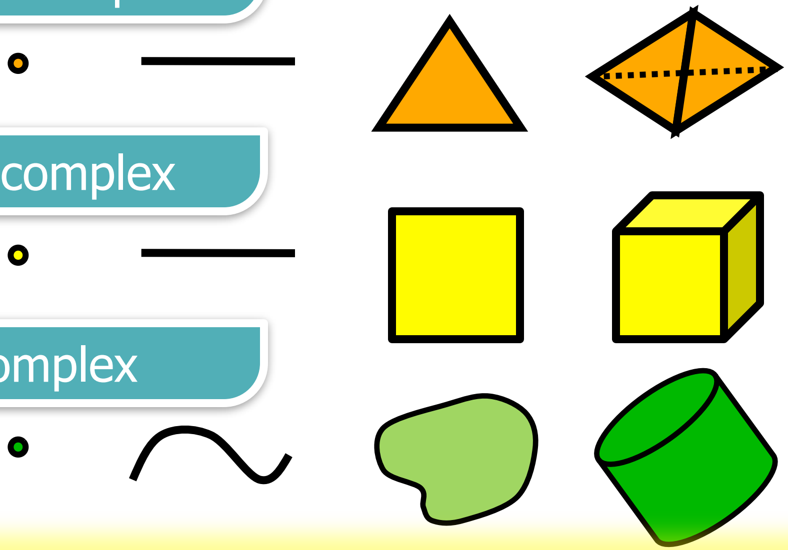
cancel

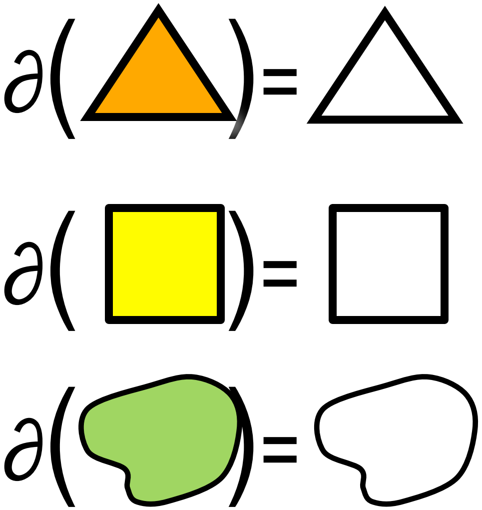
boundaries

simplicial complex

cubical complex

cell complex





theorem

cellular homology independent of cells

$$\chi = \sum_k (-1)^k \dim C_k$$

$$= \sum_k (-1)^k \dim H_k$$

mid 20th c.

example

Singular Homology

count

cancel

mid 20th c.

example

Singular Homology

count

cancel

singular complex

mid 20th c.

example

Singular Homology

count

cancel

singular complex

mid 20th c.

example

Singular Homology

count

cont. maps of k-cells

cancel

singular complex

mid 20th c.

example

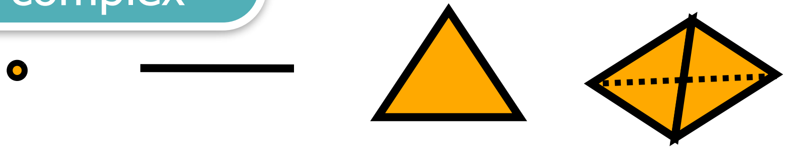
Singular Homology

count

cont. maps of k-cells

cancel

singular complex



mid 20th c.

example

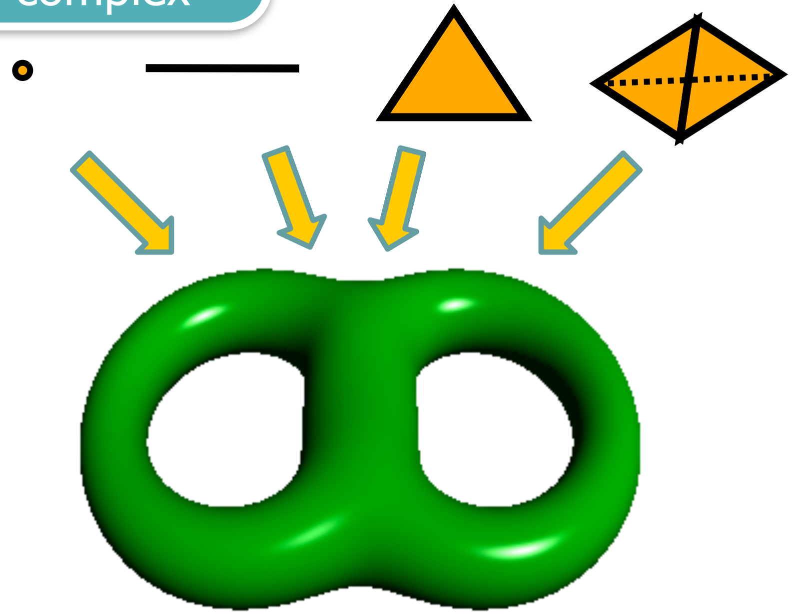
Singular Homology

count

cont. maps of k-cells

cancel

singular complex



mid 20th c.

example

Singular Homology

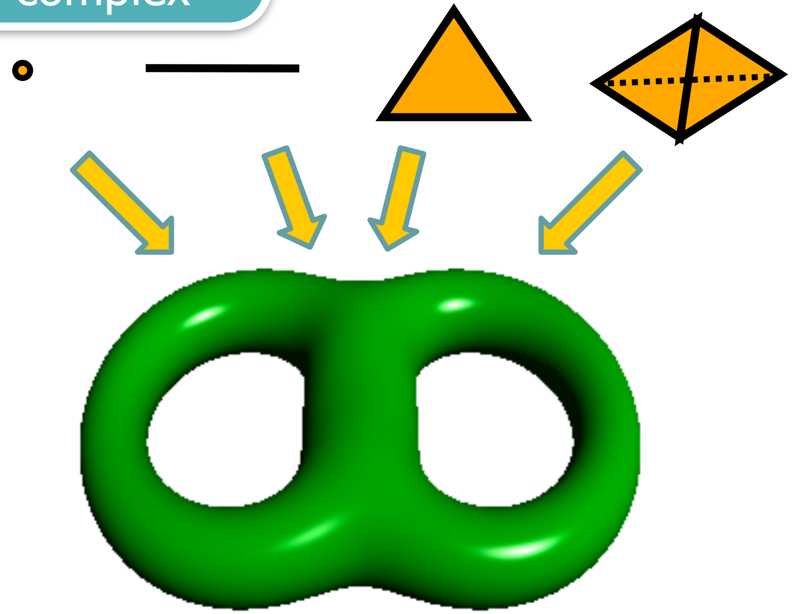
count

cont. maps of k-cells

cancel

maps of boundaries

singular complex



mid 20th c.

example

Singular Homology

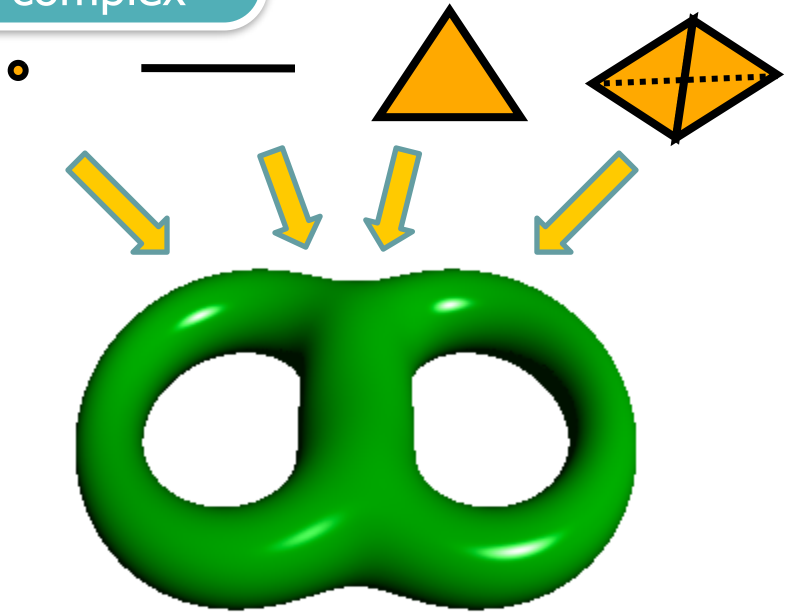
count

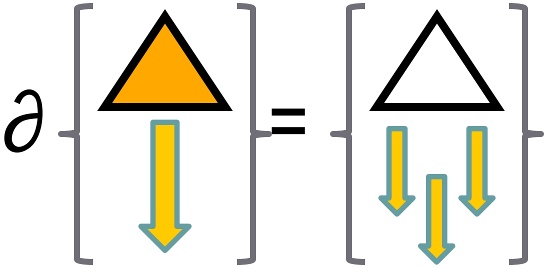
cont. maps of k-cells

cancel

maps of boundaries

singular complex





mid 20th c.

Singular Homology

example

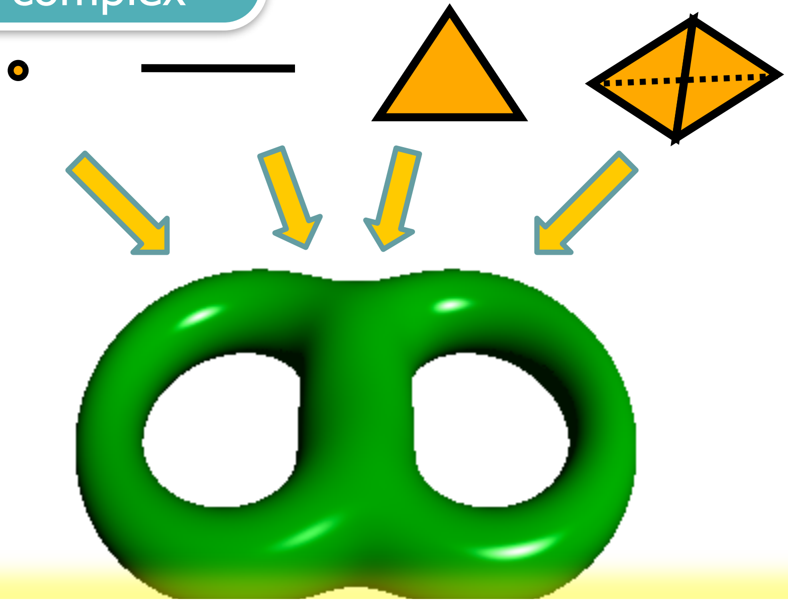
count

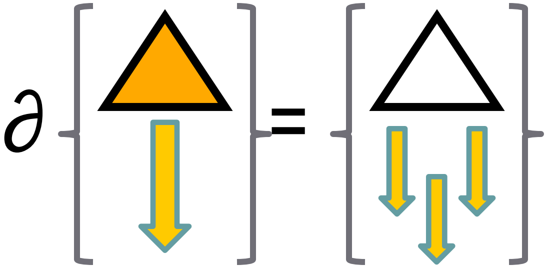
cont. maps of k-cells

cancel

maps of boundaries

singular complex





theorem

singular homology ≈ cellular homology

mid 20th c.

example

Singular Homology

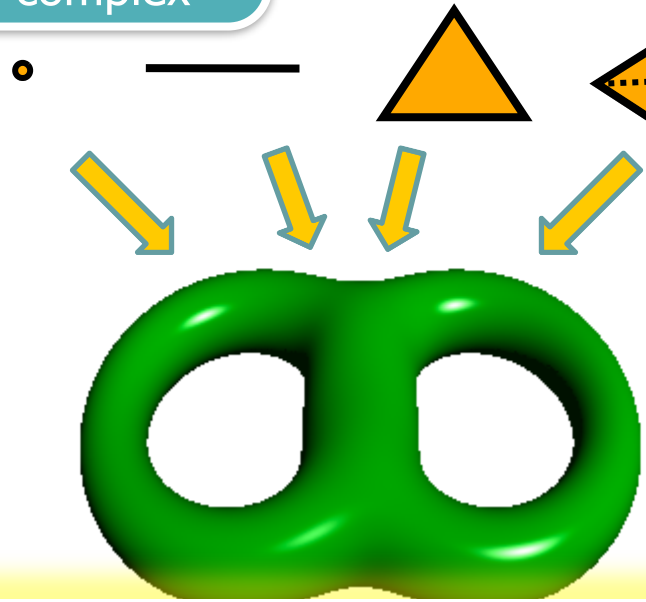
count

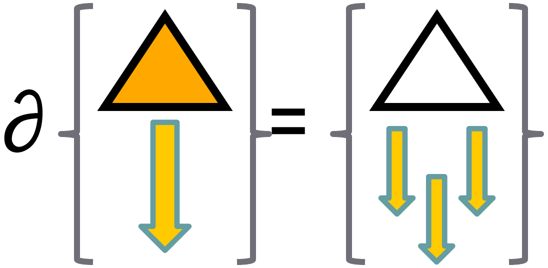
cont. maps of k-cells

cancel

maps of boundaries

singular complex





this construction yields
invariance up to
continuous deformation

theorem

singular homology ≈ cellular homology

mid 20th c.

example

Cech homology

count

cancel

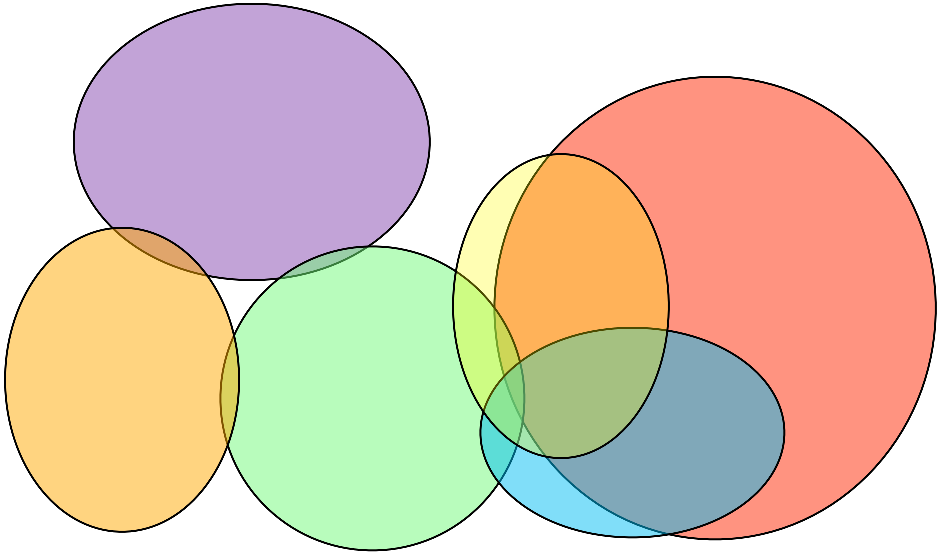
mid 20th c.

example

Cech homology

count

cancel



mid 20th c.

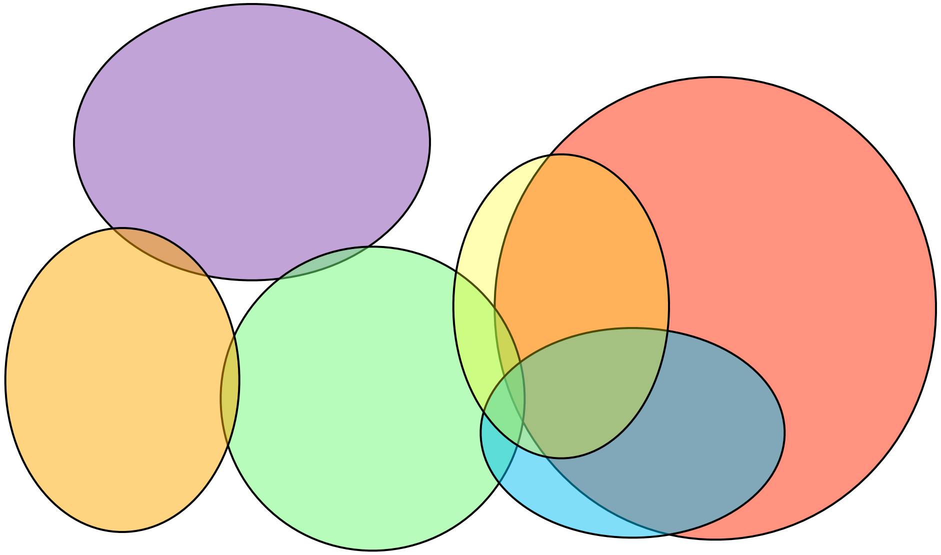
example

Cech homology

count

(convex) sets

cancel



mid 20th c.

example

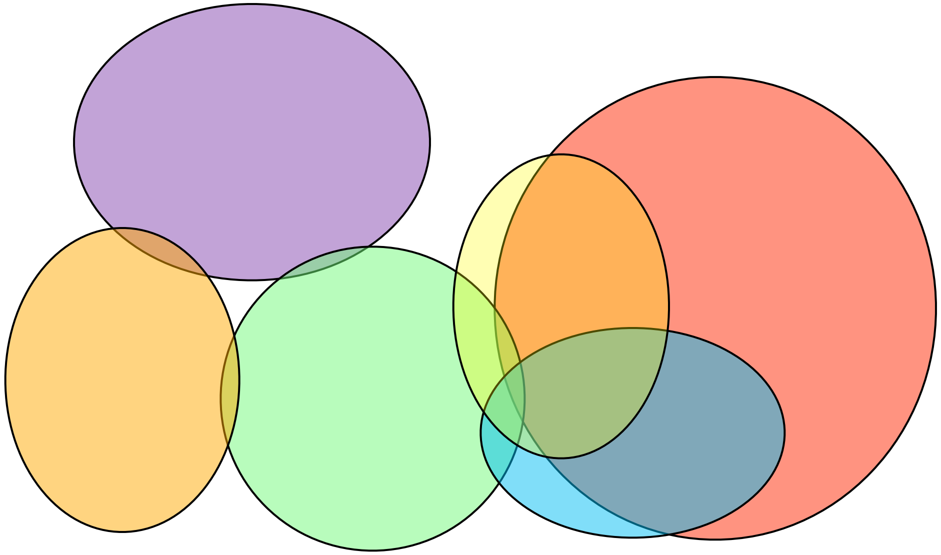
Cech homology

count

(convex) sets

cancel

intersections



mid 20th c.

example

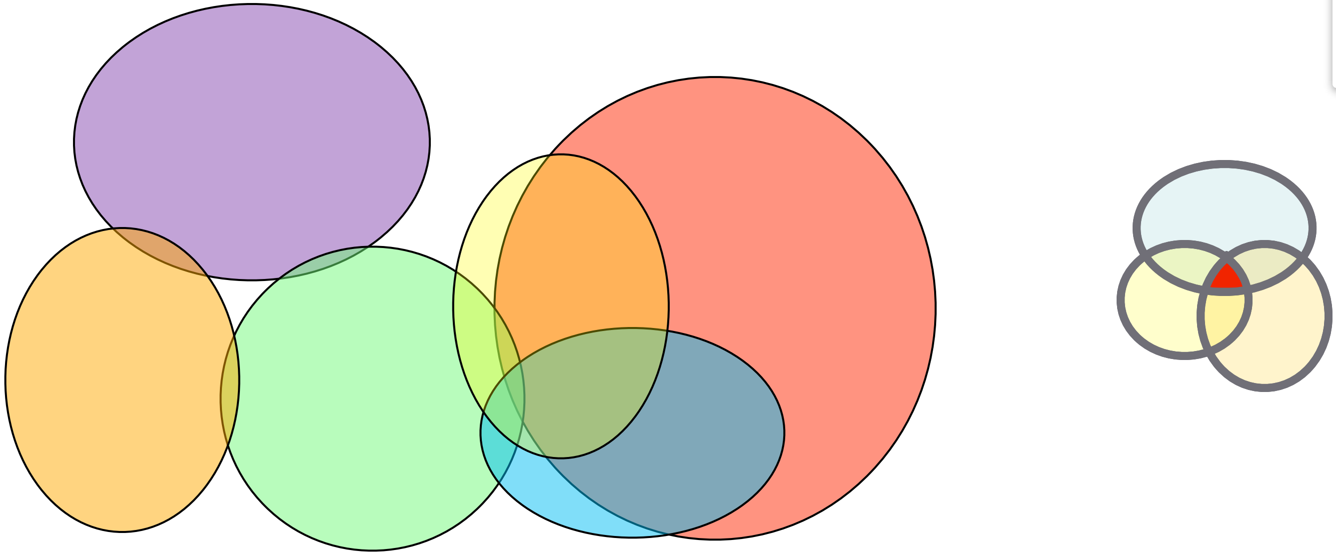
Cech homology

count

(convex) sets

cancel

intersections



mid 20th c.

example

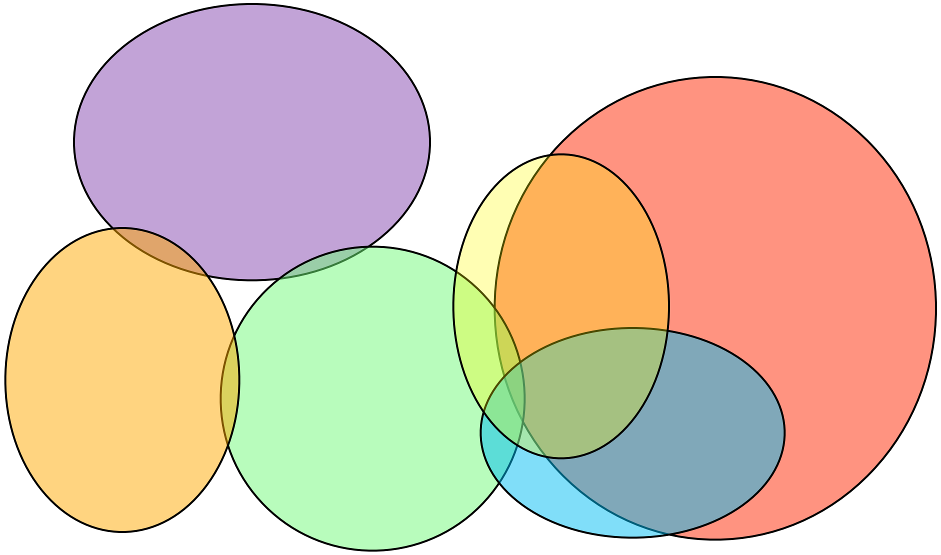
Cech homology

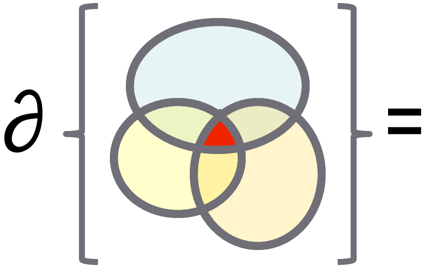
count

(convex) sets

cancel

intersections





mid 20th c.

example

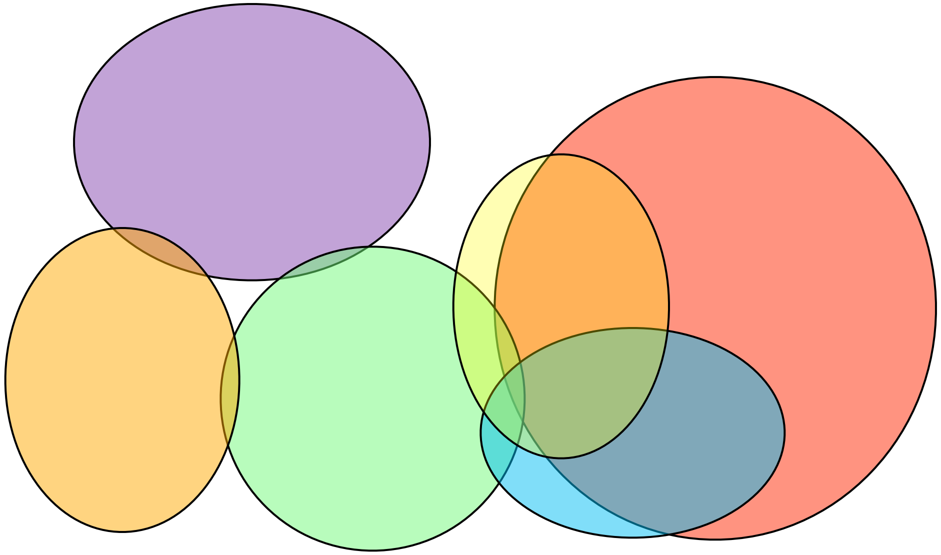
Cech homology

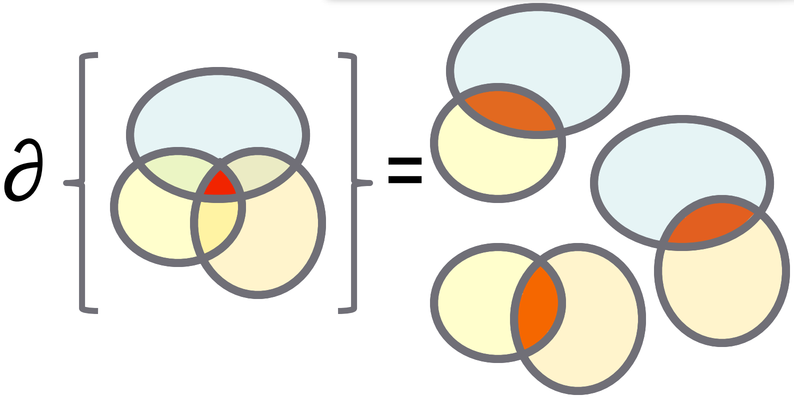
count

(convex) sets

cancel

intersections





mid 20th c.

example

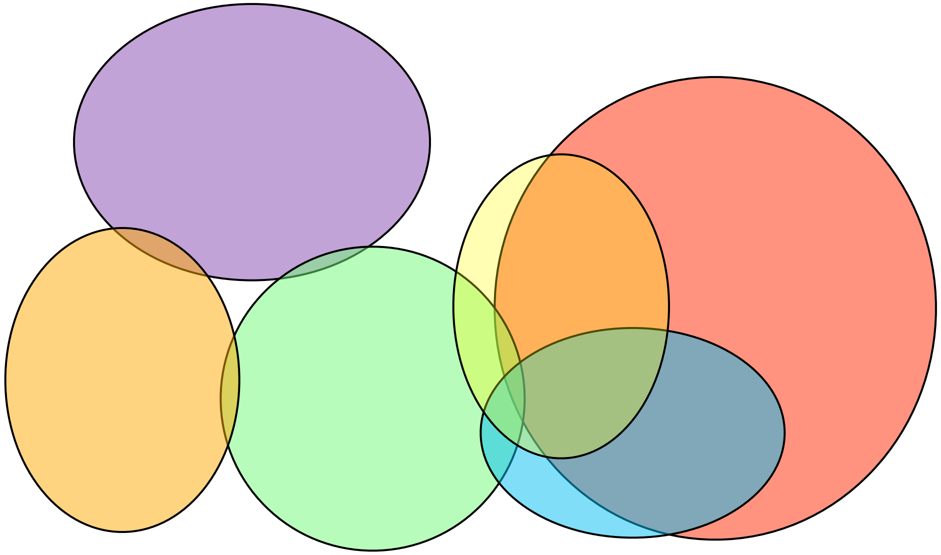
Cech homology

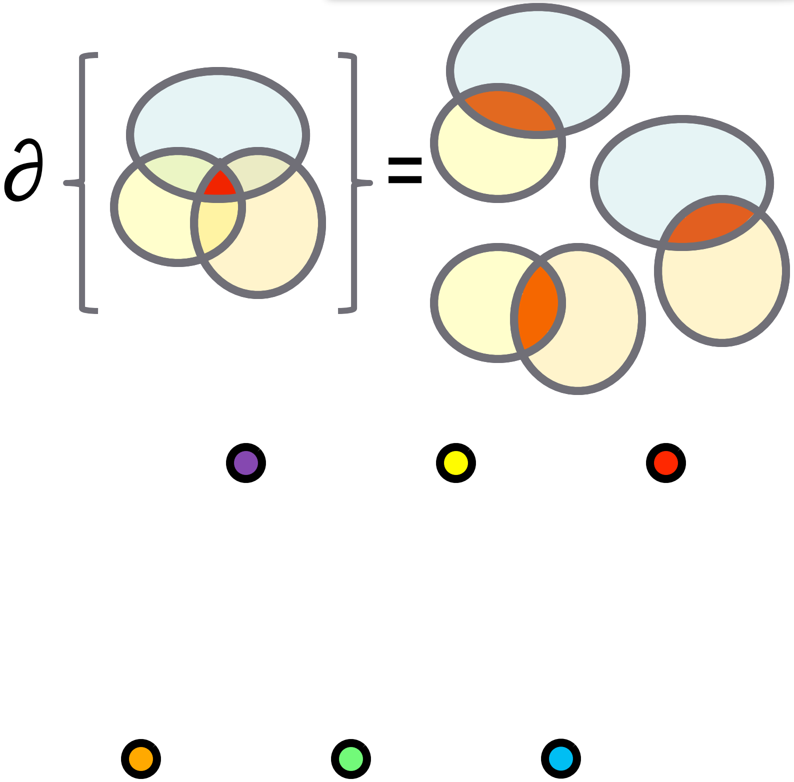
count

(convex) sets

cancel

intersections





mid 20th c.

example

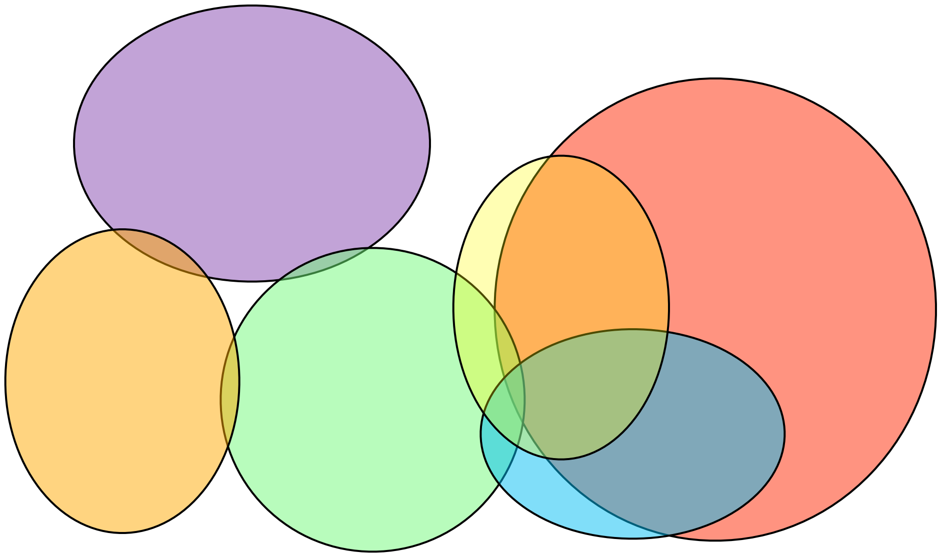
Cech homology

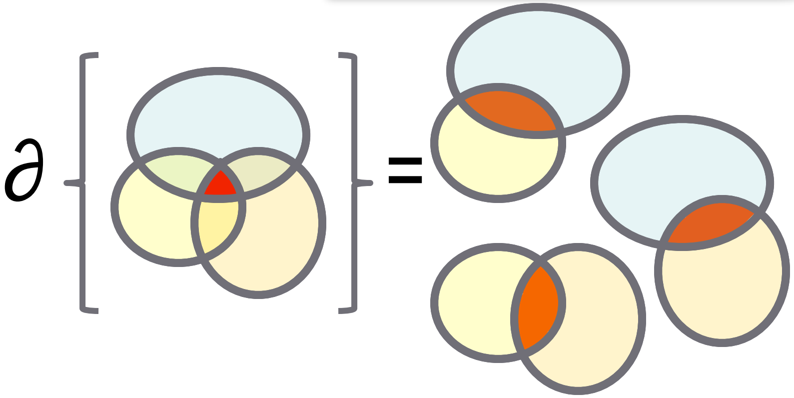
count

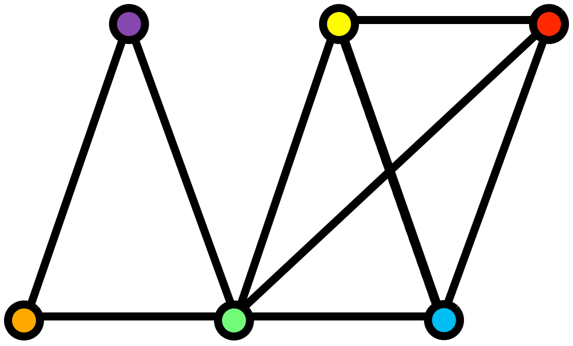
(convex) sets

cancel

intersections







mid 20th c.

example

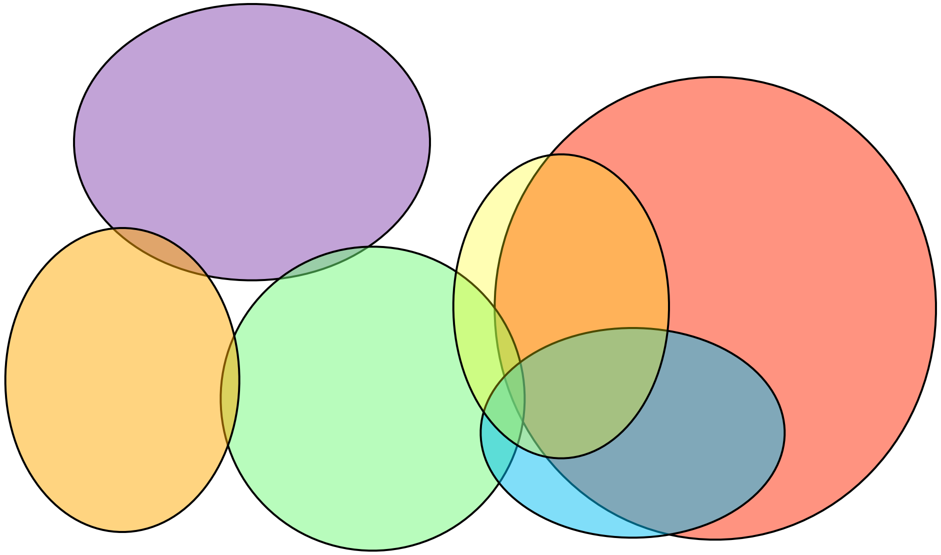
Cech homology

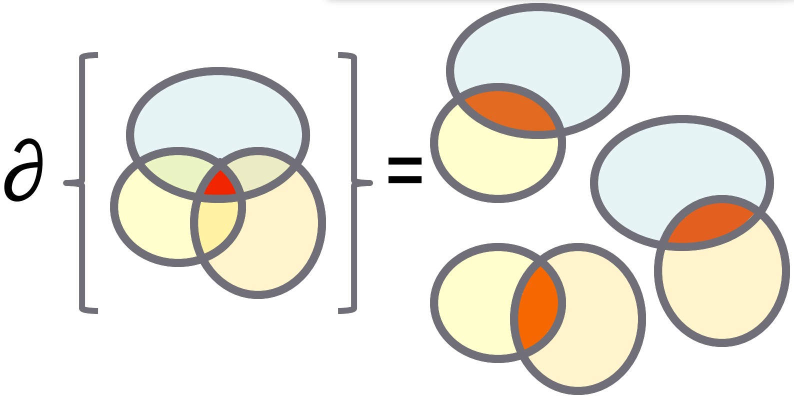
count

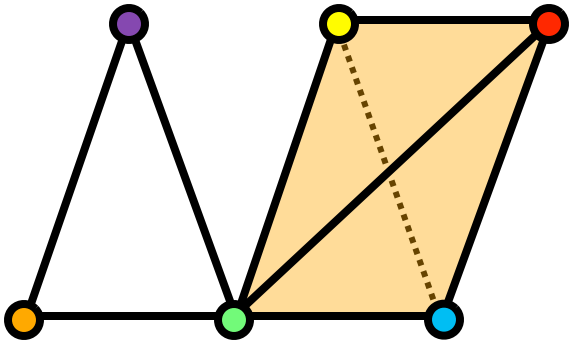
(convex) sets

cancel

intersections







mid 20th c.

example

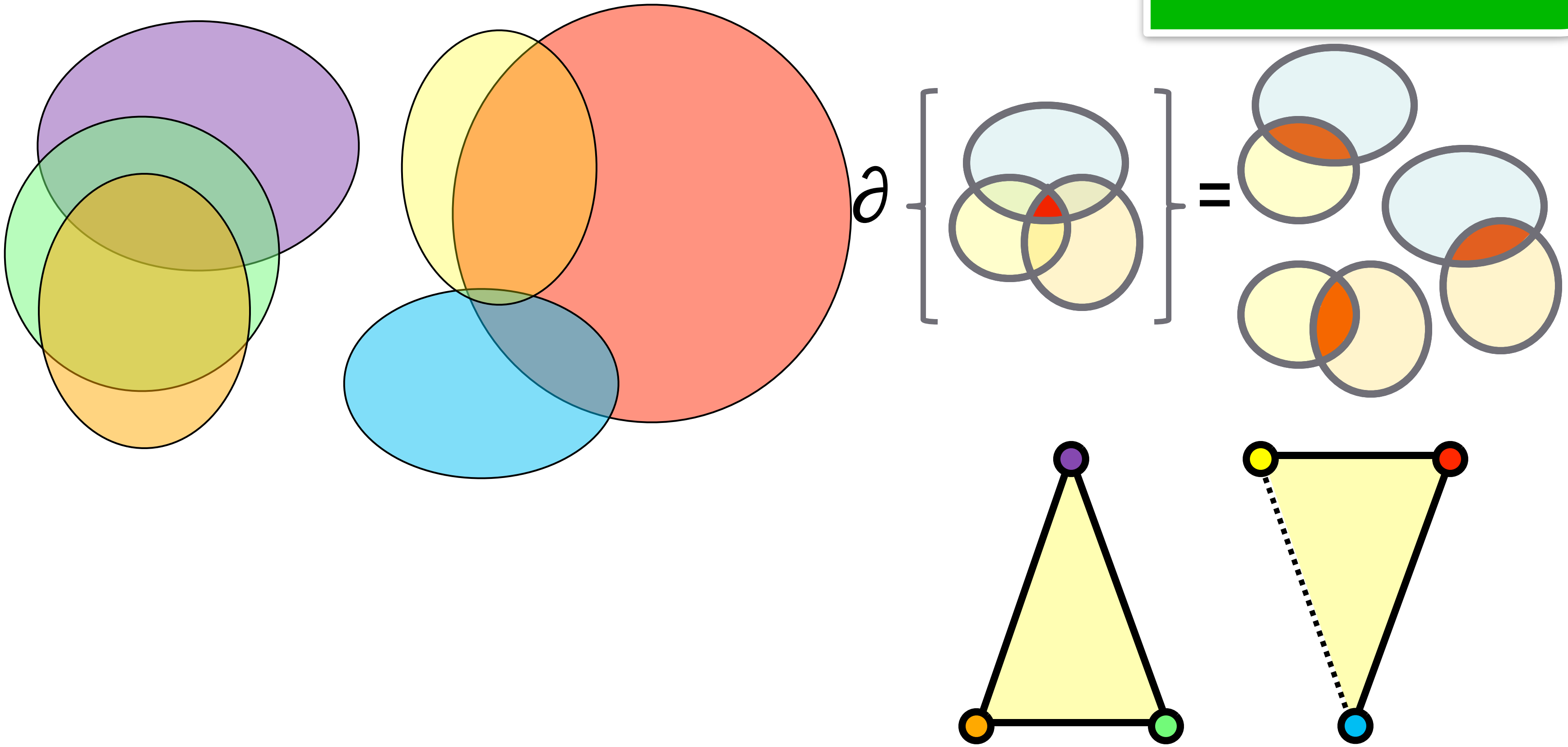
Cech homology

count

(convex) sets

cancel

intersections



mid 20th c.

example

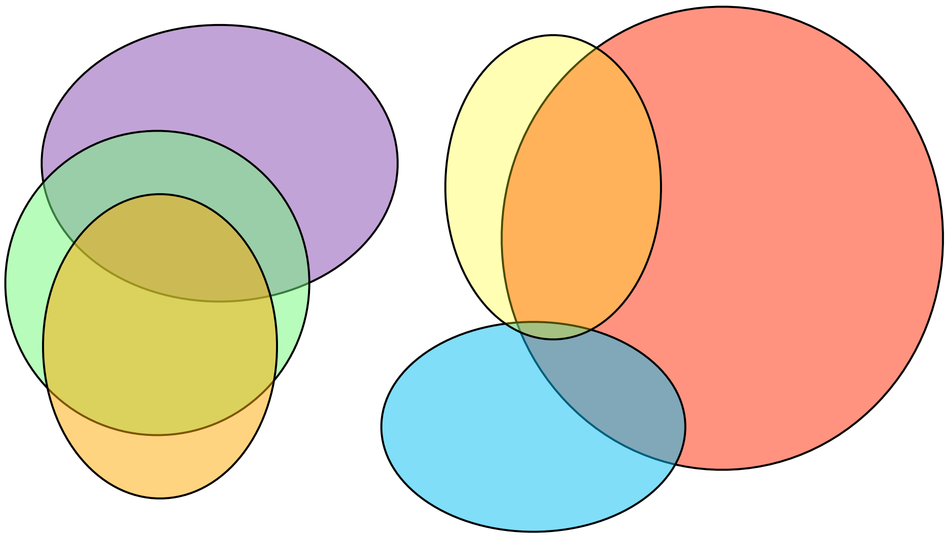
Cech homology

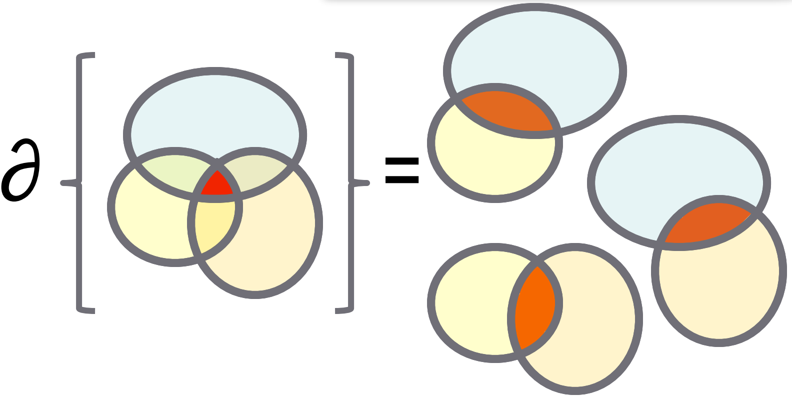
count

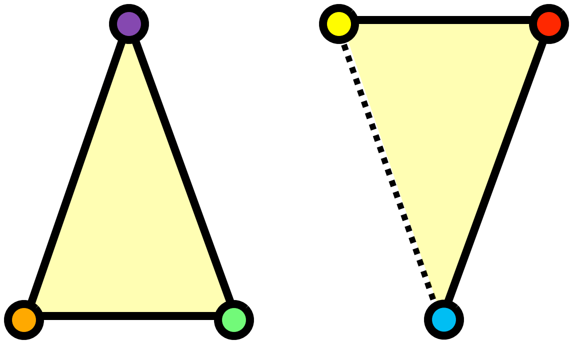
(convex) sets

cancel

intersections







theorem

Cech homology ≈ homology of union

mid 20th c.

example

Morse homology

count

cancel

mid 20th c.

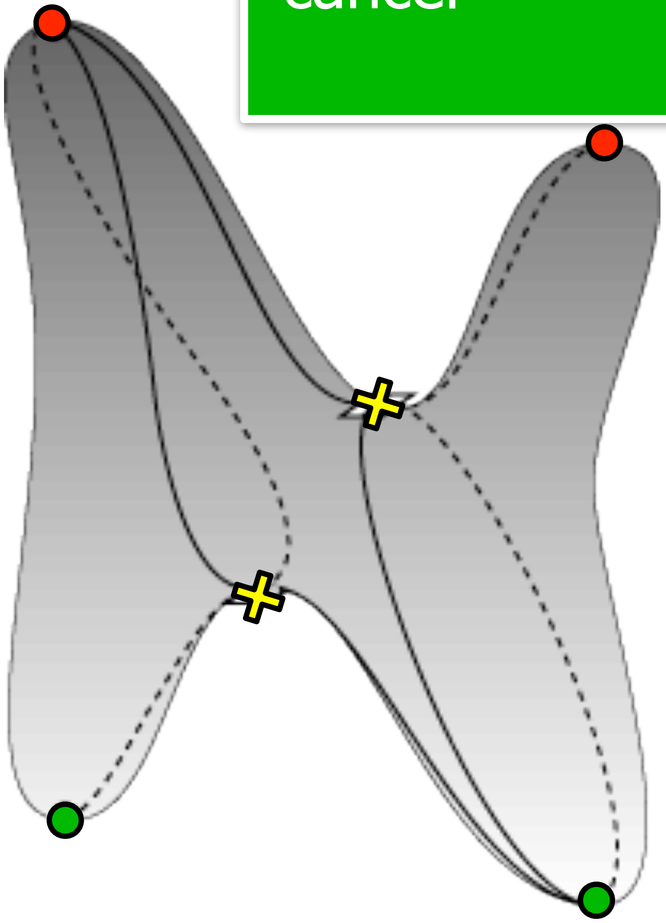
example

Morse homology

count

cancel

Consider gradient flow of a (Morse) function f:M → R



mid 20th c.

example

Morse homology

count

fixed points of -∇f

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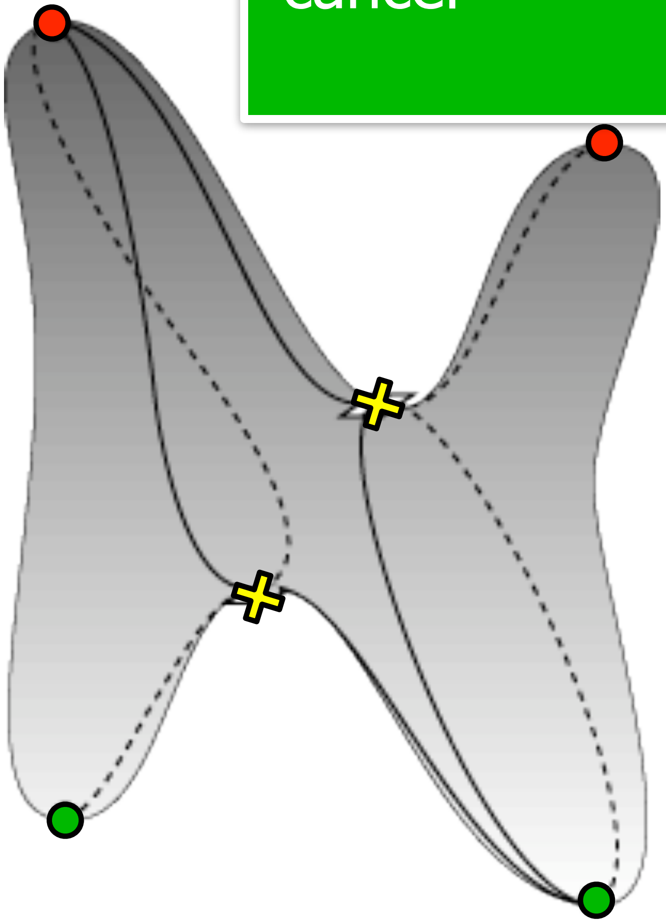

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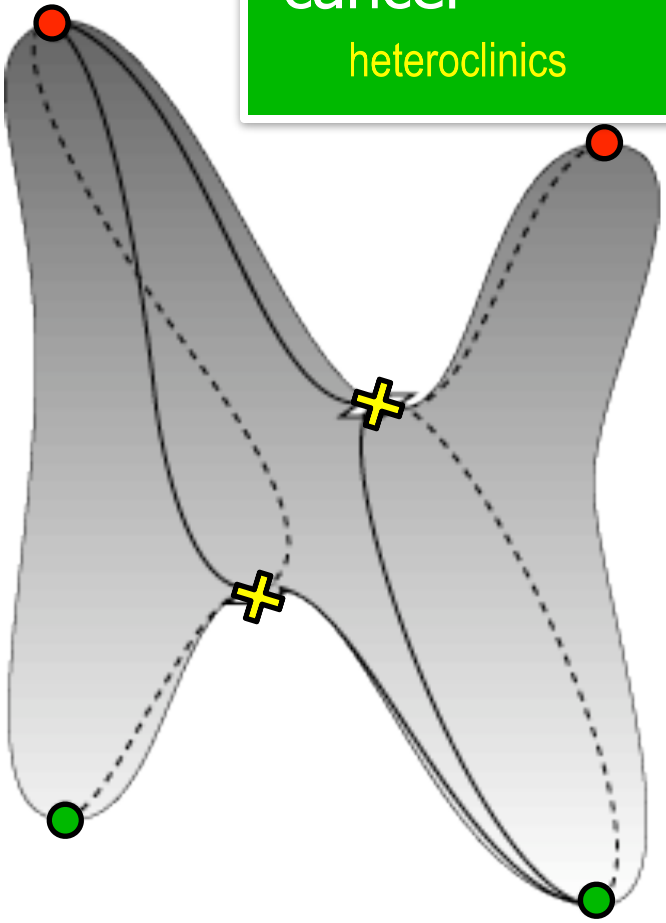

mid 20th c.

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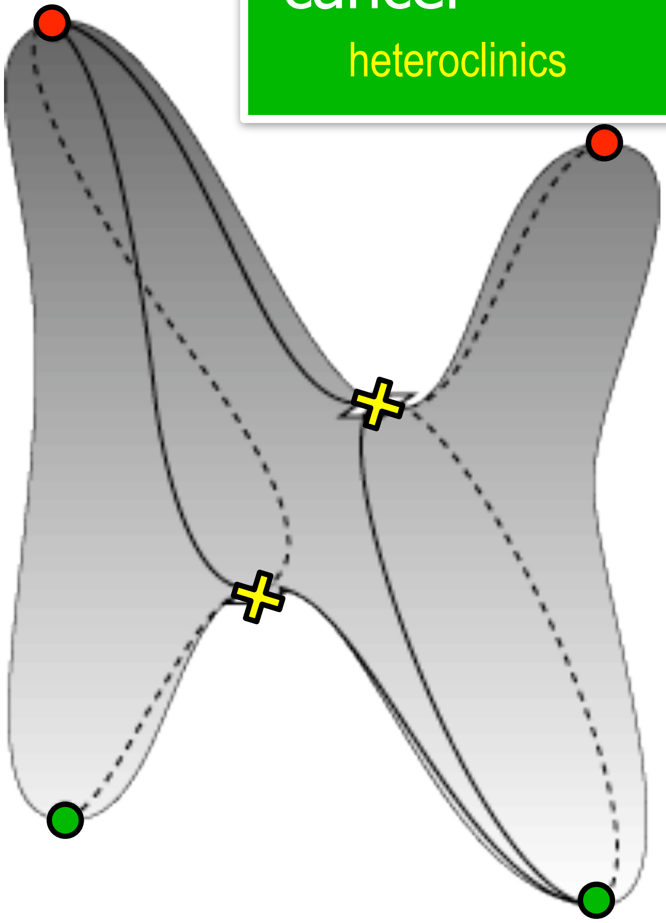
Consider gradient flow of a (Morse) function f:M → R

C_k counts fixed points whose unstable manifold has dimension k



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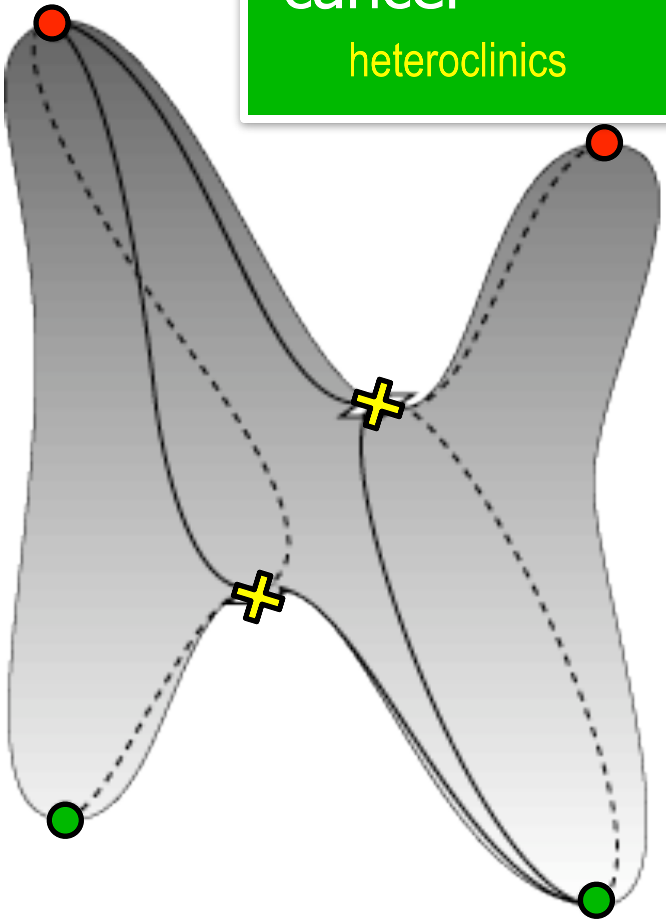

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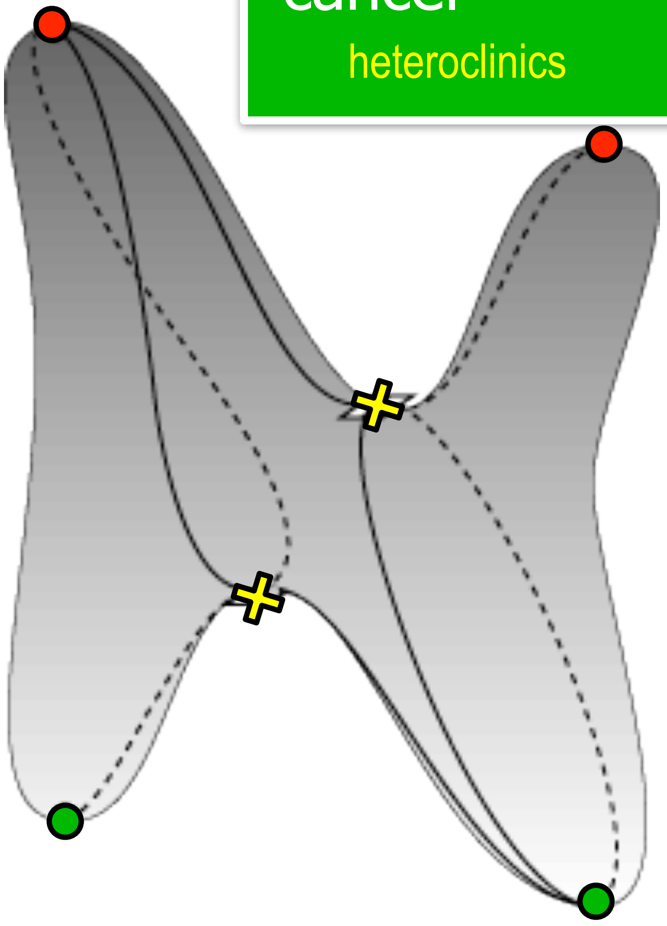

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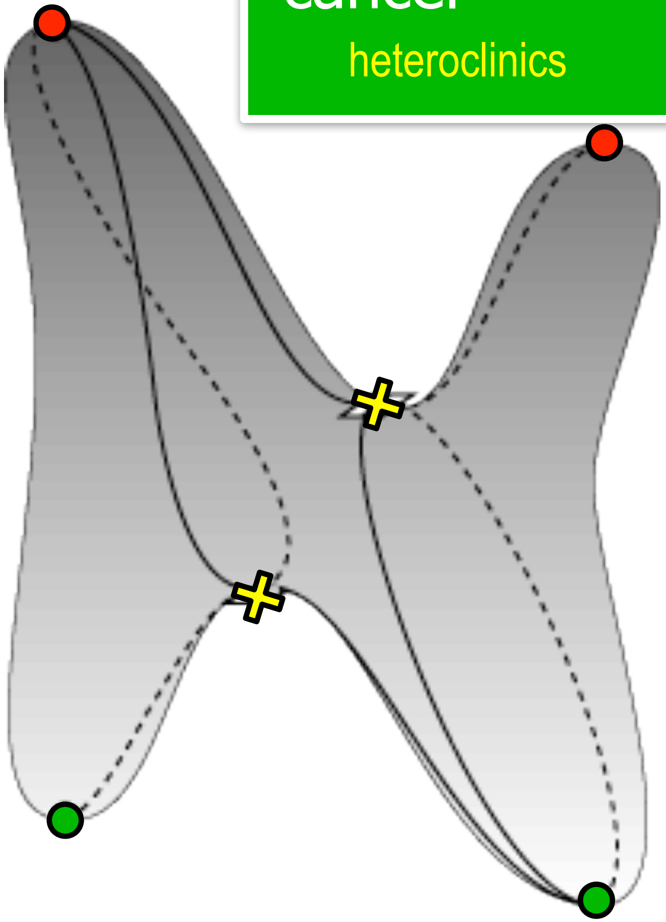

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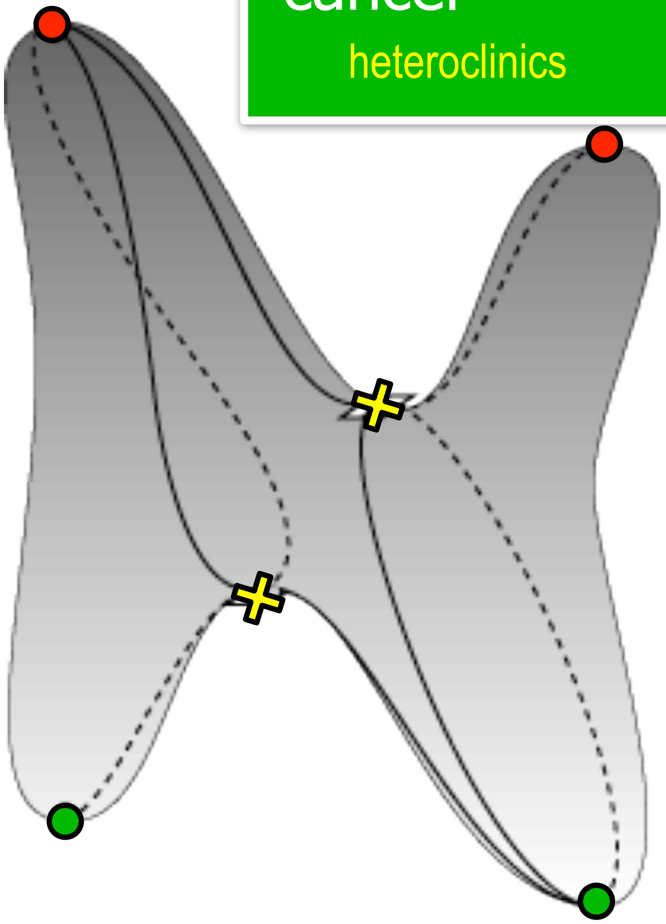

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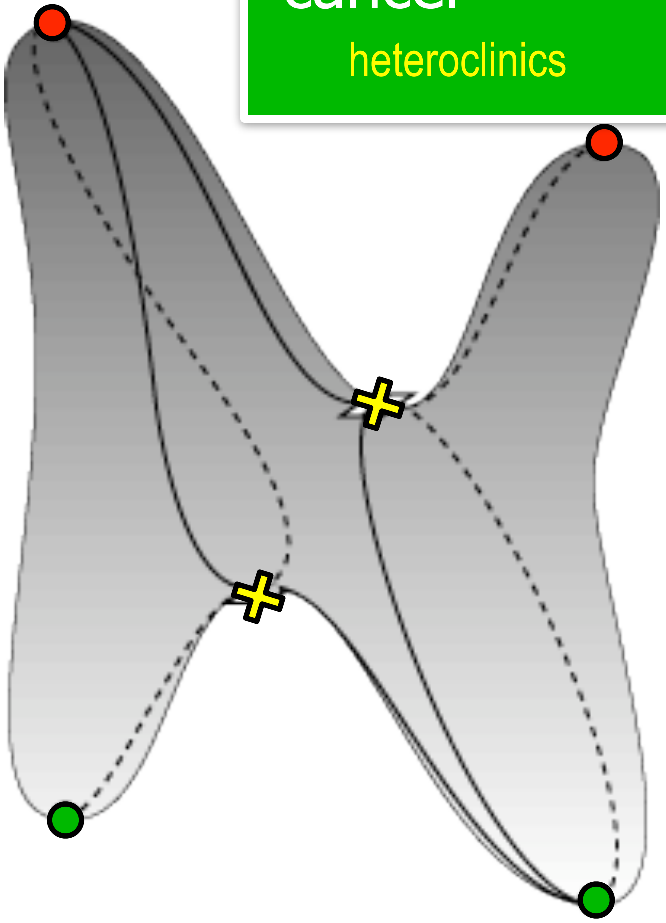

theorem

Morse homology of f ≈ homology of M

late 20th c.

example

Conley/Floer theory

count

invariant sets

cancel

connecting orbits

late 20th c.

example

Conley/Floer theory

count

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Conley

late 20th c.

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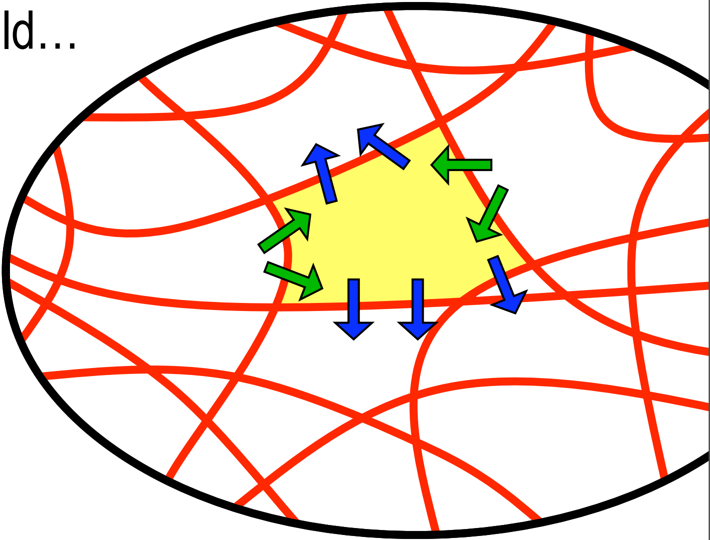
Conley

no need to be nondegenerate, gradient, or on a manifold…



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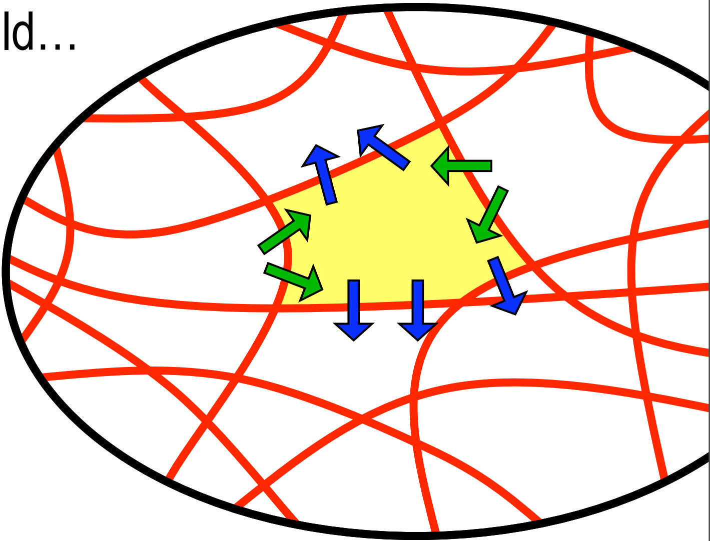

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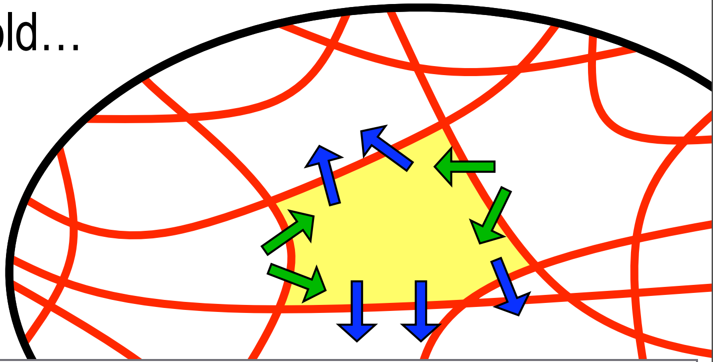
useful! experimental data analysis, biology, PDEs, …
computable! good algorithms for Conley homology



Sunday, May 17

MS9
Topology and Computations in Dynamics

Organizer: Jean-Philippe Lessard
Vrije Universiteit Amsterdam, The Netherlands
Jan Bouwe Van Den Berg
VU University, Amsterdam

late 20th c.

example

Conley/Floer theory

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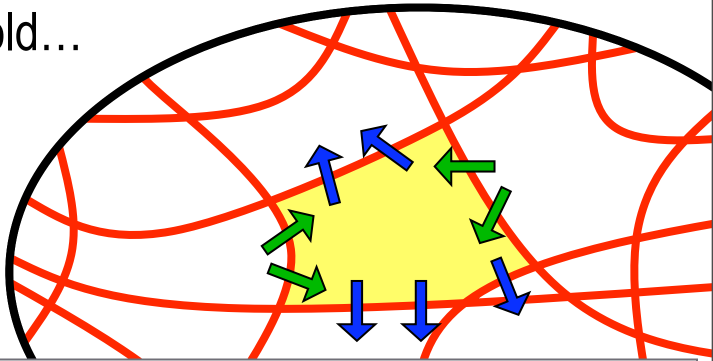

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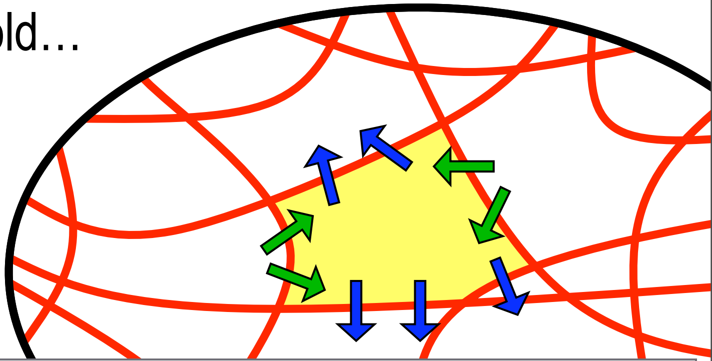
Floer

no need to be finite dimensional…



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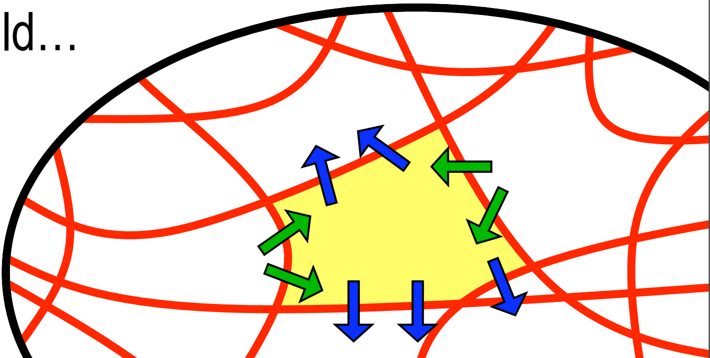
no need to be finite dimensional…

computation is a challenge!



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Homology

Homology

Count

Homology

Count

Cancel

Homology

Count

Cancel

Local-to-Global Algebra

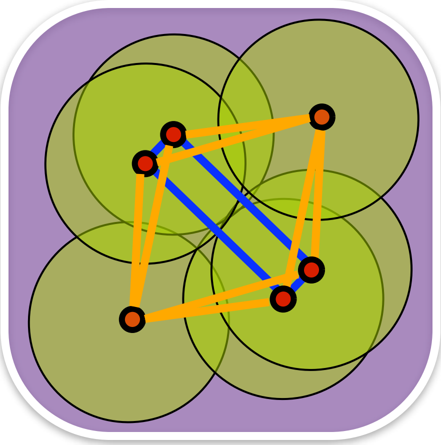
exact sequences

functoriality

excision

gluing

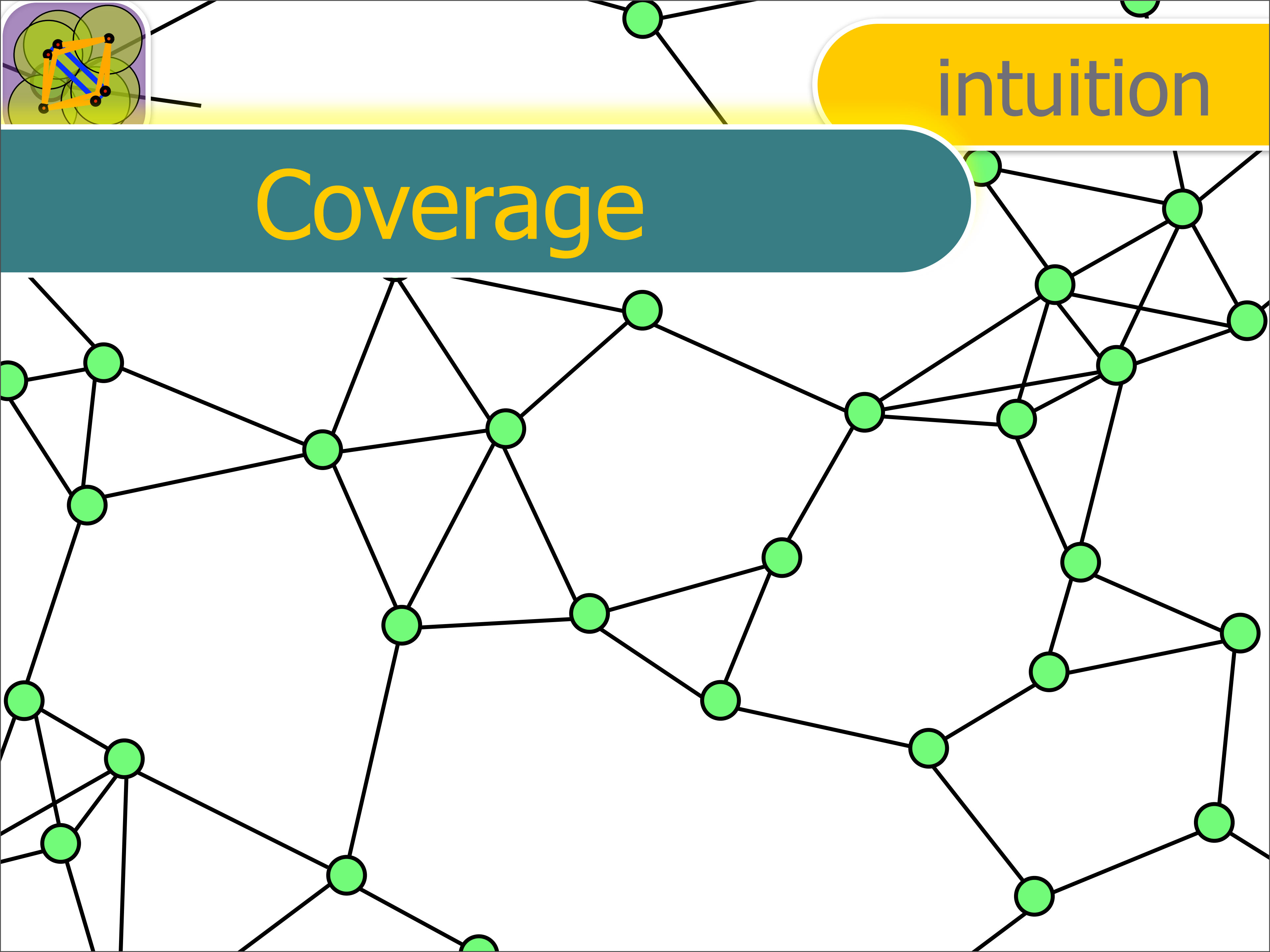
Coverage



Coverage

Coverage

intuition



Coverage

Coverage problems in sensor networks

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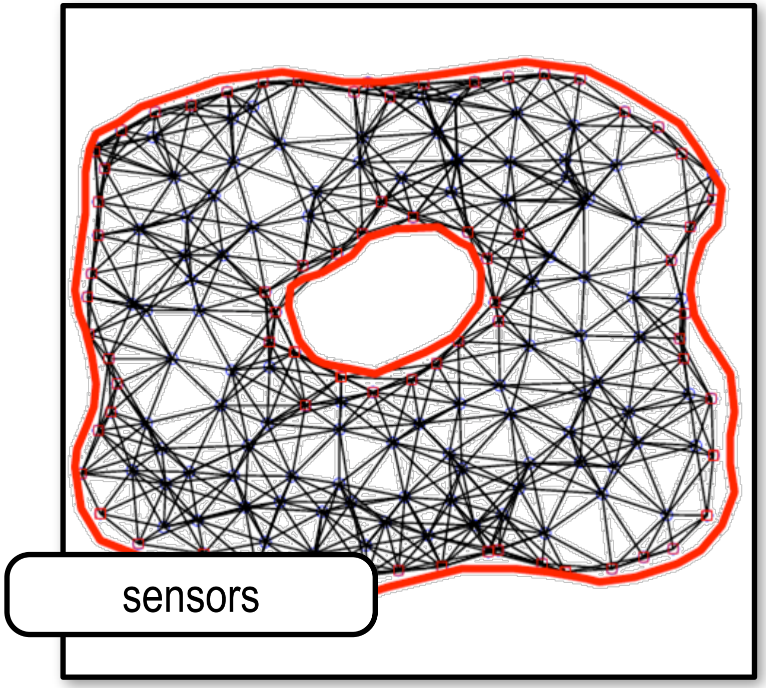

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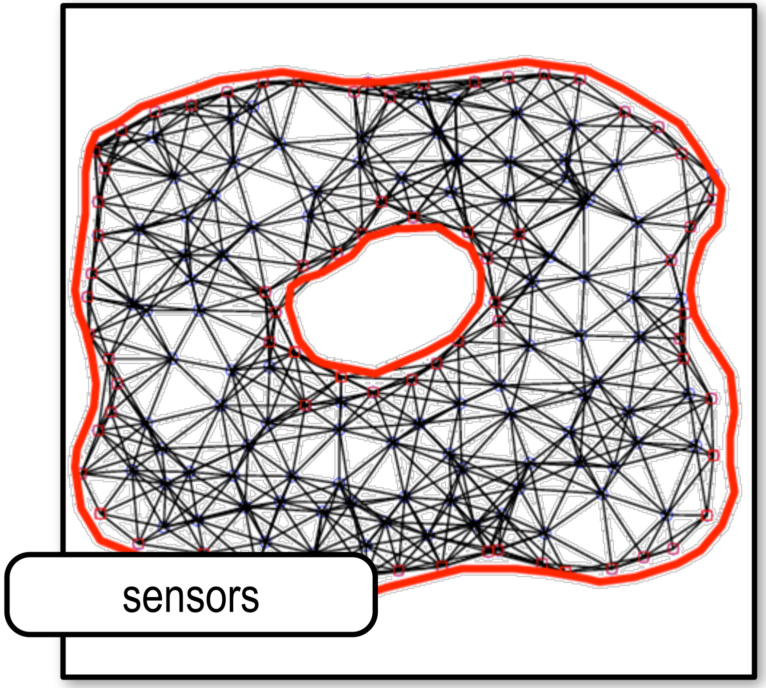

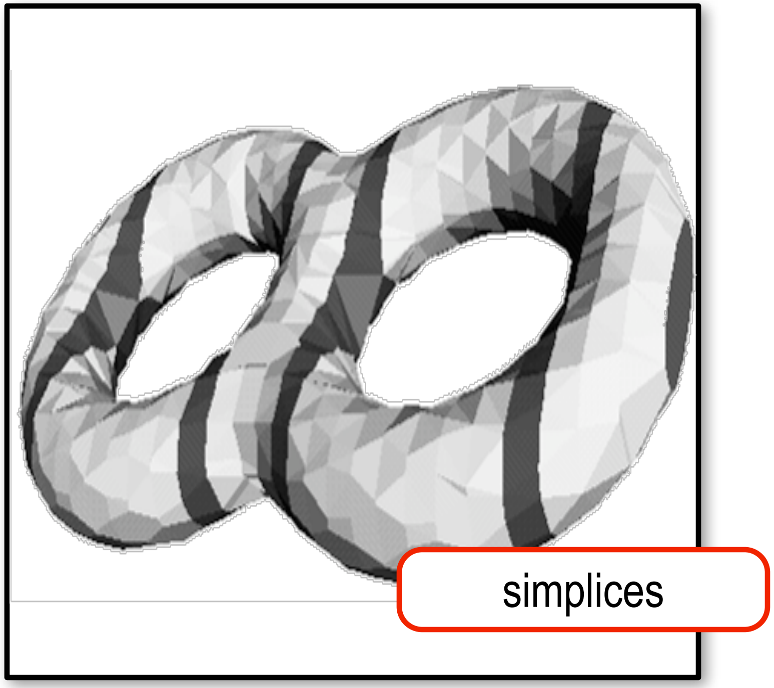

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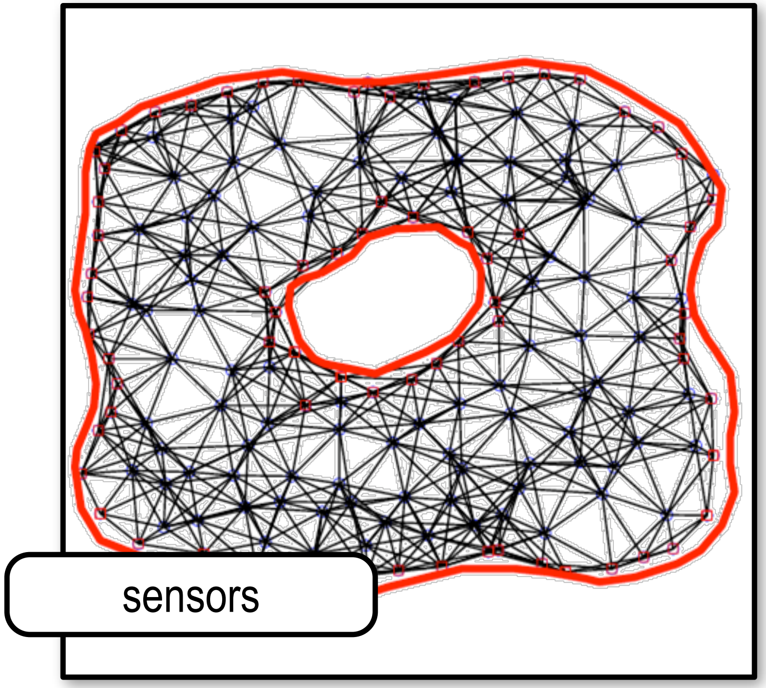

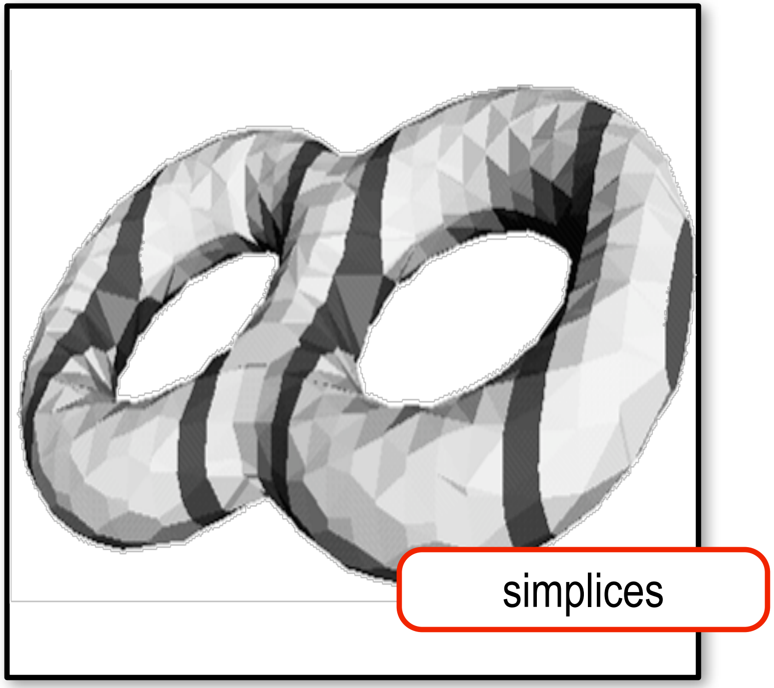

sensors and simplices each have knowledge only of their identities and of their local connectivity...

Coverage

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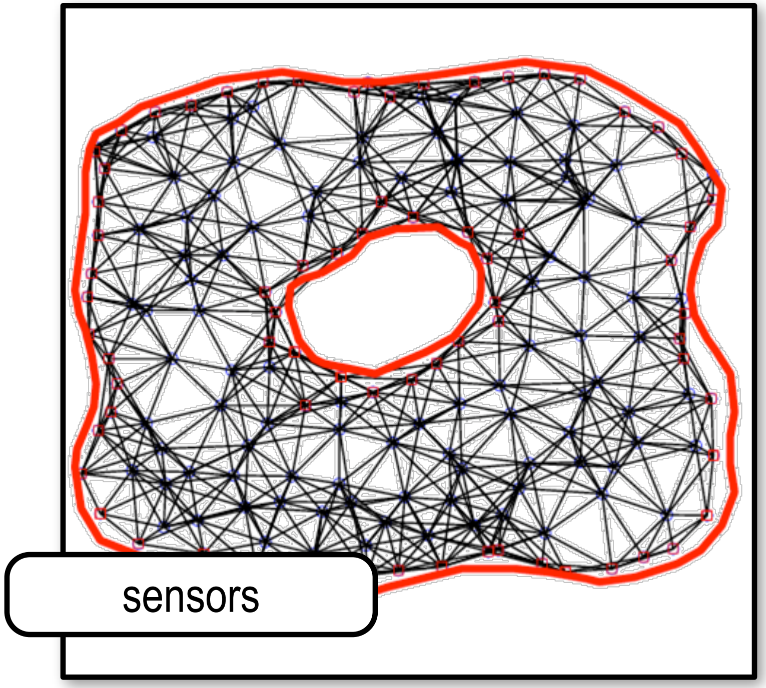
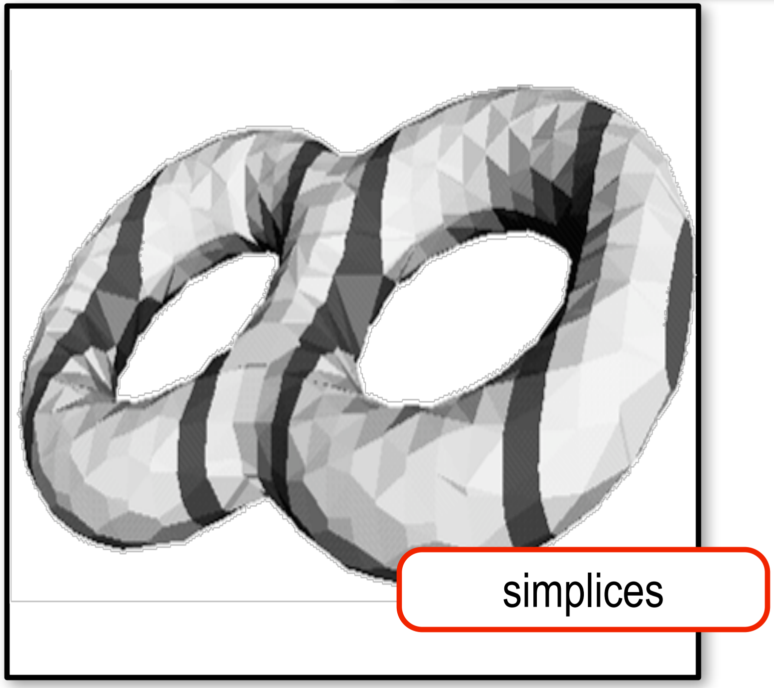




sensors and simplices each have knowledge only of their identities and of their local connectivity...

intuition

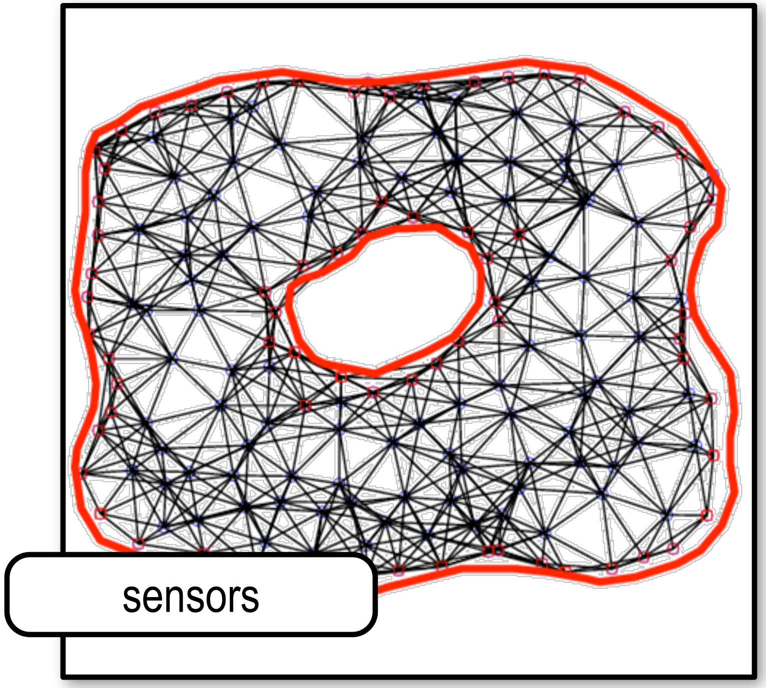
count

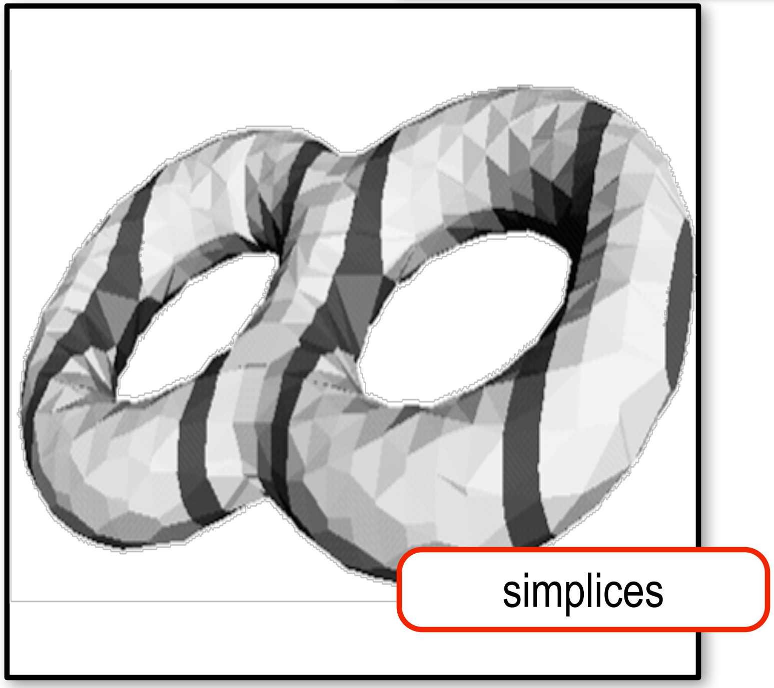
nodes

cancel

Coverage

Coverage problems in sensor networks


sensors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communication

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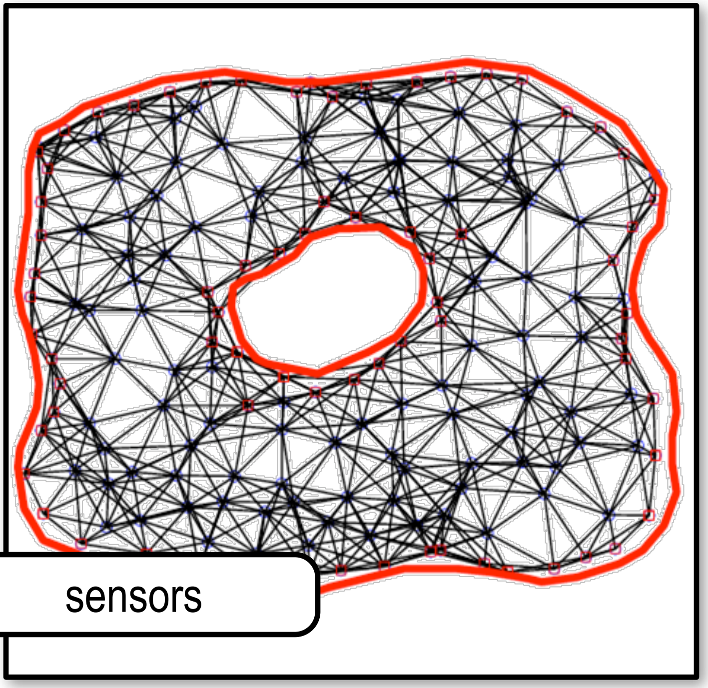
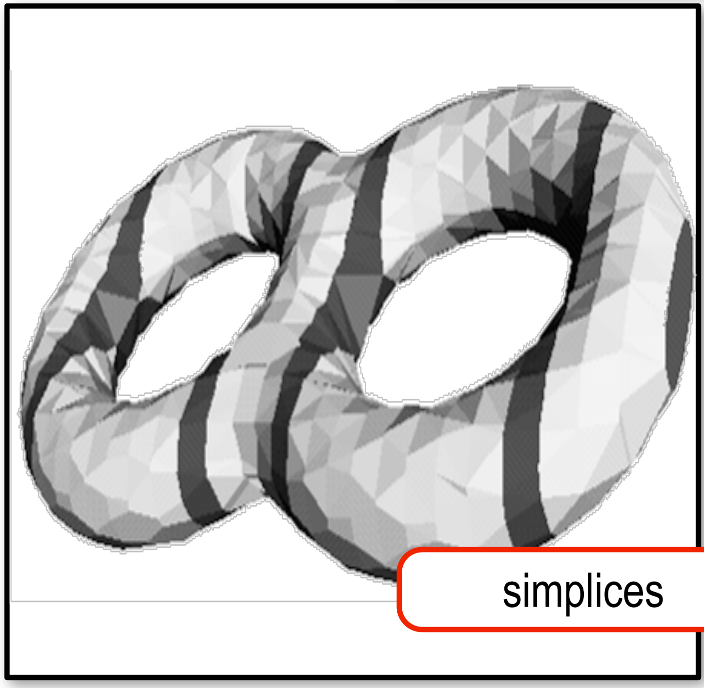
Coverage problems in sensor networks



sensors



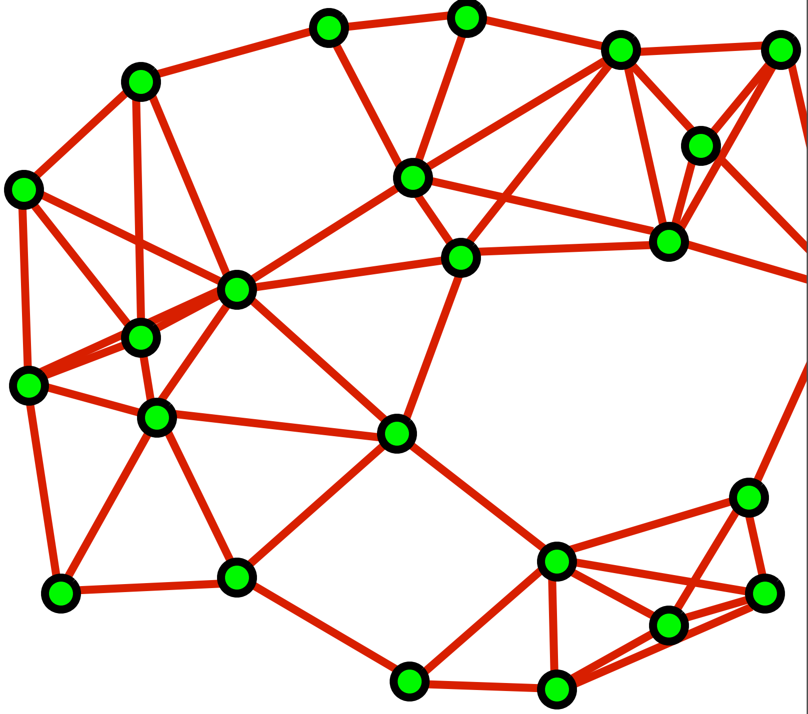
simplices

sensors and simplices each have knowledge only of their identities and of their local connectivity...

networks and complexes

given node id’s, local communication links

count nodes & cancel via signal connectivity



networks and complexes

given node id's, local communication links

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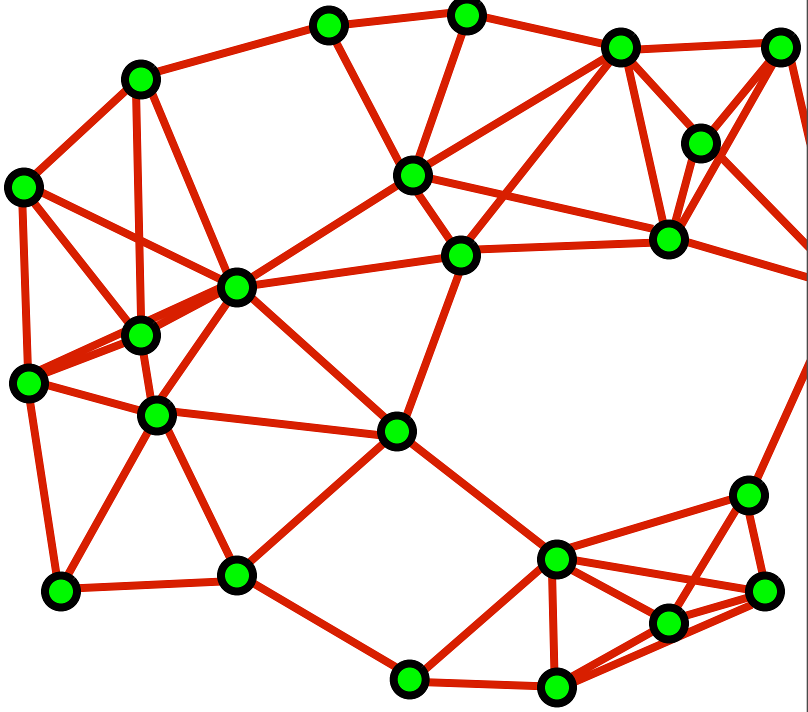
$$C_0 \leftarrow C_1 \leftarrow C_2 \leftarrow C_3 \leftarrow \ldots$$

[nodes] [pairs]



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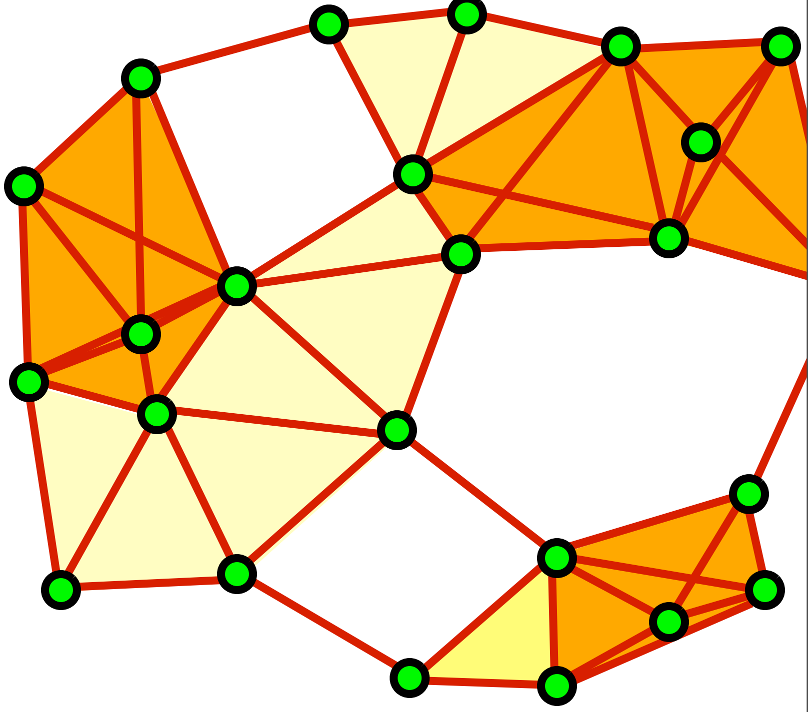
[nodes] [pairs] [triples] [quads]



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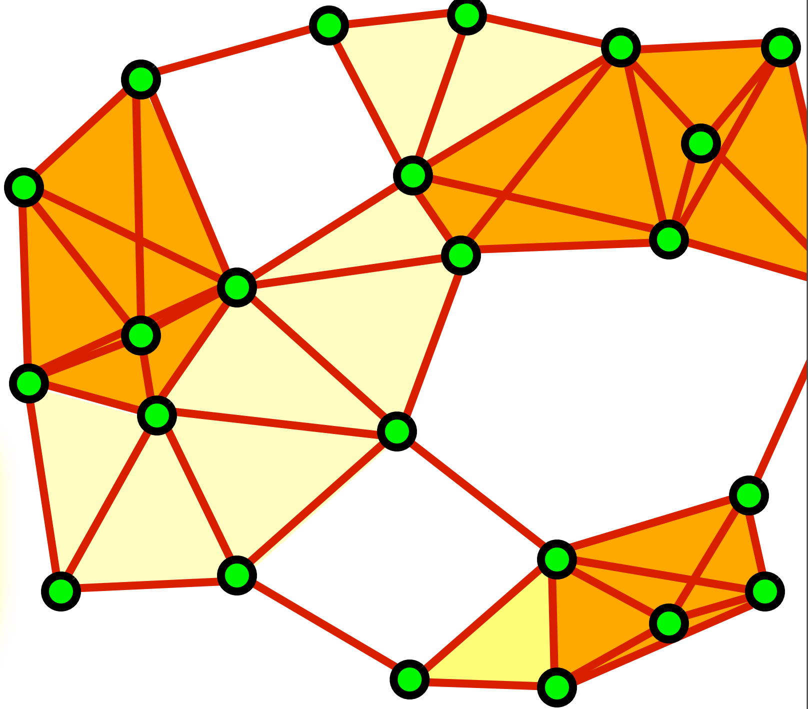
definintion

the flag complex of a network
is the maximal simplicial completion



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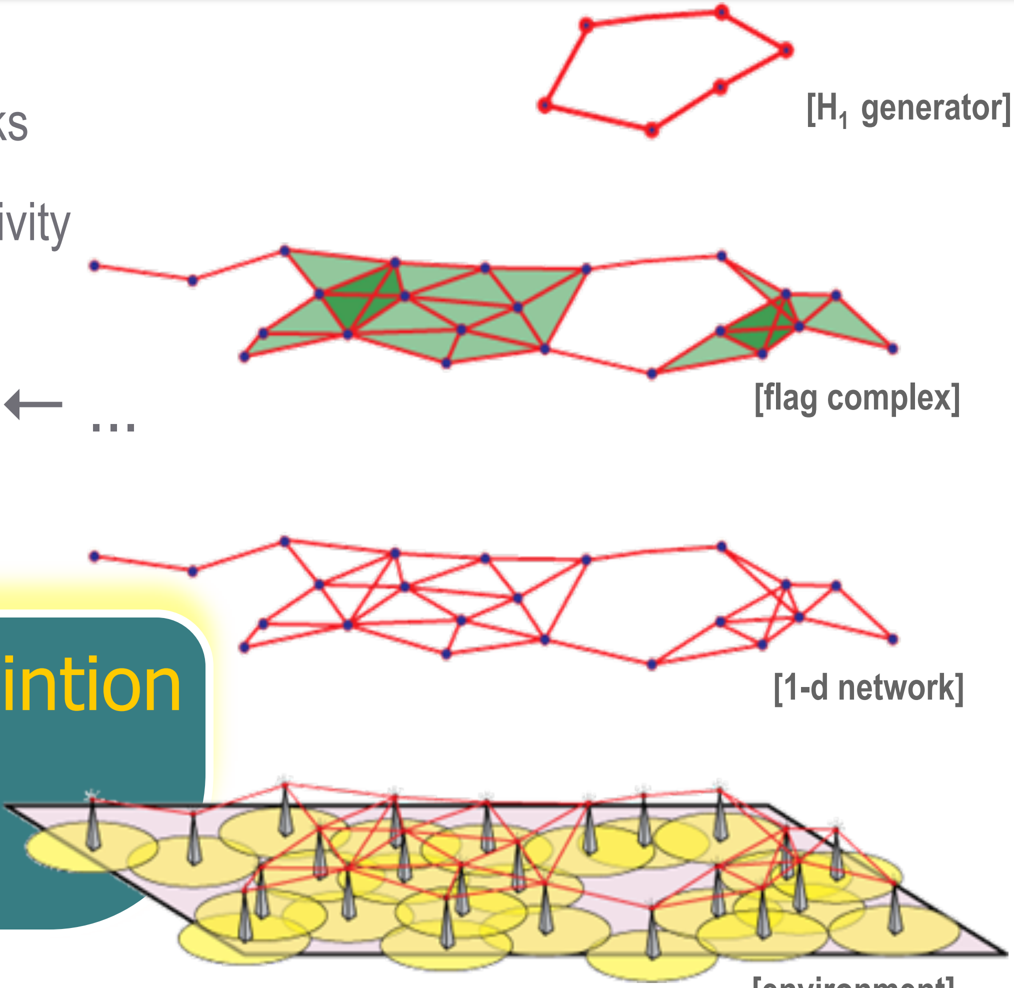

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homology converts local higher-order network connectivity into global structure...

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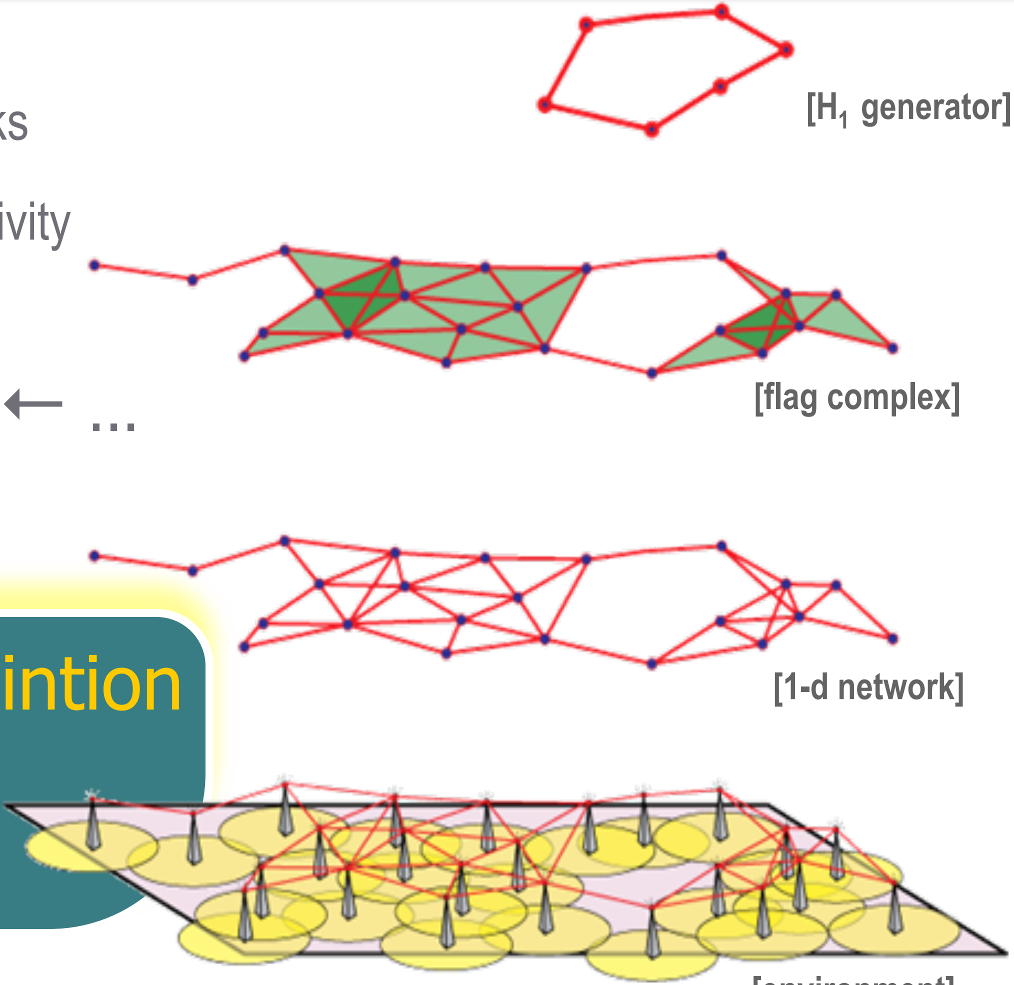

definintion

the flag complex of a network
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homology converts local higher-order network connectivity into global structure...

...without the need for coordinates; density assumptions; uniform distributions, etc.

coverage assumptions

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1. compact polygonal domain D in R^2

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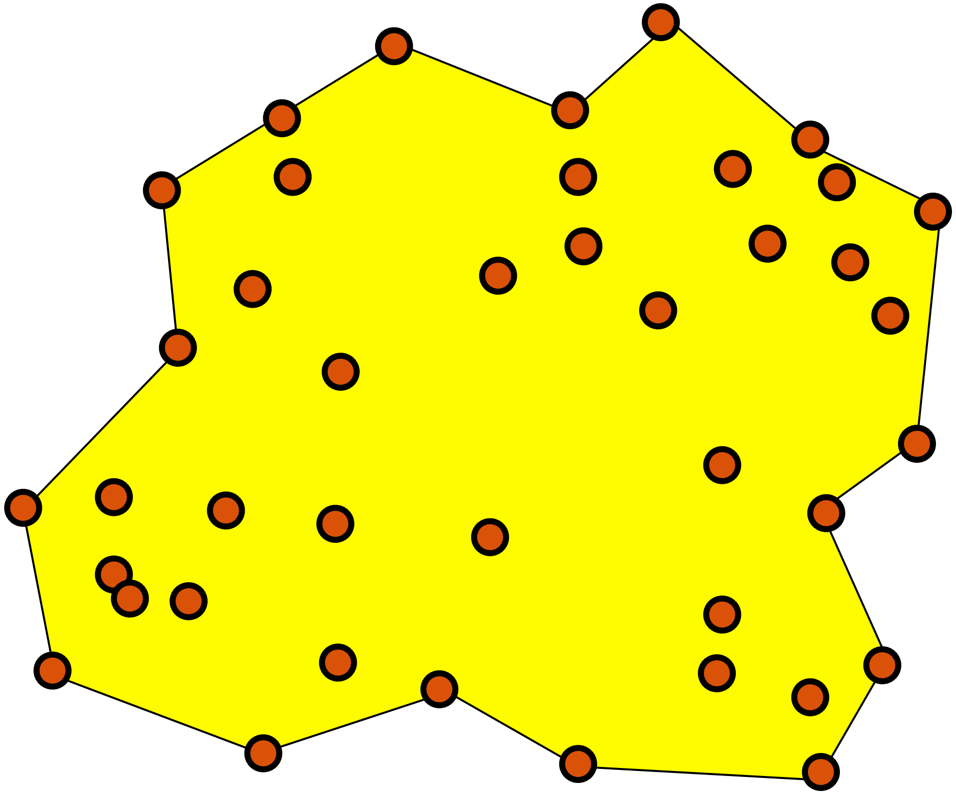
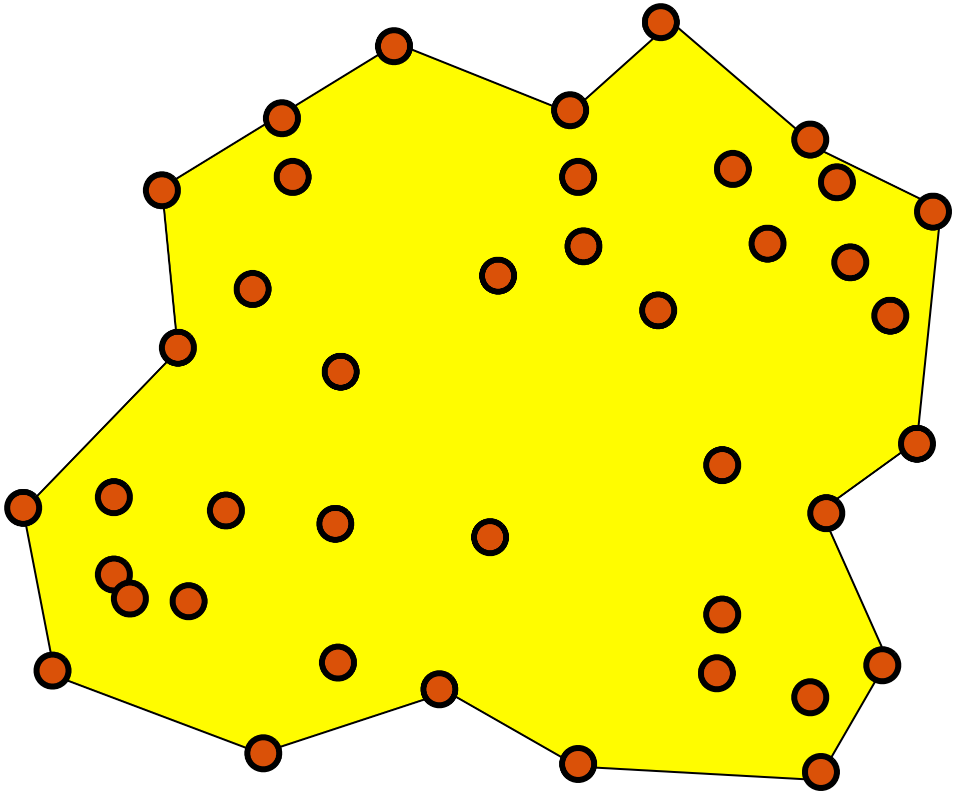


coverage assumptions

1. compact polygonal domain D in R^2
2. nodes broadcast unique id's to neighbors



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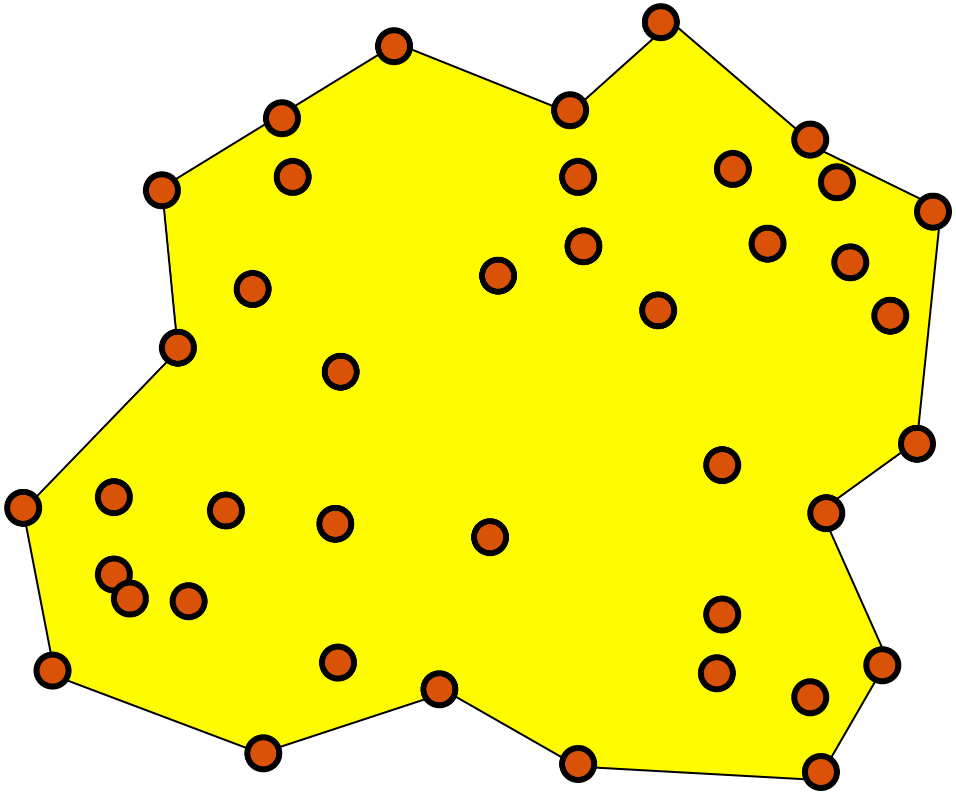
3. coverage regions of a 2-simplex of connected nodes contain the convex hull



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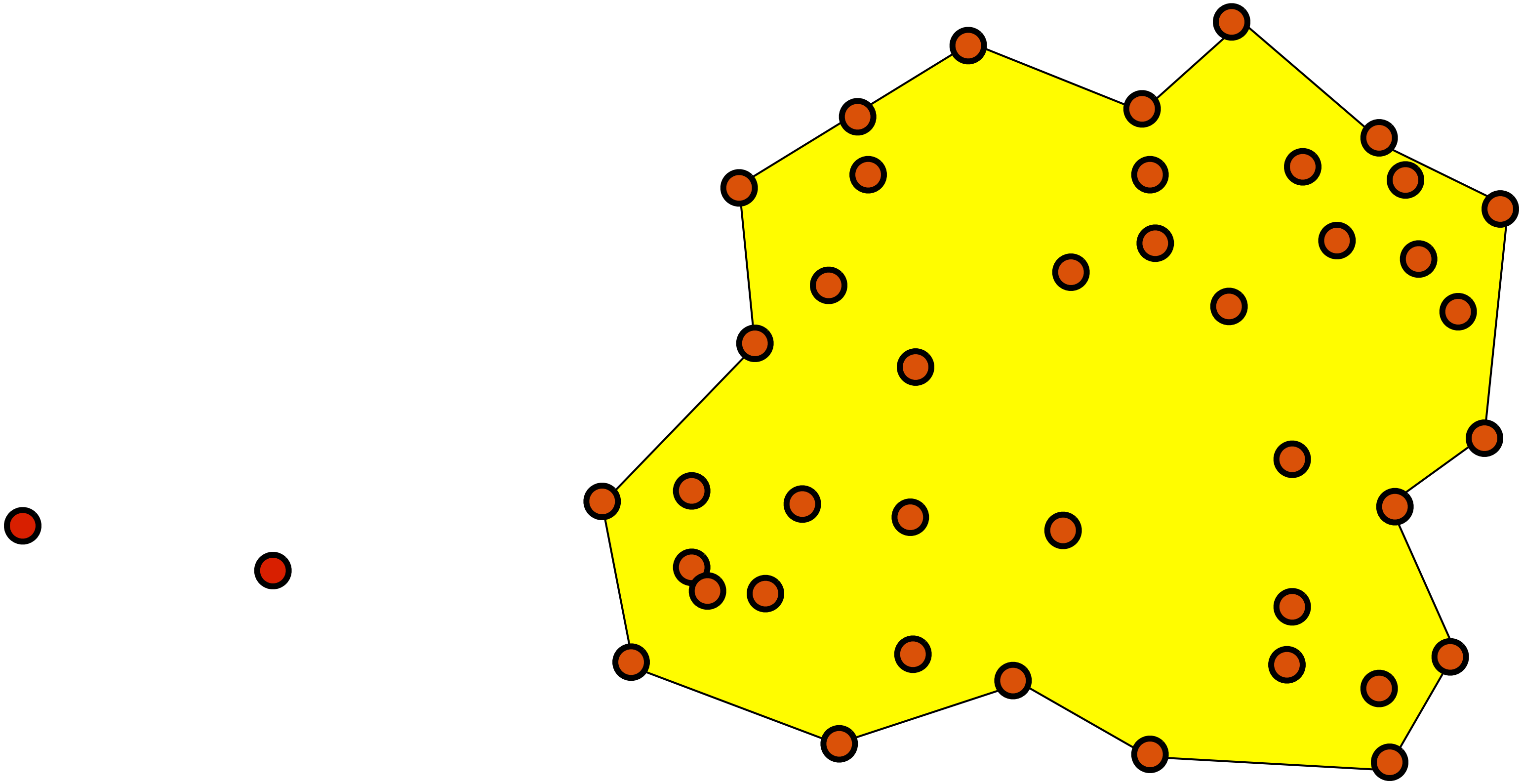

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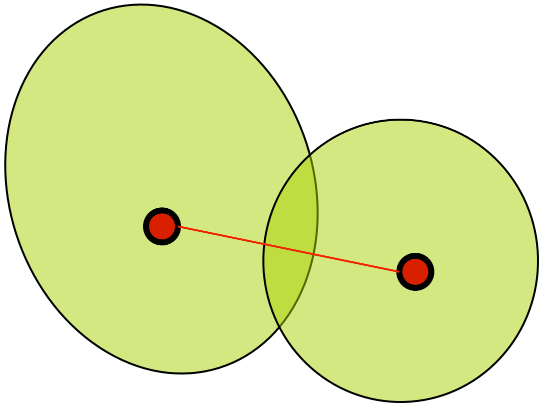

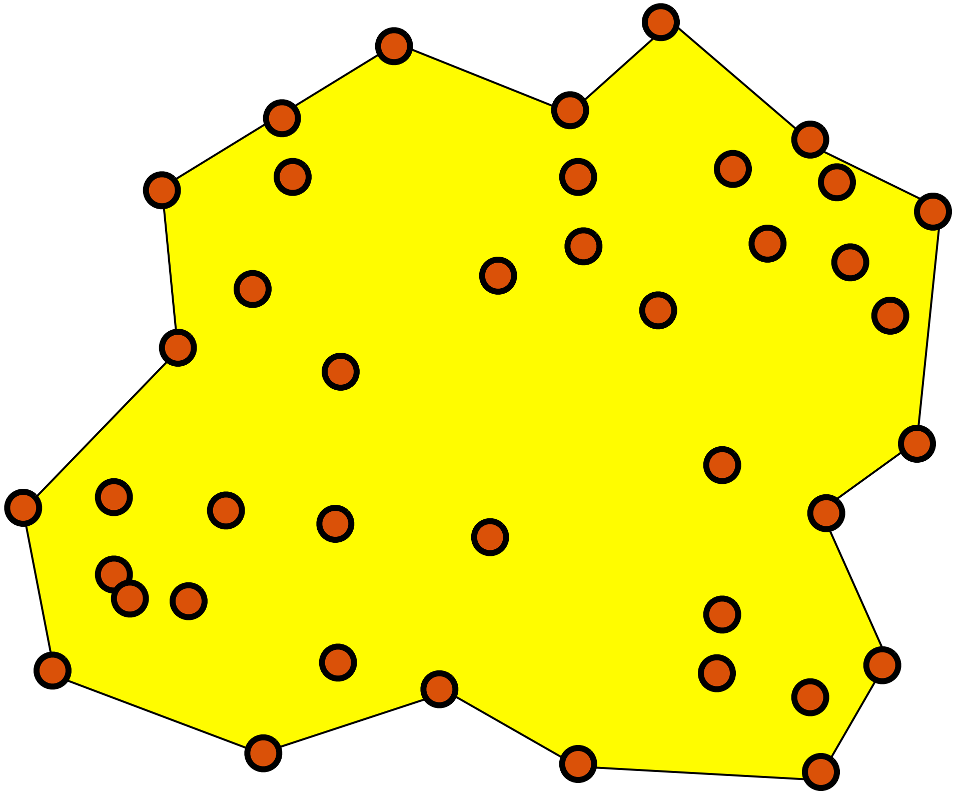

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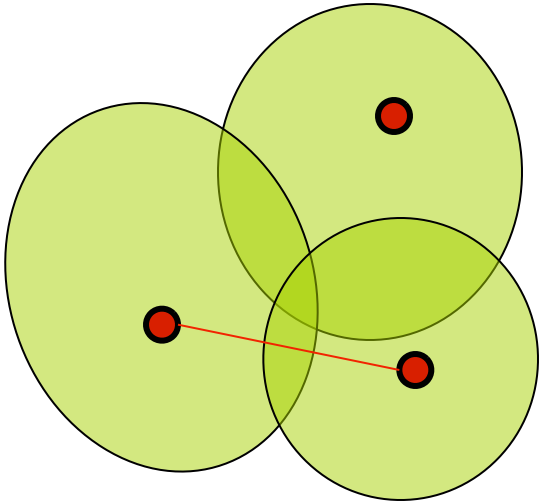

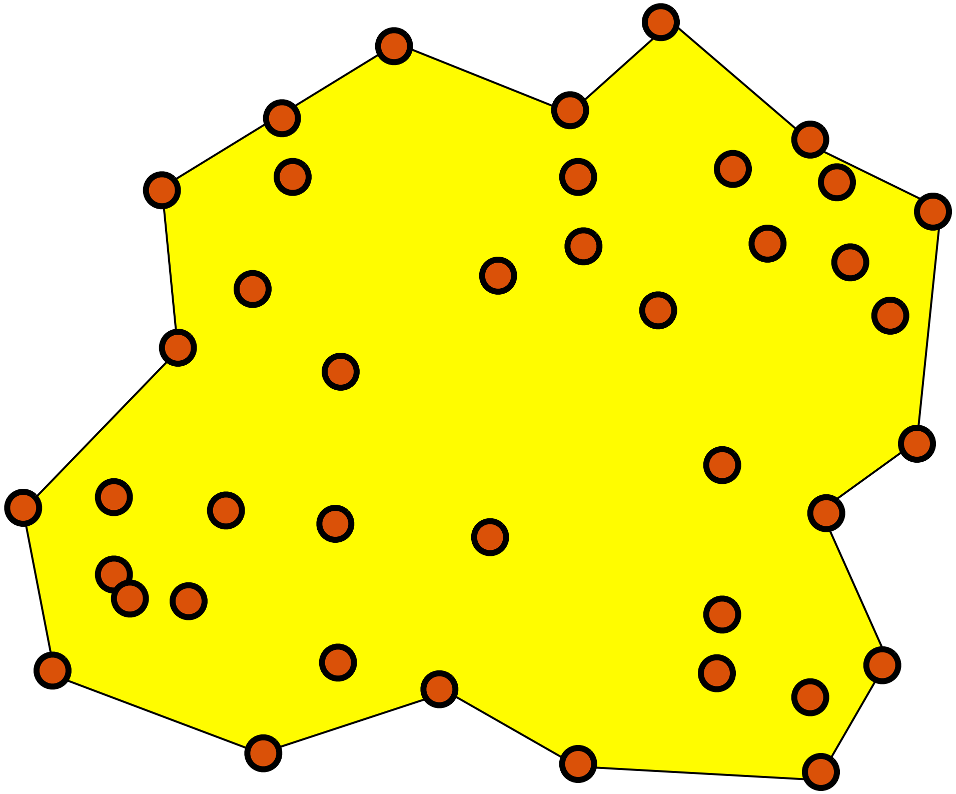

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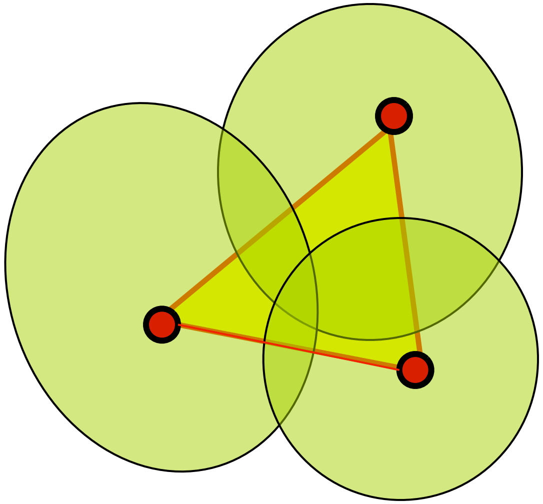

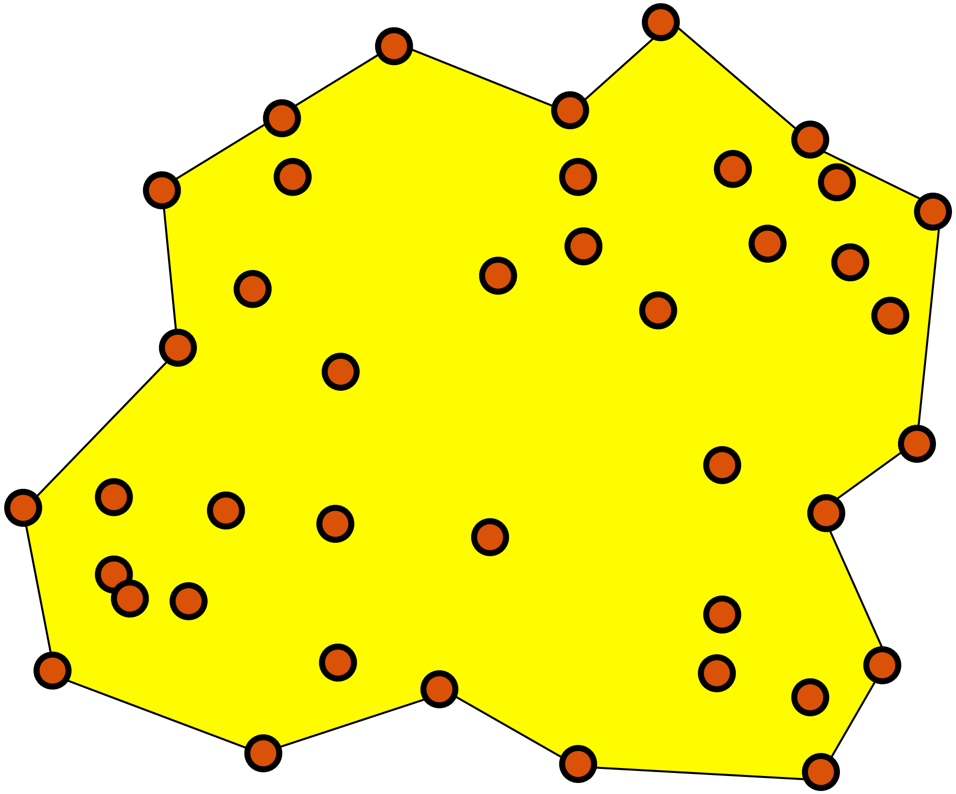

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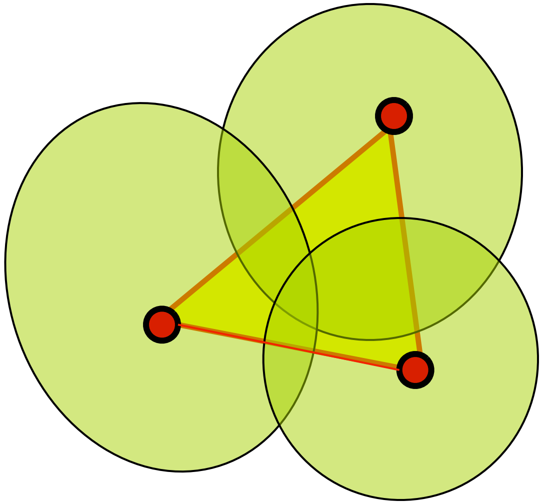

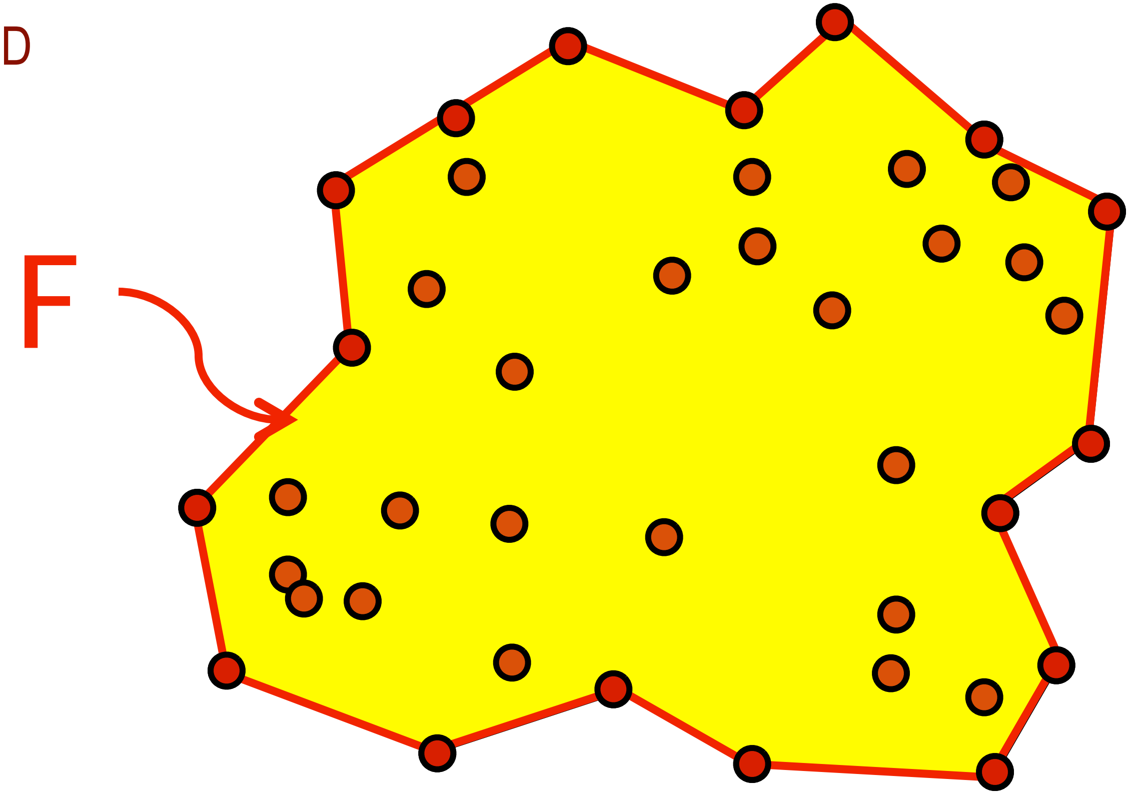

homological coverage

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Theorem [DG]: under above assumptions, the sensor network covers the domain without gaps if there exists $[\alpha]$ in $H_2(R, F)$ with $\partial\alpha\neq 0$

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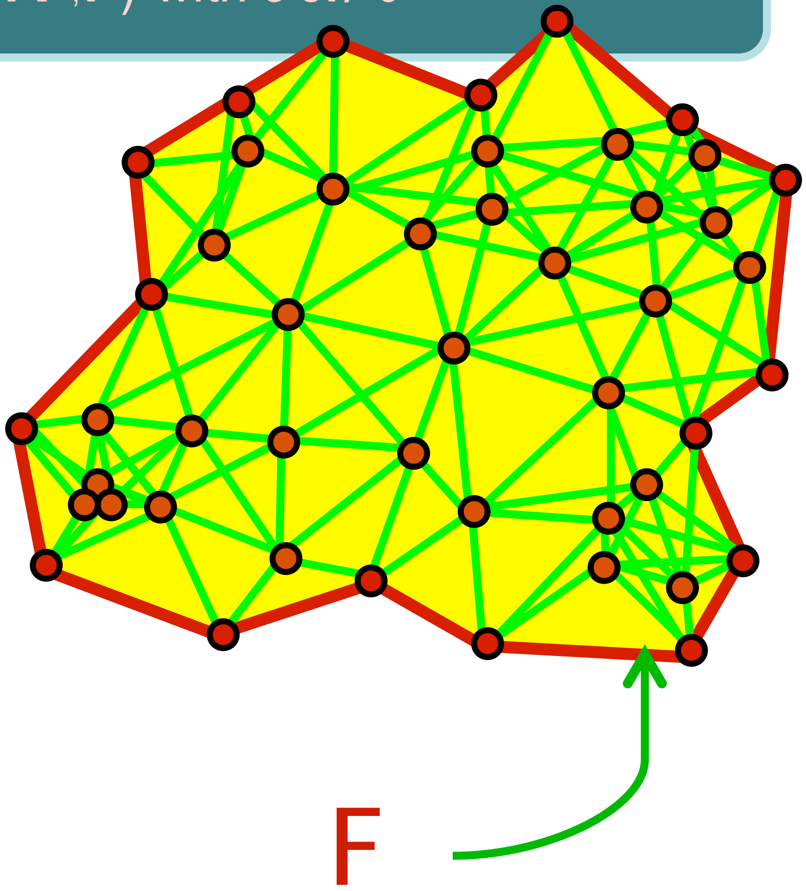
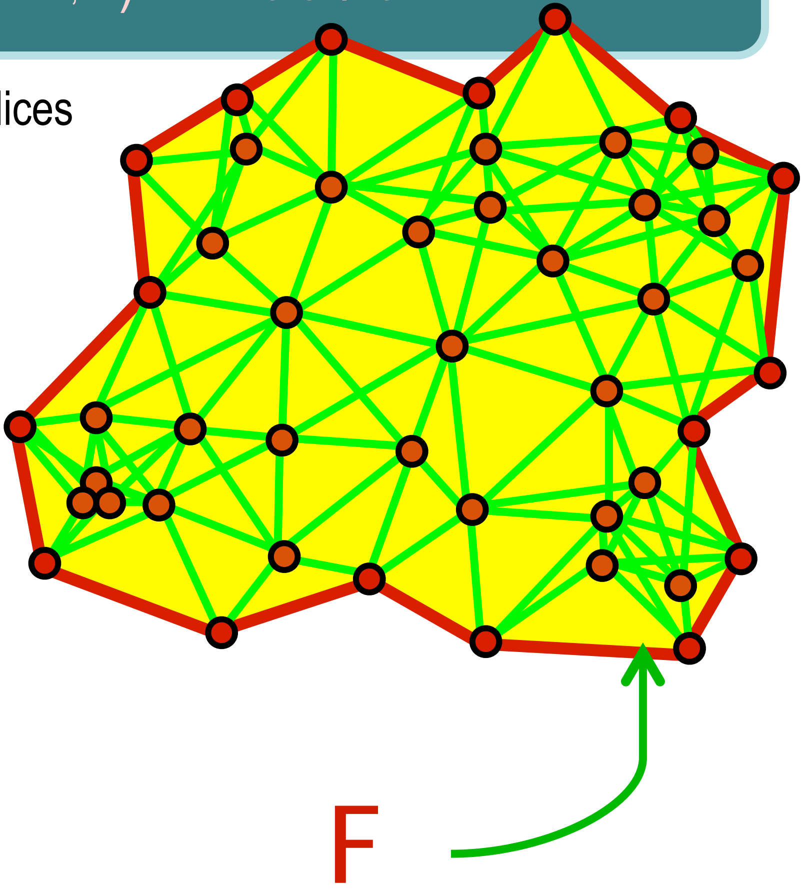


homological coverage

Theorem [DG]: under above assumptions, the sensor network covers the domain without gaps if there exists $[\alpha]$ in $H_2(R ,F)$ with $\partial\alpha\neq 0$

intuition: $[\alpha]$ “triangulates” the domain with covered simplices



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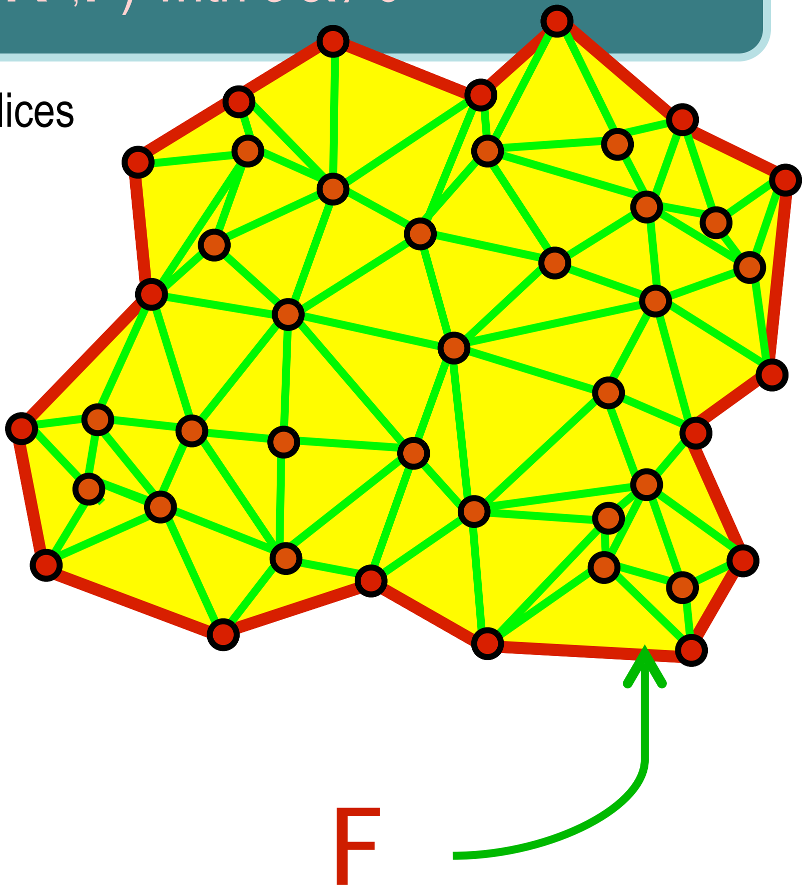

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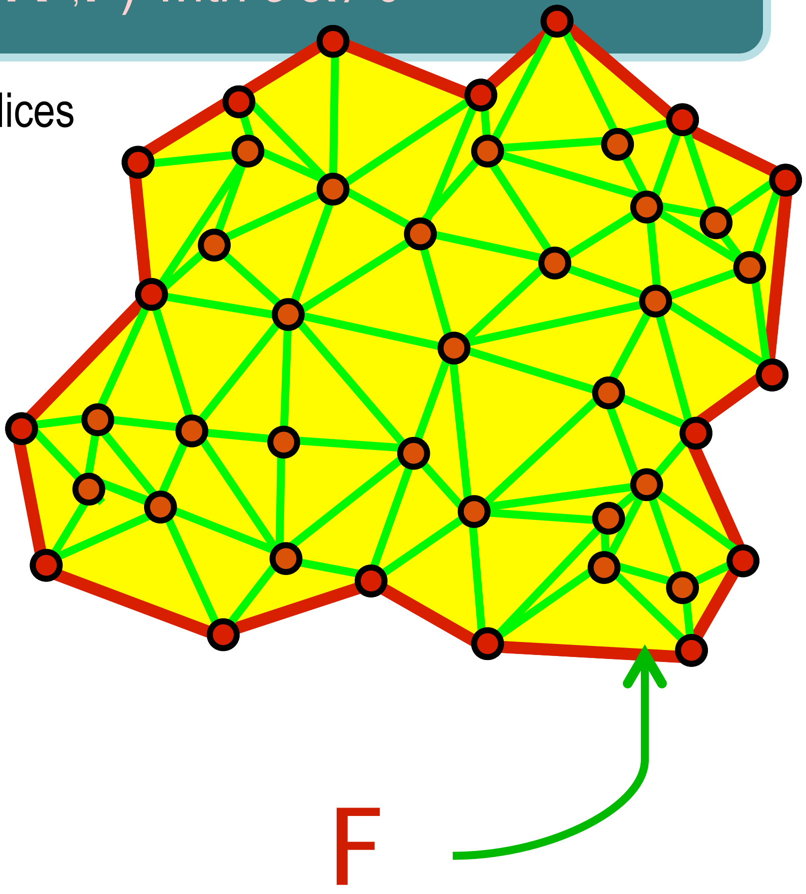

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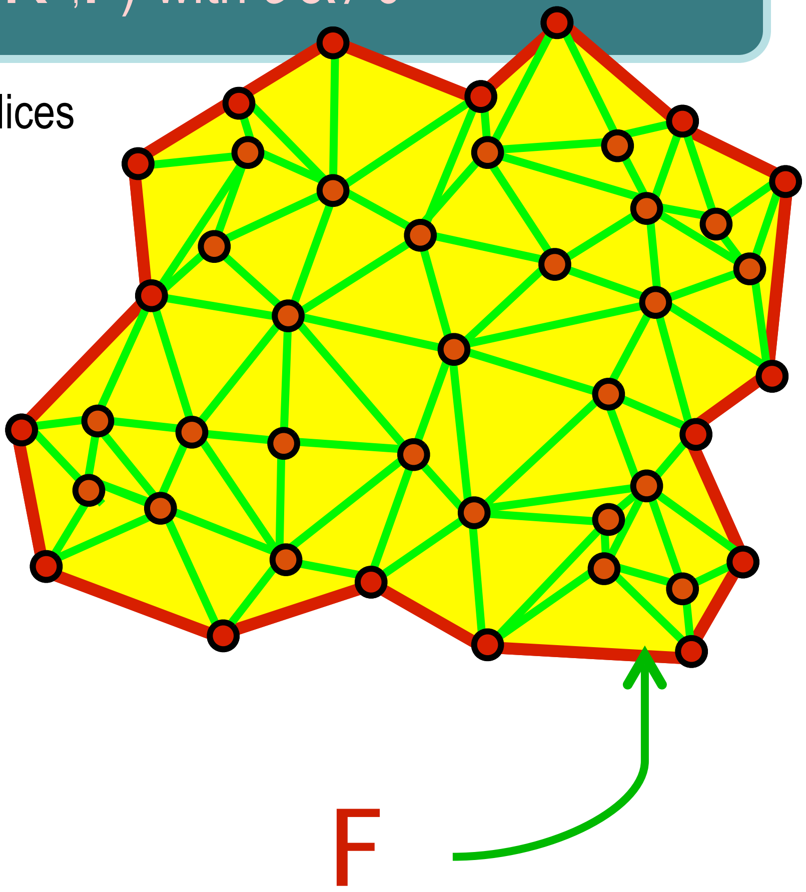

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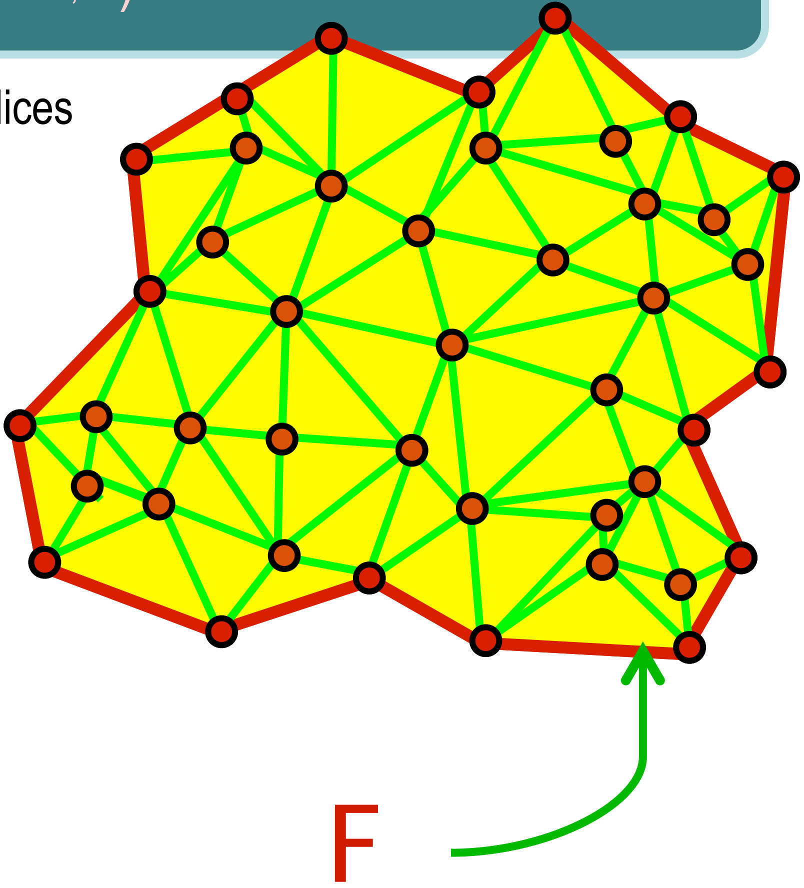

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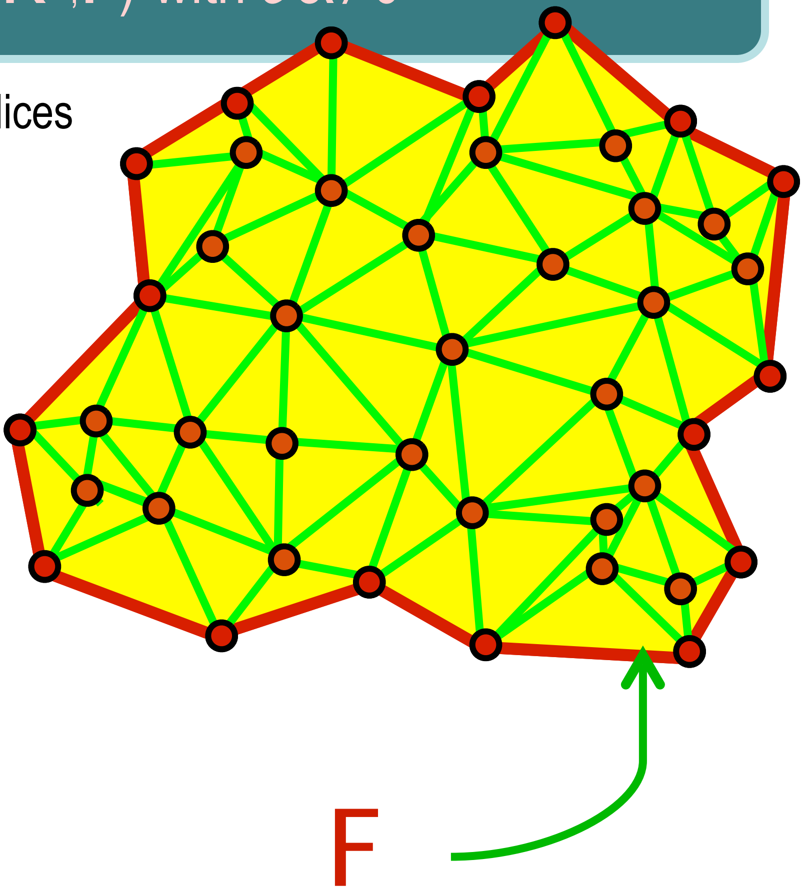

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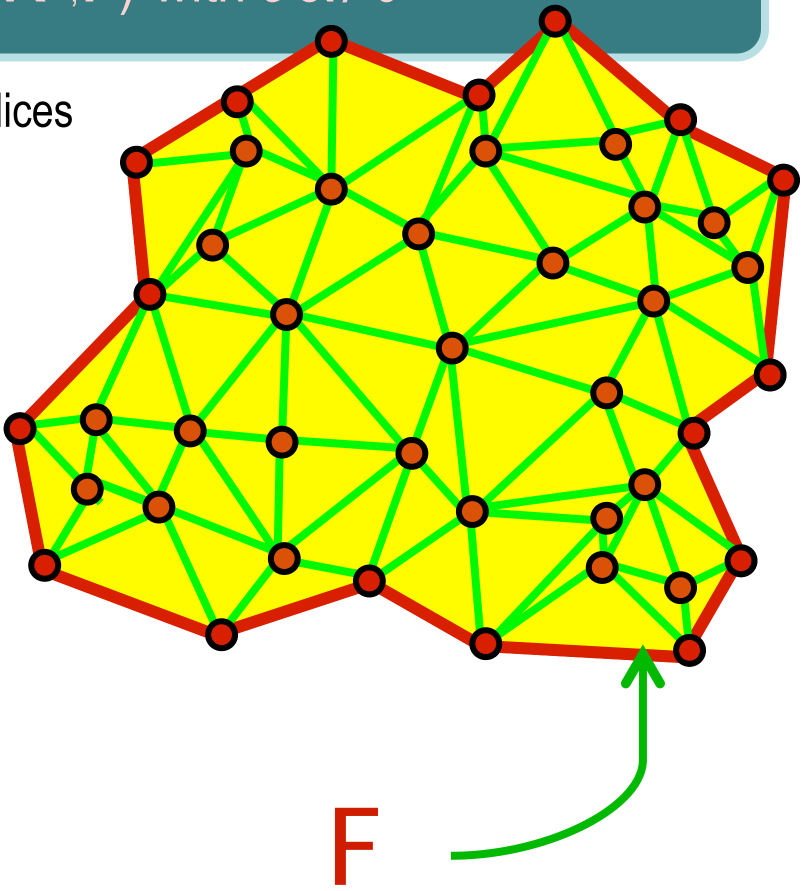

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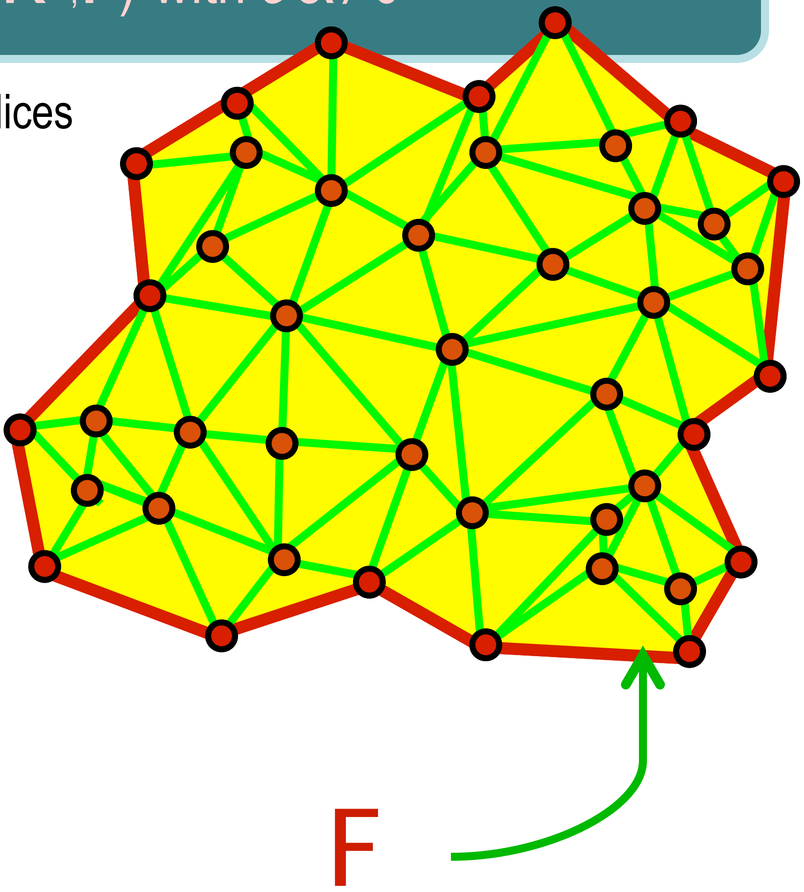

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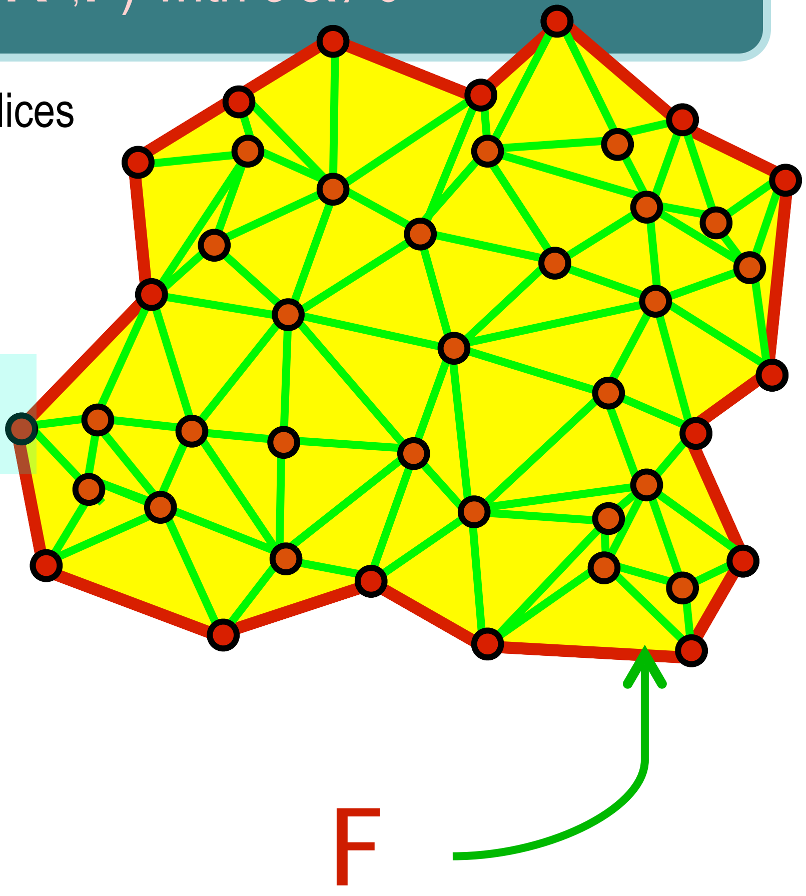

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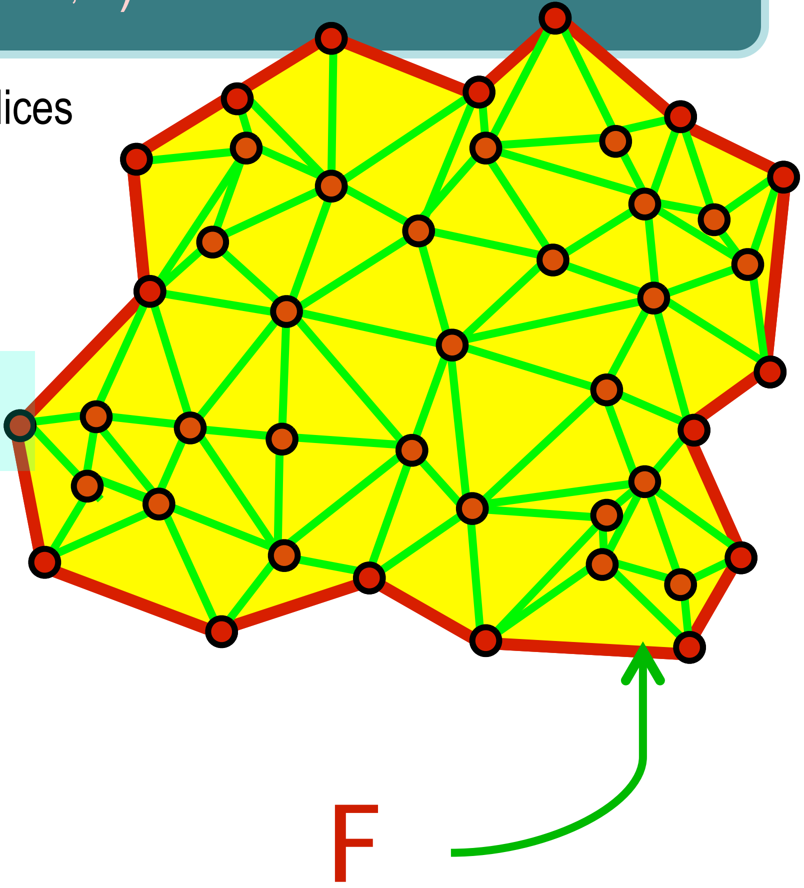

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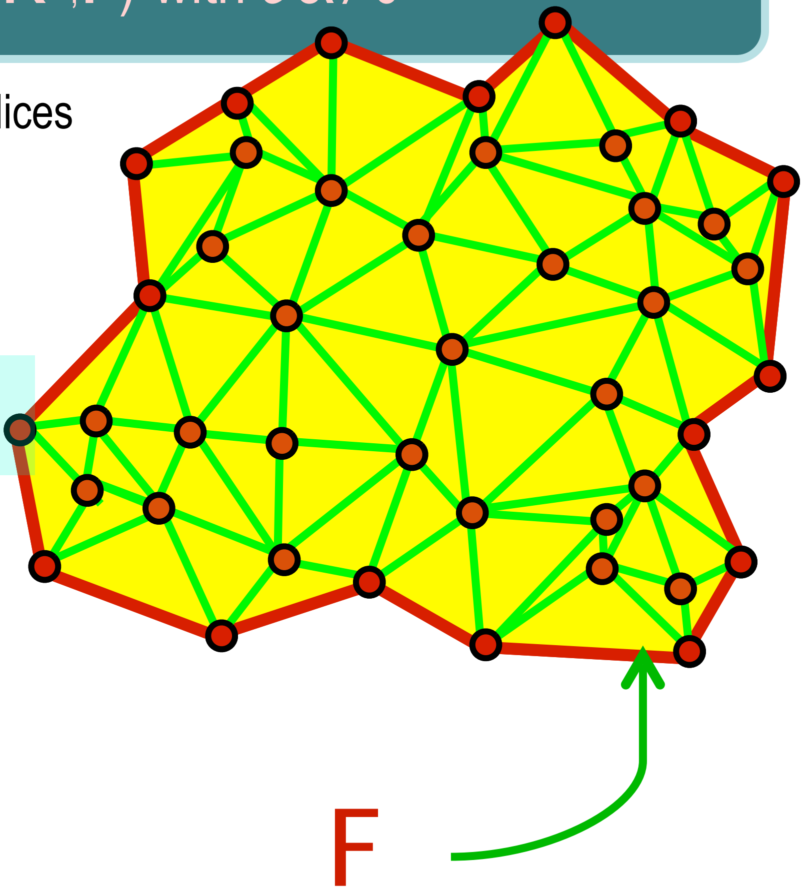

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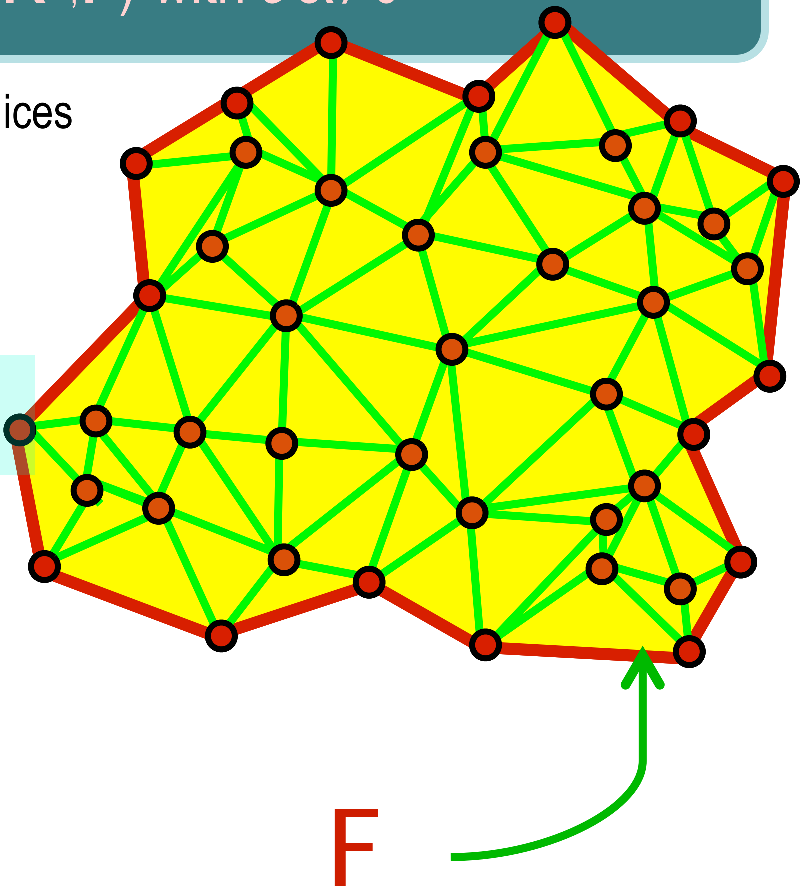

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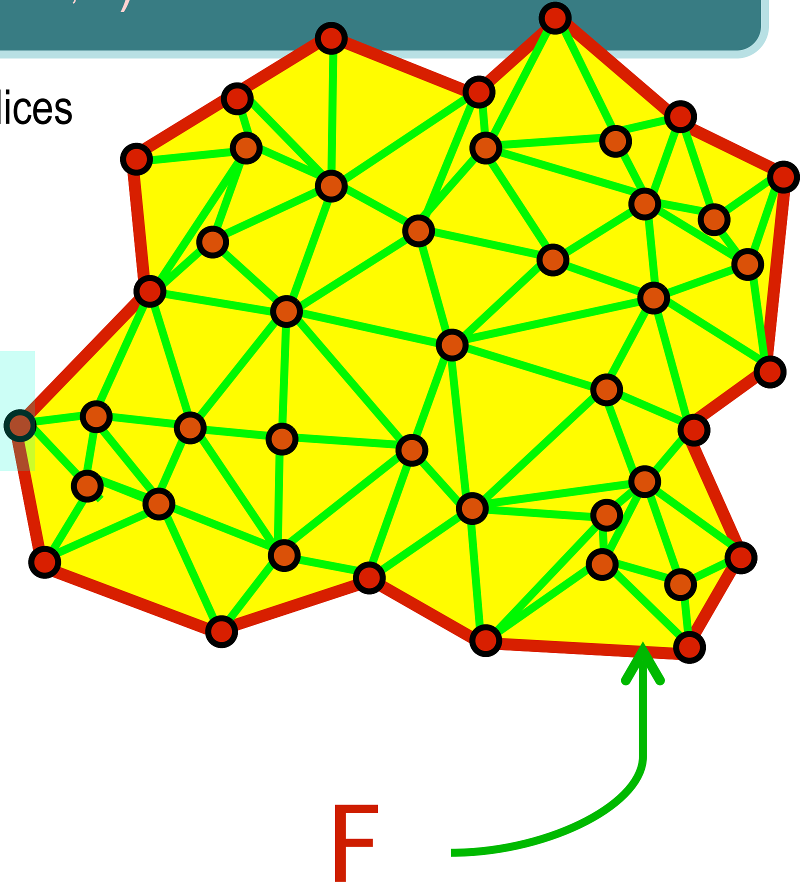

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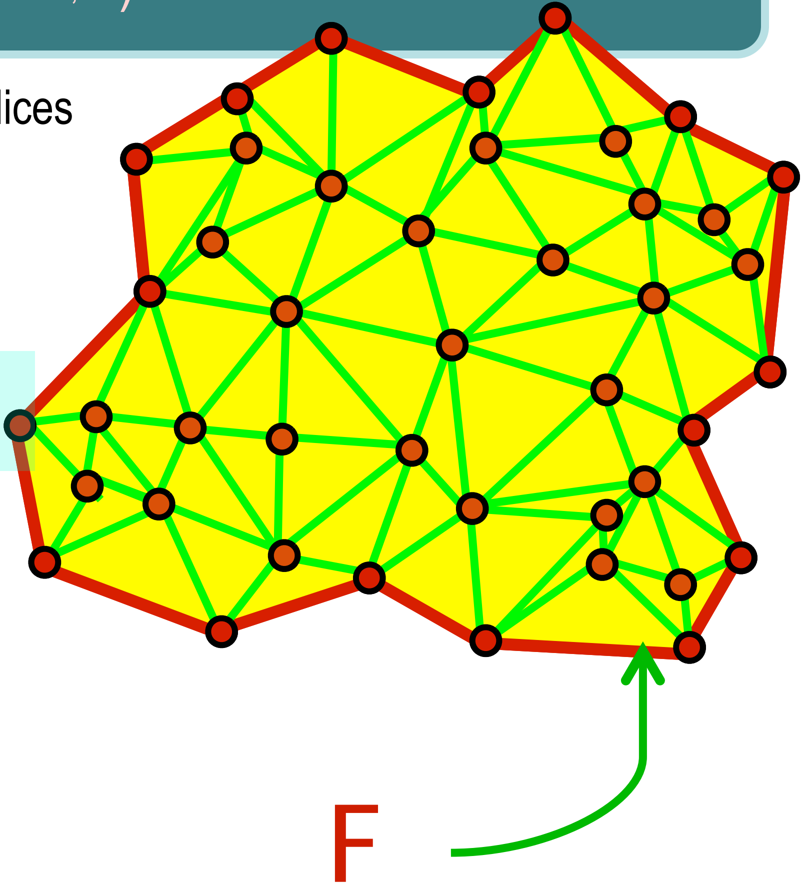

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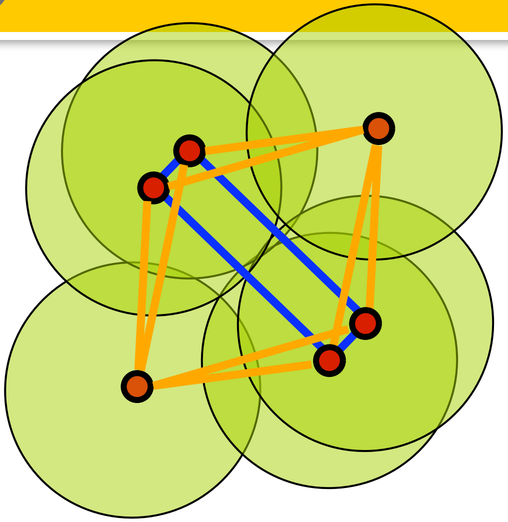

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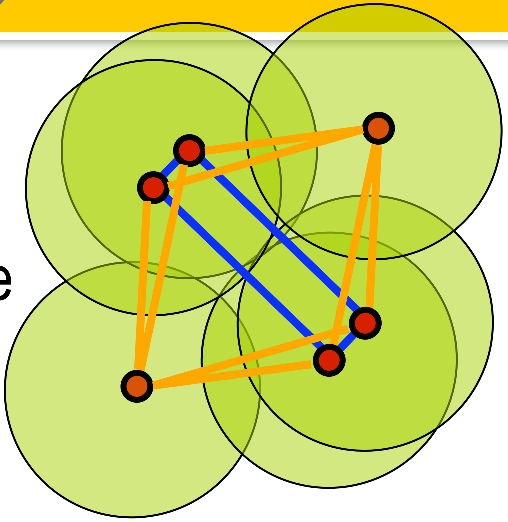

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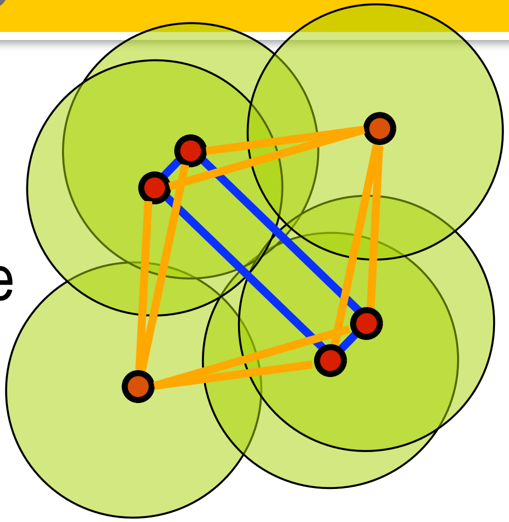

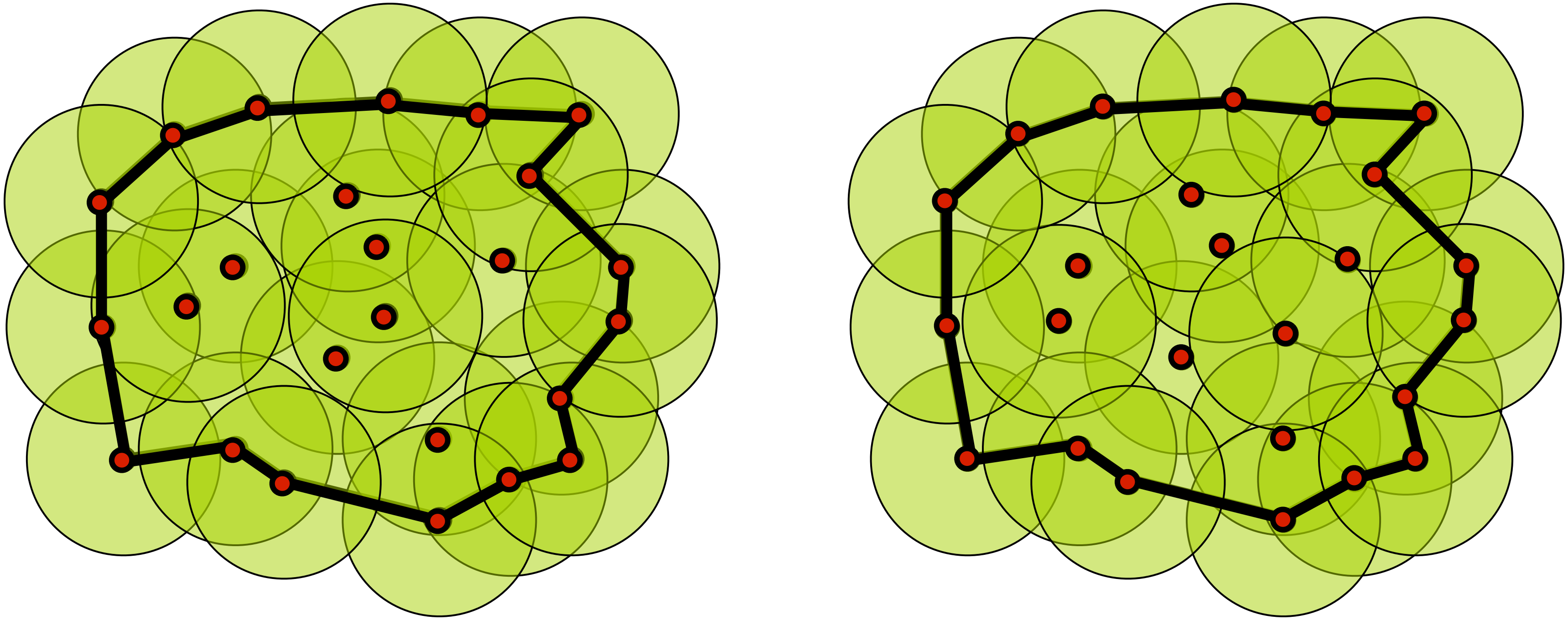

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Computation via PLEX [de Silva, 2006]

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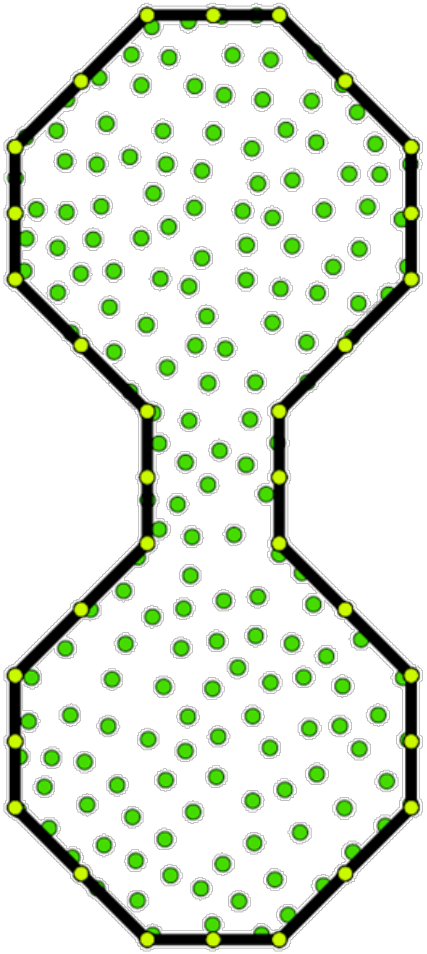

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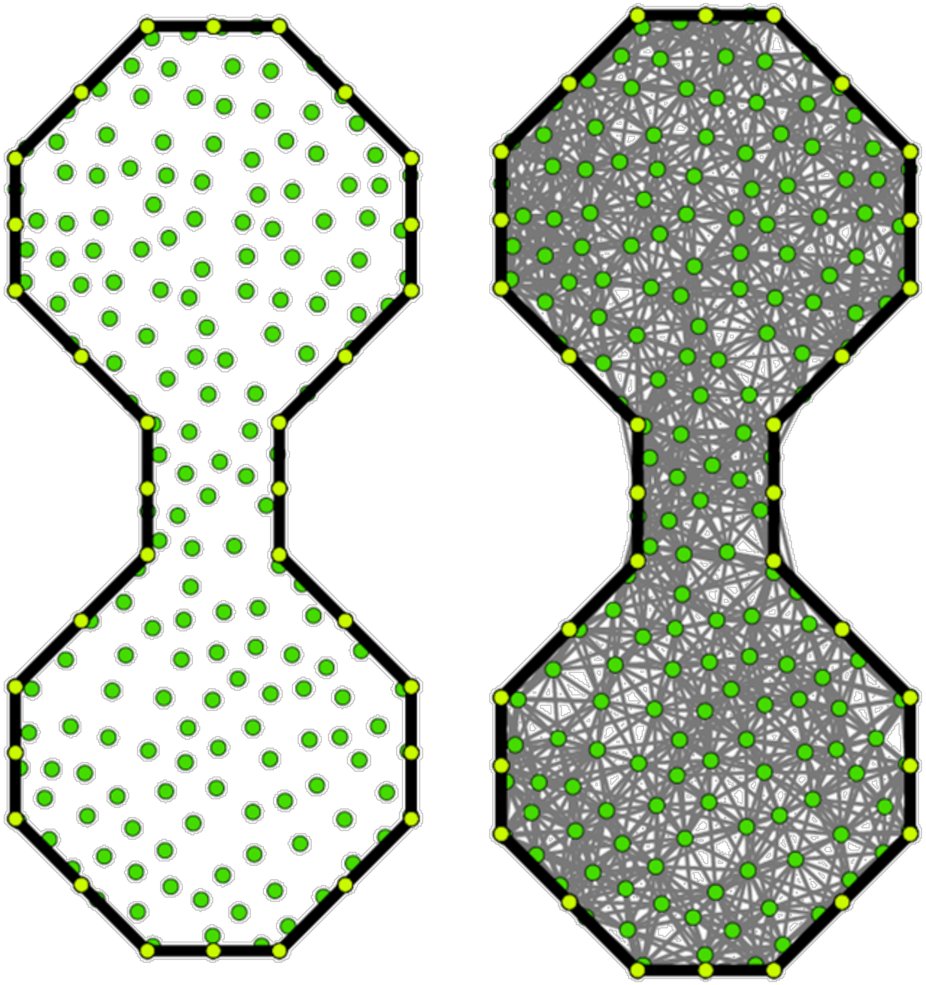

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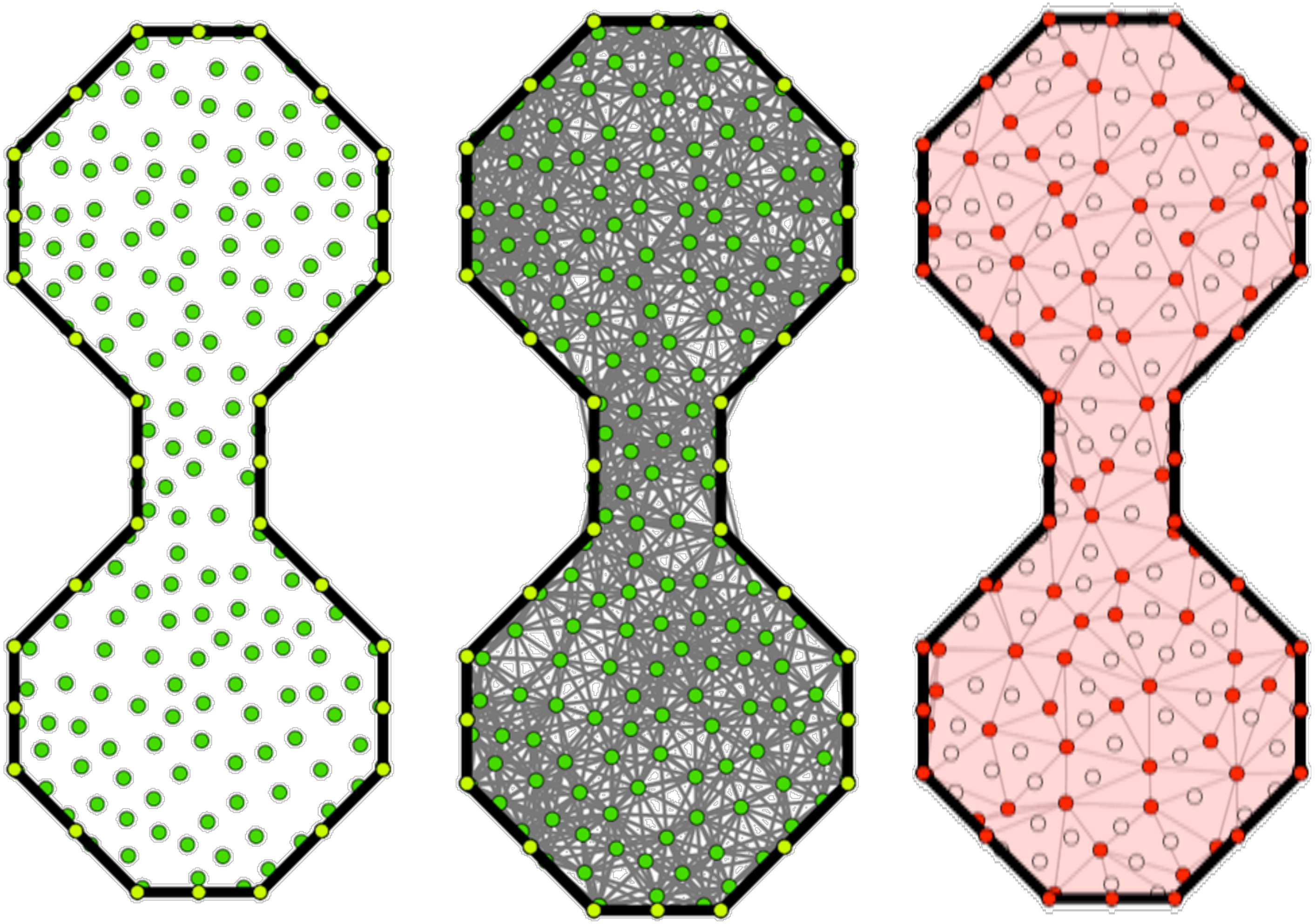

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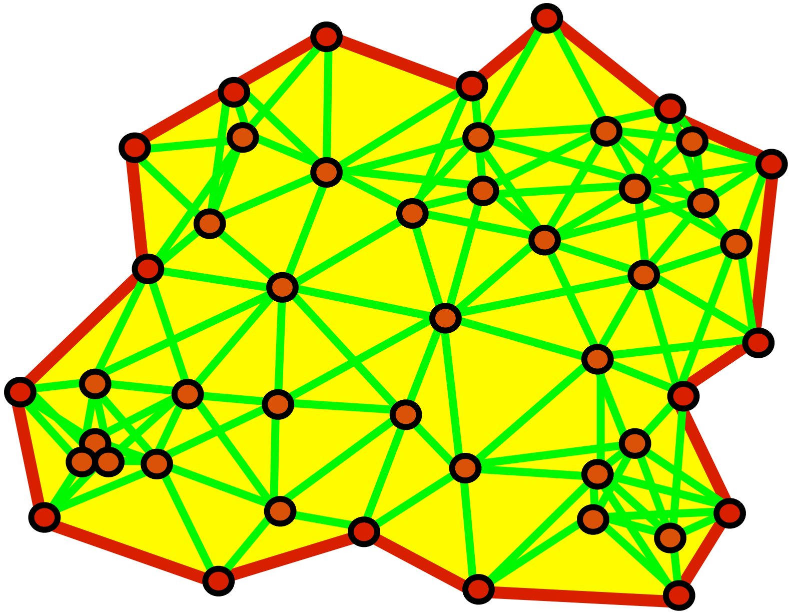

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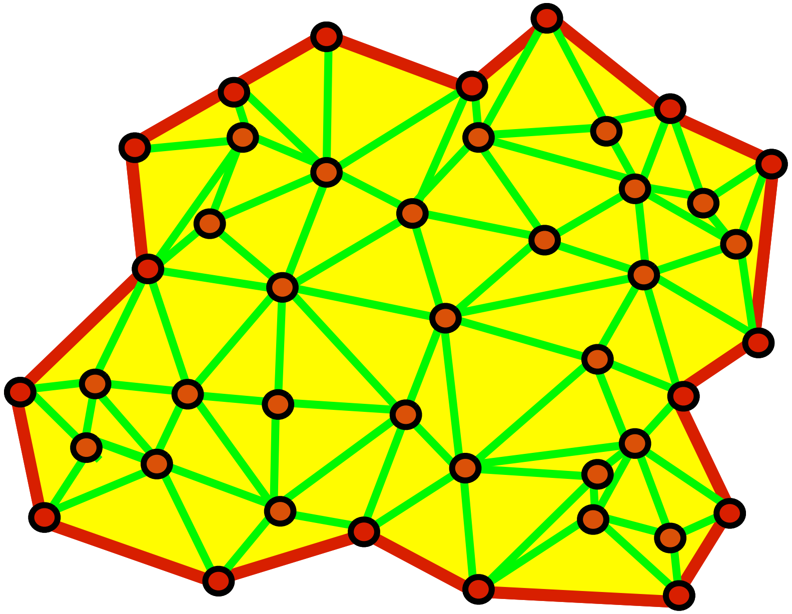

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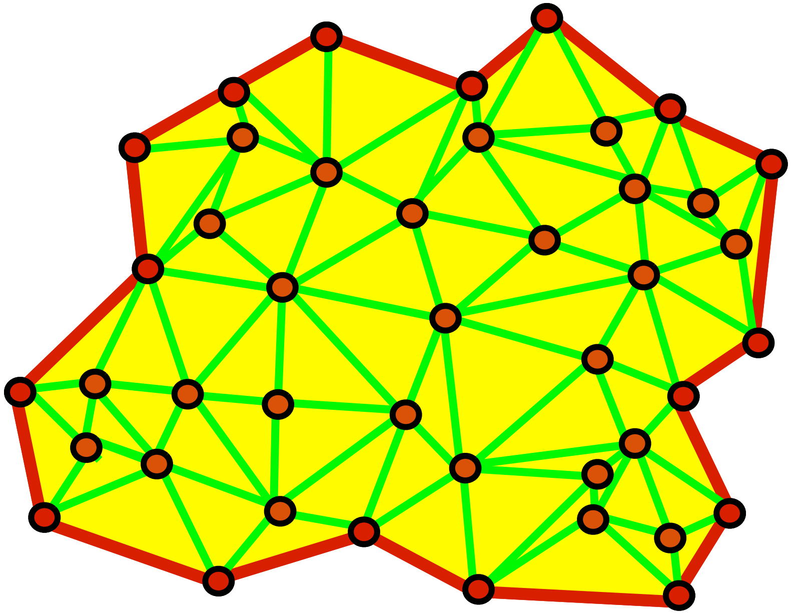

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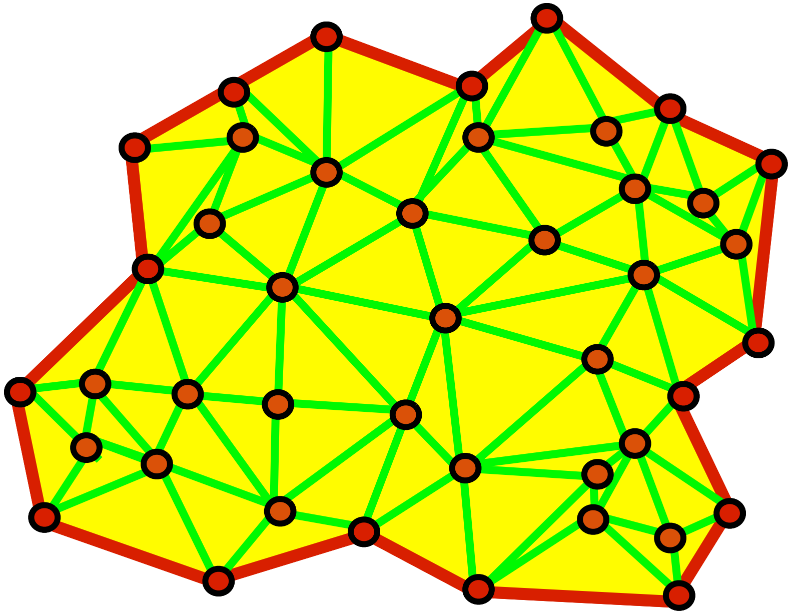

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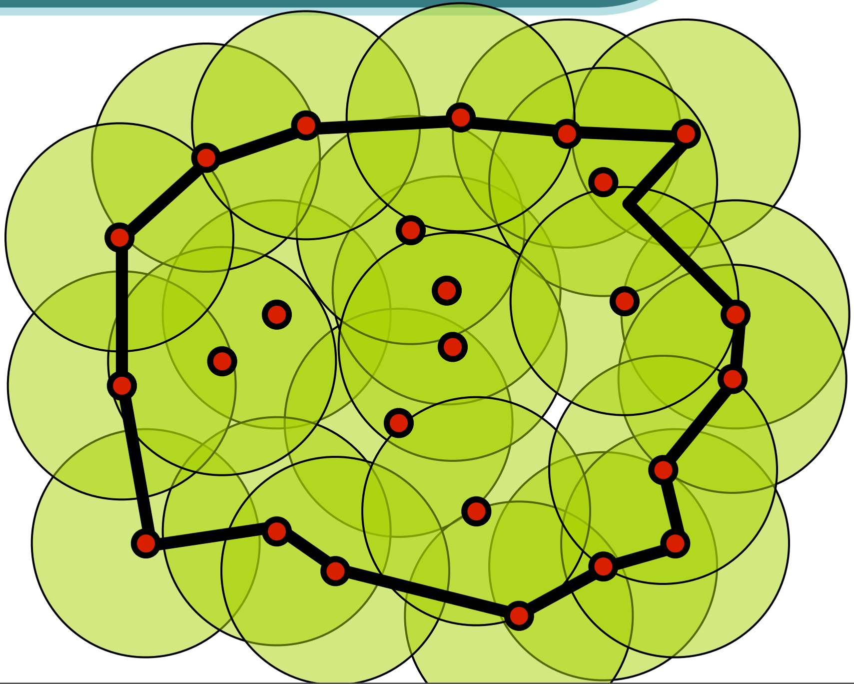

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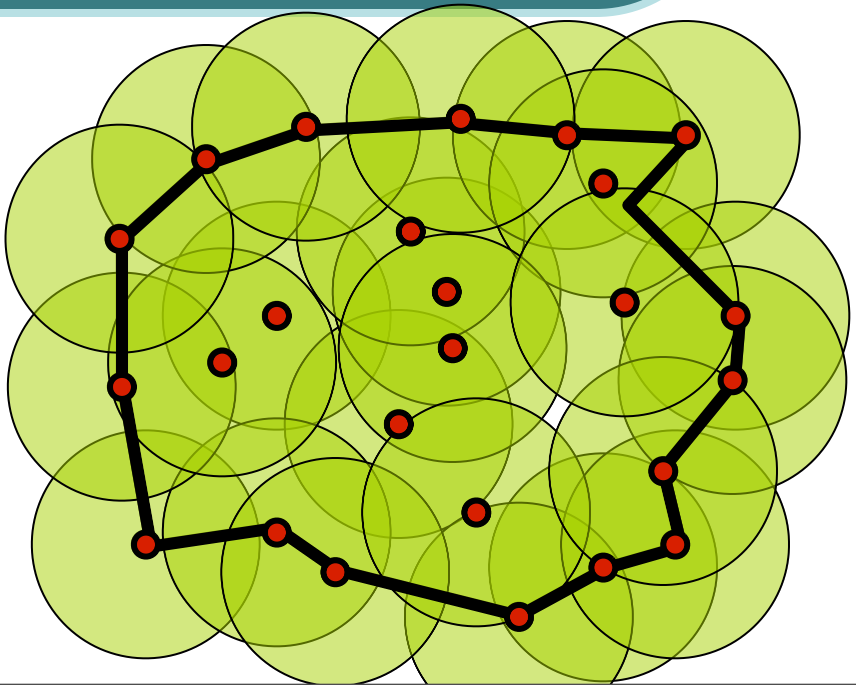

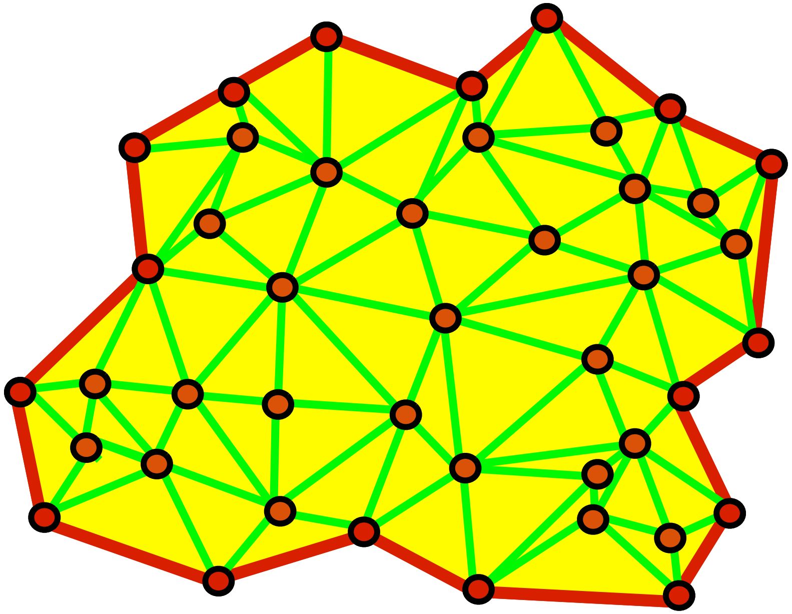

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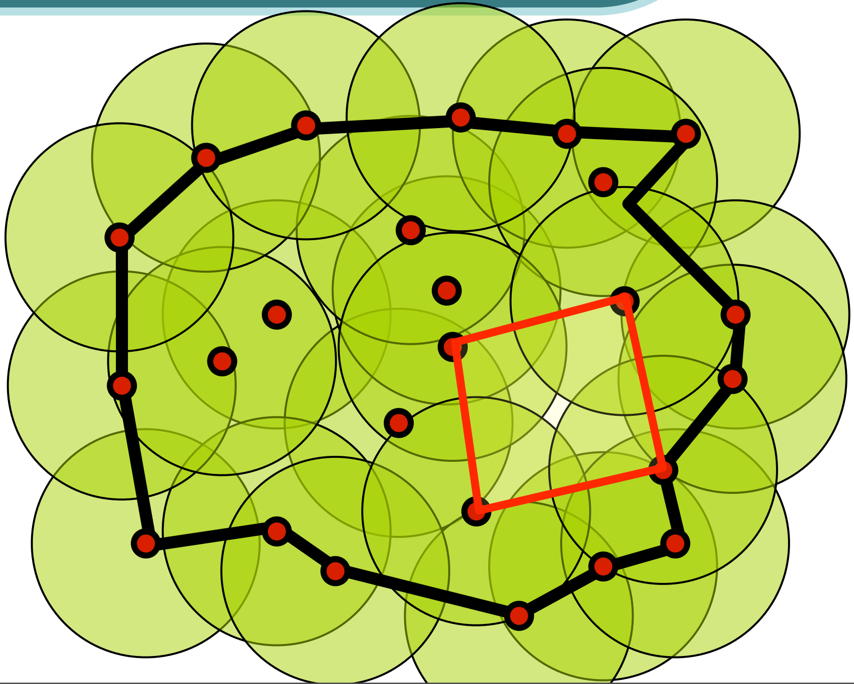

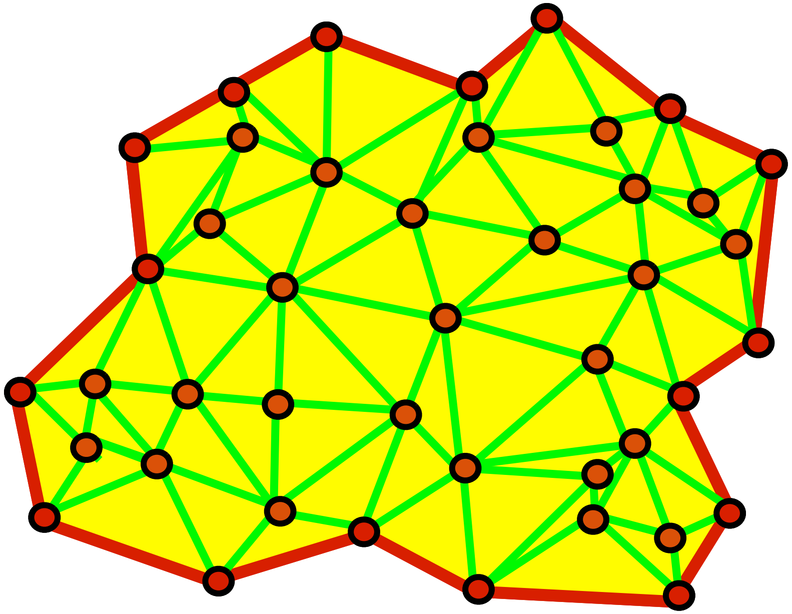

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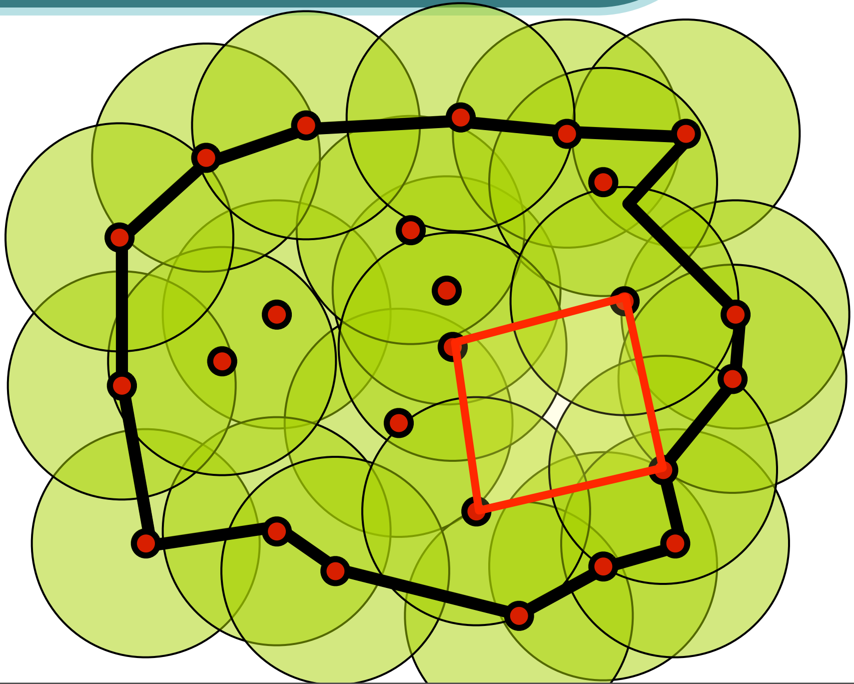

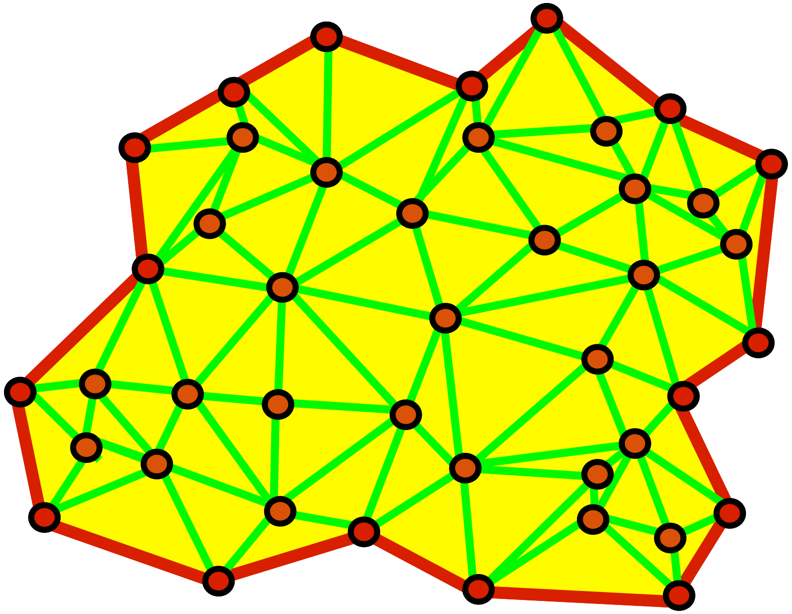

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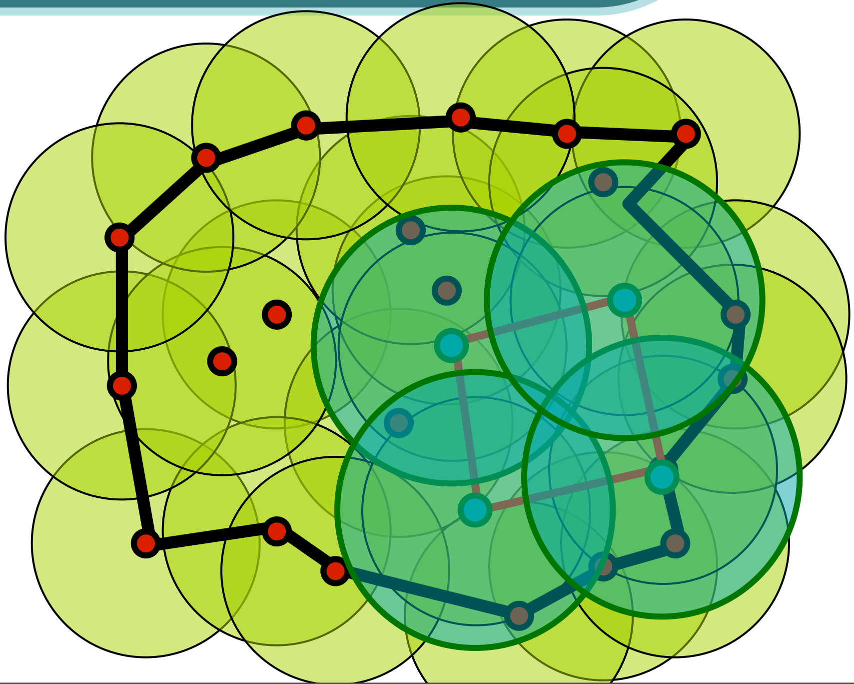

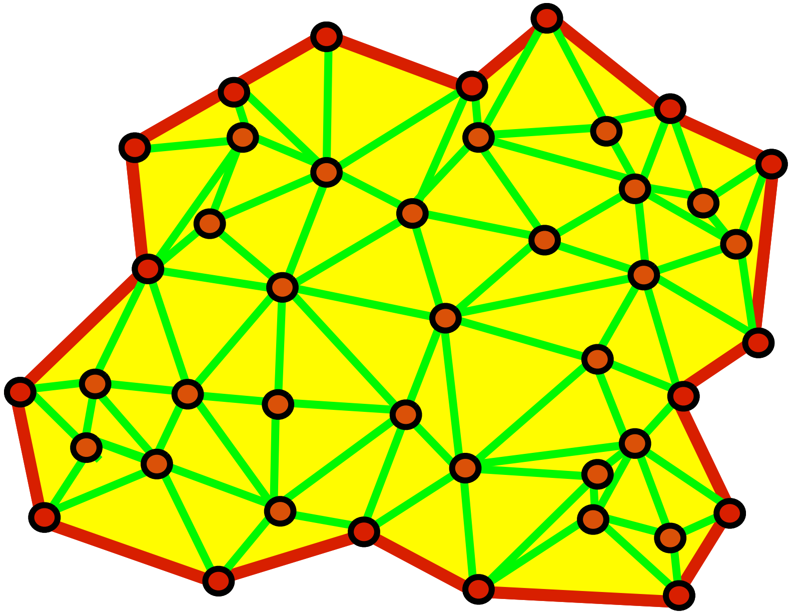

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Mrozek et al.

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$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1 \xleftarrow{\partial} C_2 \xleftarrow{\partial} C_3 \xleftarrow{\partial} \dots$$

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$$\boxed{\frac{dc}{dt} = - \mathcal{L}_k\, c(t)}$$

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Egerstedt & Muhammad

$$\frac{dc}{dt} = - \mathcal{L}_k \, c(t)$$

dynamics of heat flow is globally asymptotically stable iff $H_k = 0$

Mrozek et al.

distributed algebraic algorithms…

coverage: distributed

question: is the computation distributable?

use dynamics…

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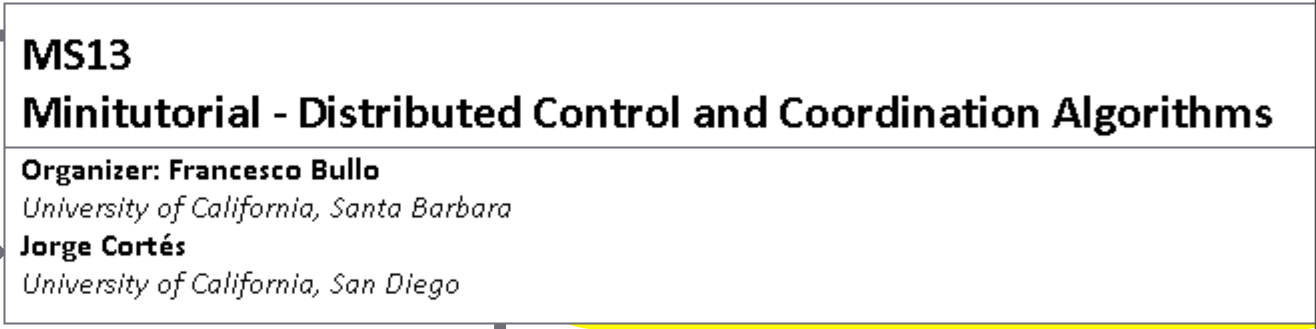

MS13

Minitutorial - Distributed Control and Coordination Algorithms

Organizer: Francesco Bullo
University of California, Santa Barbara
Jorge Cortés
University of California, San Diego

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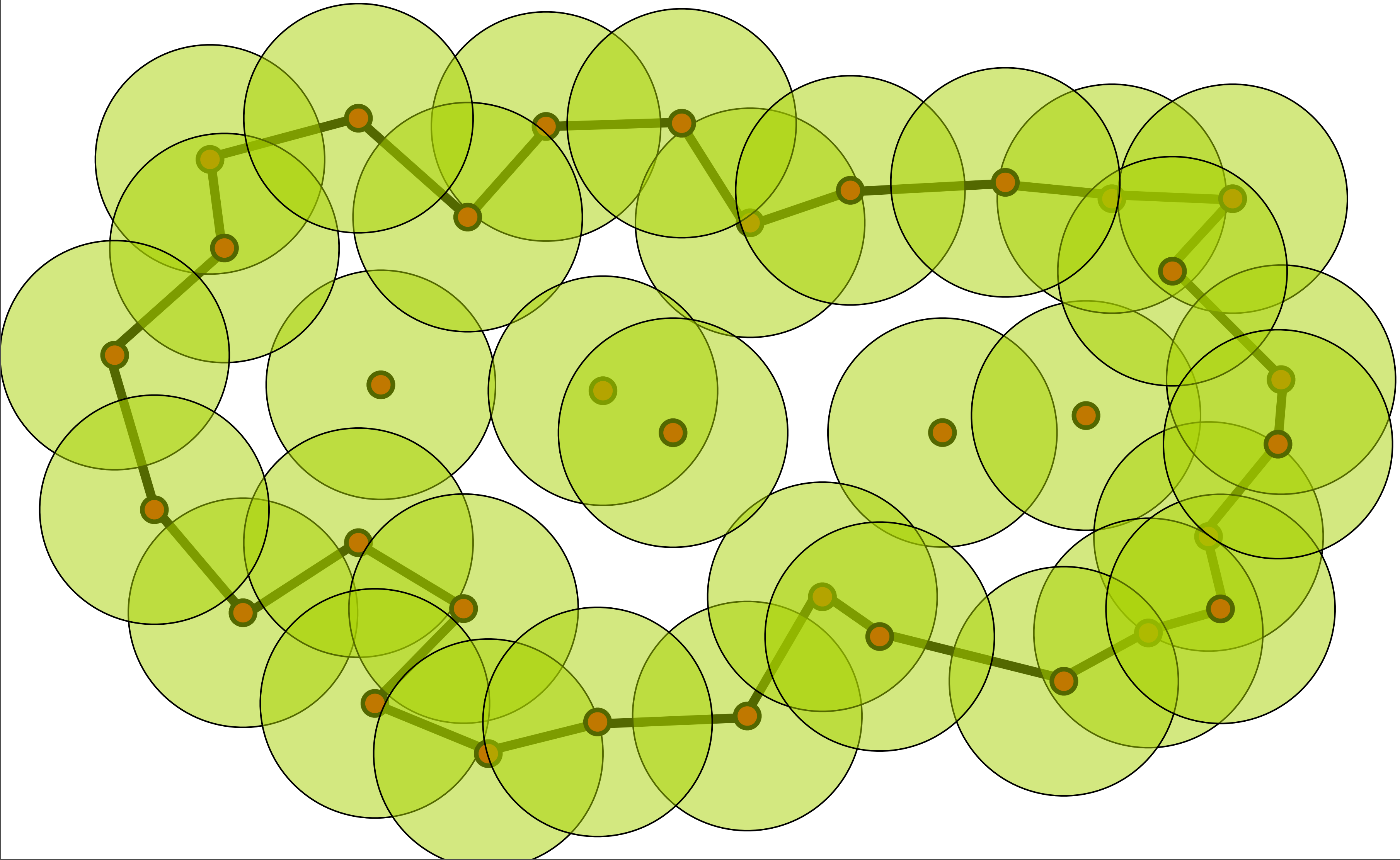

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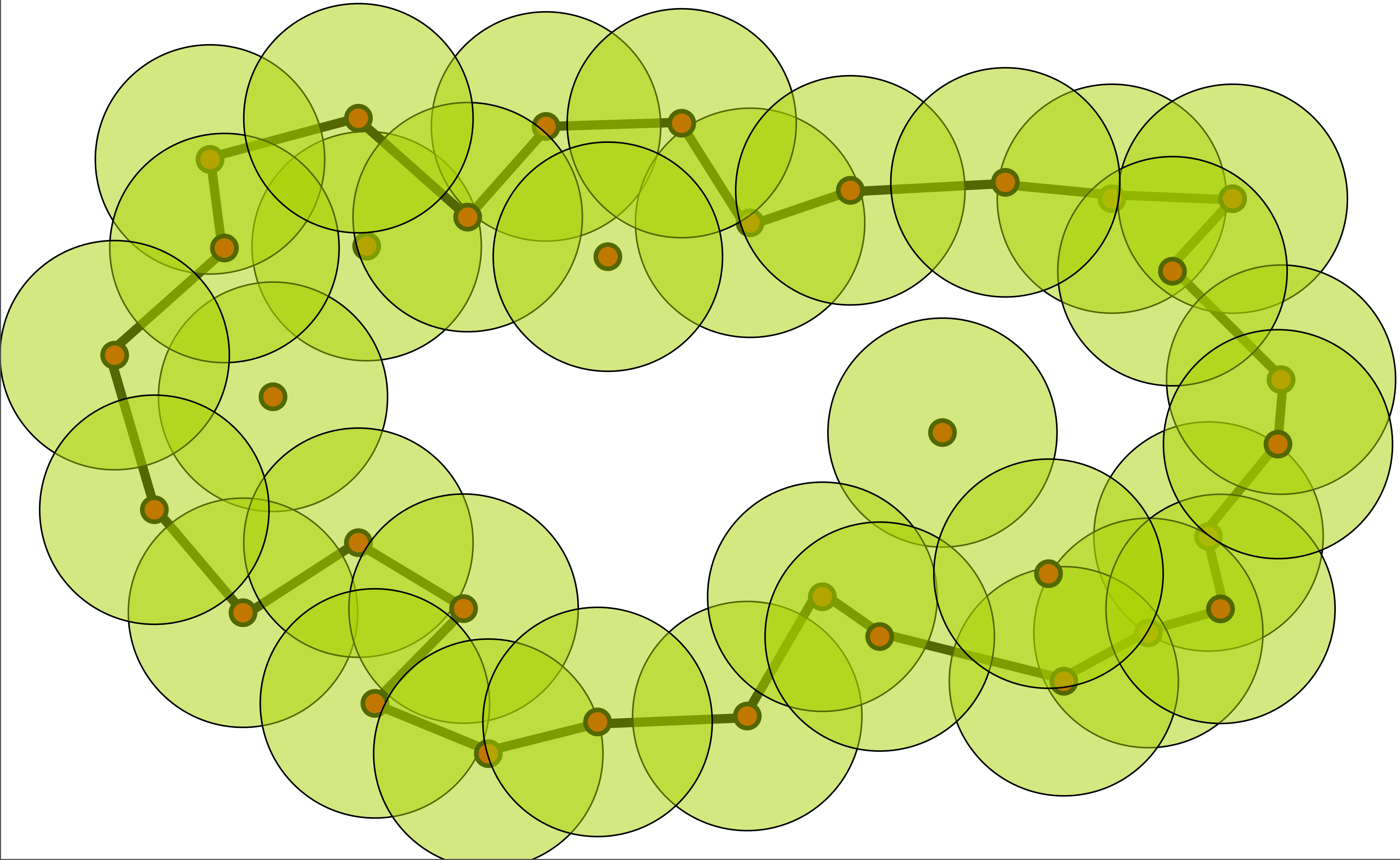

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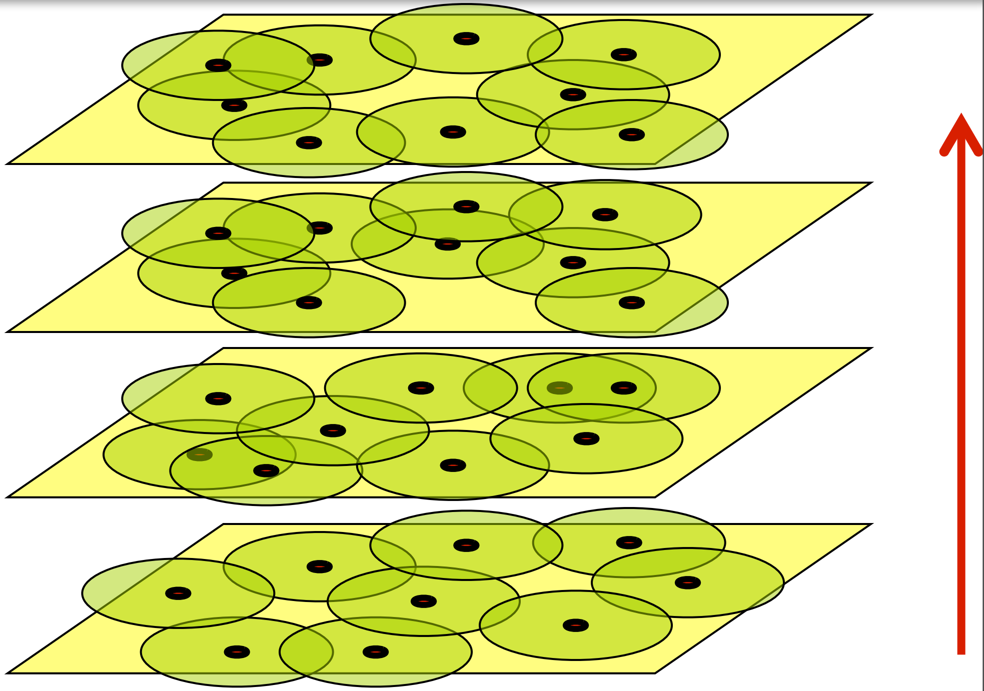
Given a sequence of updates to network graph (not too coarse; keep boundary fixed)

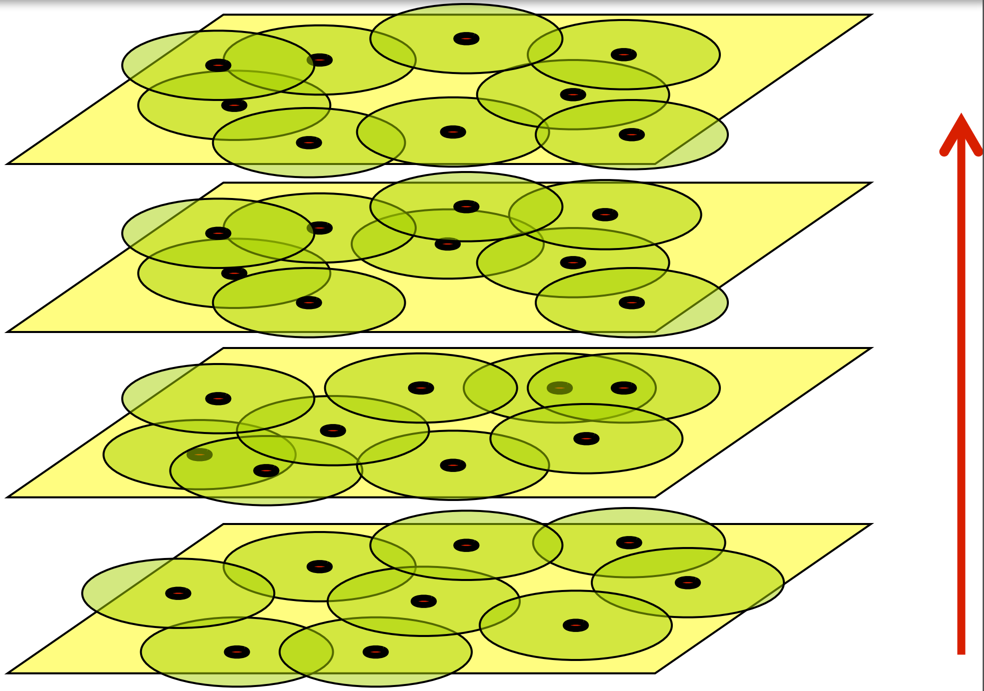


coverage: dynamic

Given a sequence of updates to network graph (not too coarse; keep boundary fixed)

Construct complexes

$R_{i,}$ i=1...N



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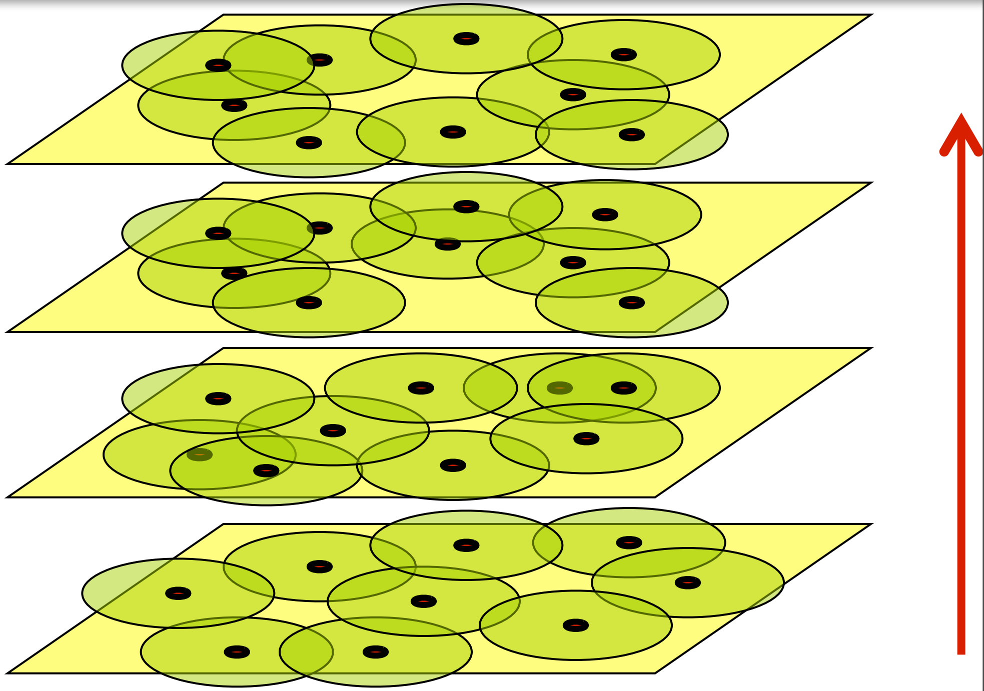

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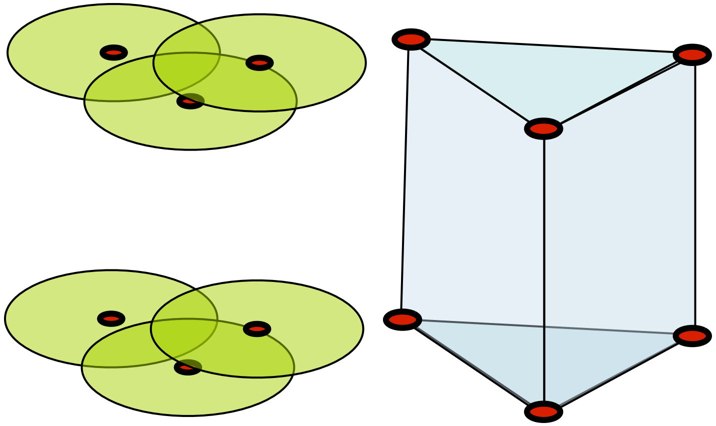
There is a natural way to “amalgamate” or stitch these complexes together to build a “hocolim” that admits a homological criterion for capture...



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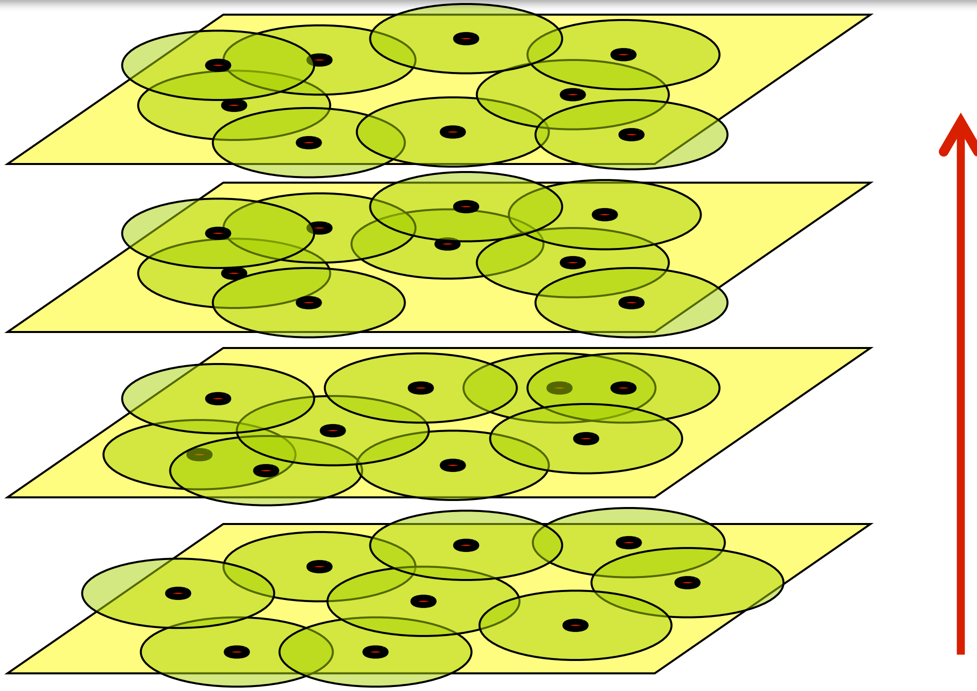

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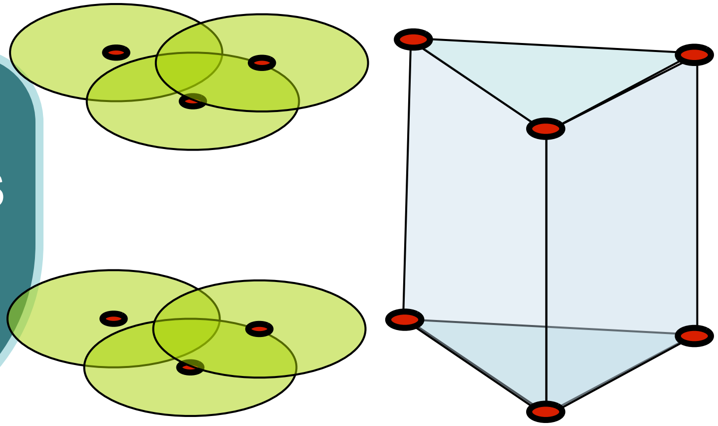
Theorem [DG]: there are no evaders in the mobile network if the homotopy colimit satisfies the homological coverage criterion

([α] in H_2 with $\partial\alpha \neq 0$ on F)



Homological Coverage

moral

Homological Coverage

a network is the skeleton of
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moral

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$$0 \xleftarrow{\partial} C_0 \xleftarrow{\partial} C_1$$

Homological Coverage

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moral

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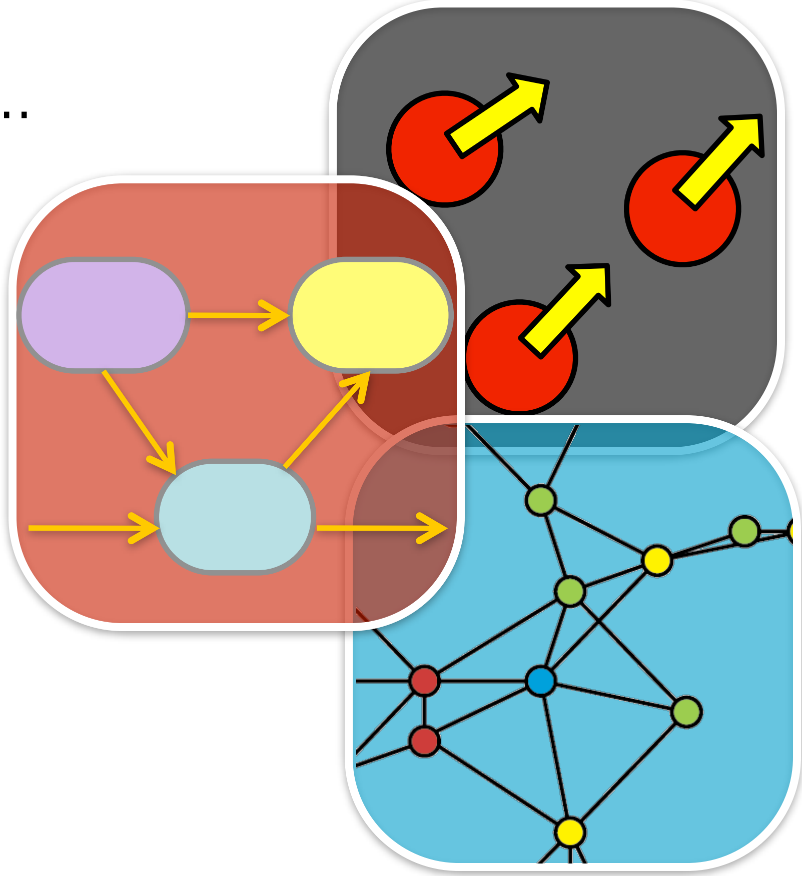
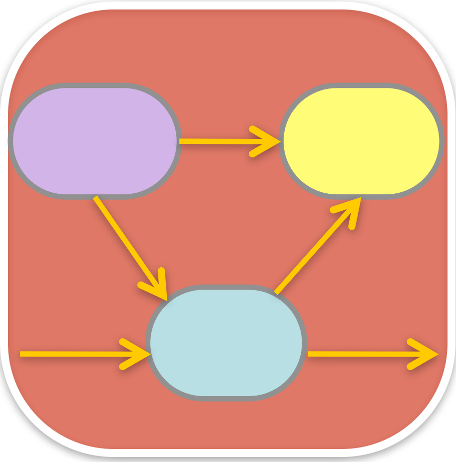
Deleted Scenes

homological methods in networks…



teaser

Persistence



Persistence

the proof of a theorem I won’t state…

teaser

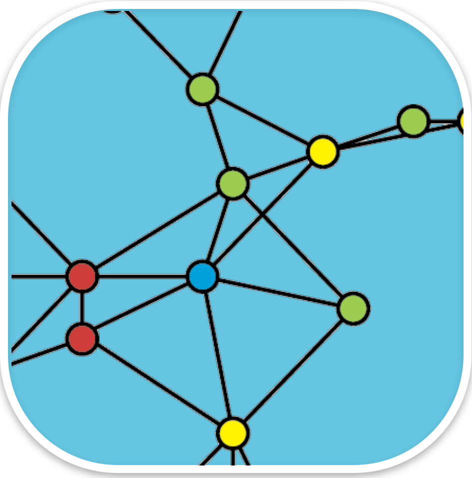
Persistence

the proof of a theorem I won't state…

$$\begin{tikzcd}
 & H_d(R_s) \arrow[r, "j_*"] \arrow[d, "i_*"] & H_d(R_s, F_s) \arrow[r, "\partial_*"] \arrow[d, "i_*"] \arrow[rd, bend left, "= 0"] & H_{d-1}(F_s) \arrow[d, "i_*"] \\
C_w \arrow[rd] & H_d(R_m) \arrow[l] \arrow[r, "j_*"] \arrow[d, "i_* = 0"] & H_d(R_m, F_m) \arrow[r, "\partial_*"] \arrow[d, "i_*"] & H_{d-1}(F_m) \arrow[d, "i_*"] \\
 & H_d(R_w) \arrow[r, "j_*"] & H_d(R_w, F_w) \arrow[r, "\partial_*"] & H_{d-1}(F_w)
\end{tikzcd}$$

teaser

Sheaf Integration



teaser

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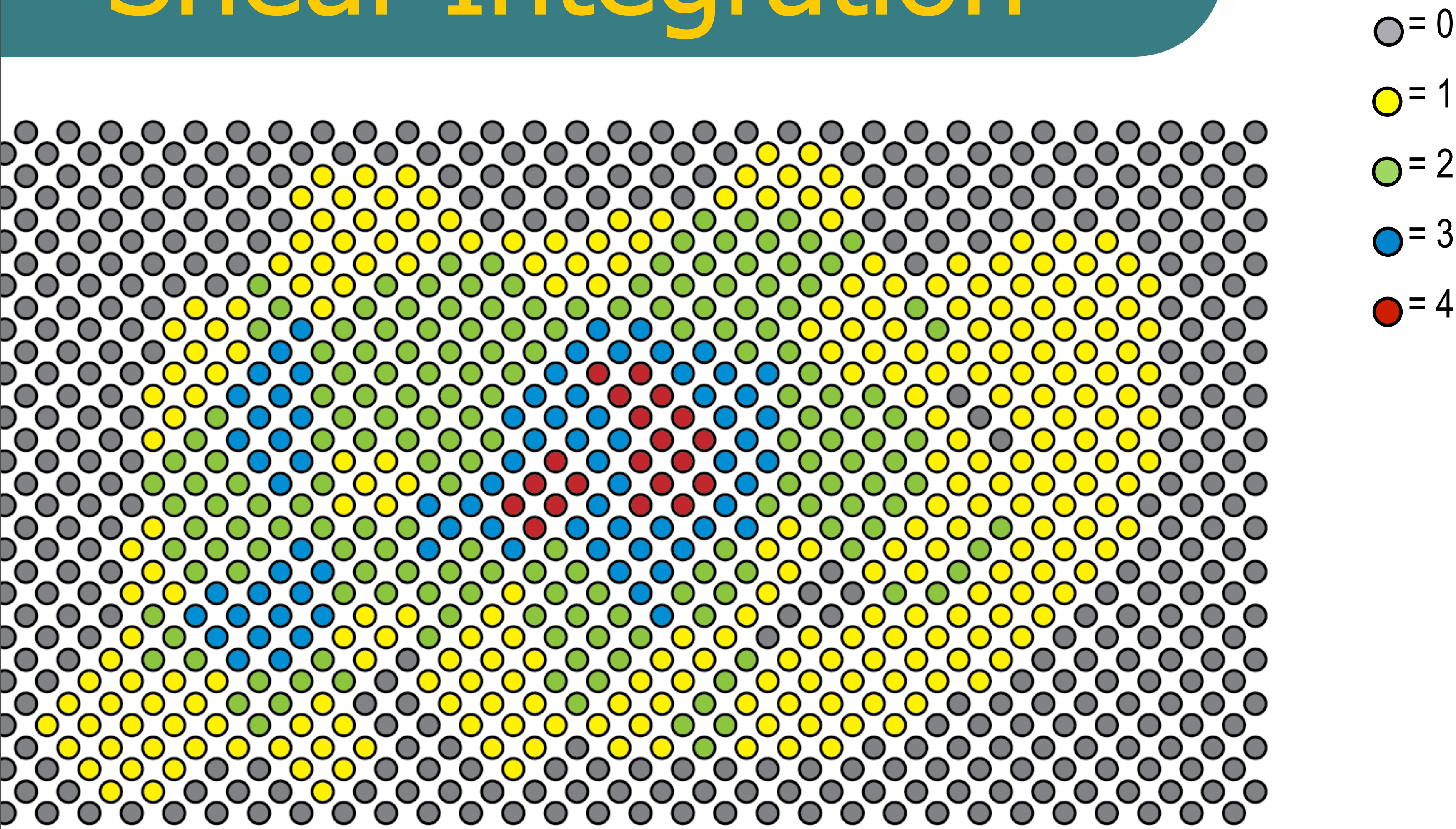

teaser

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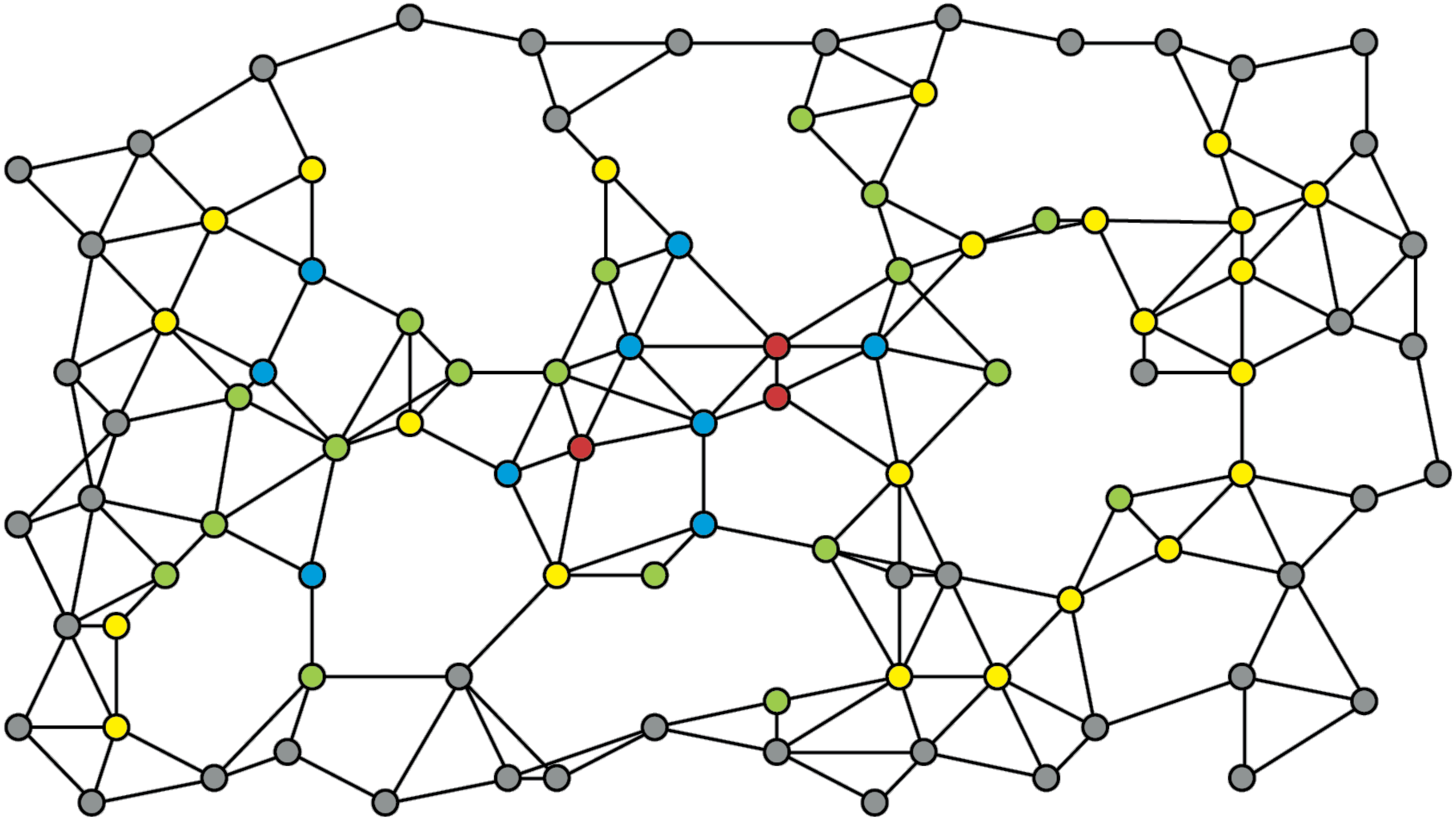


⚪ = 0

🟡 = 1

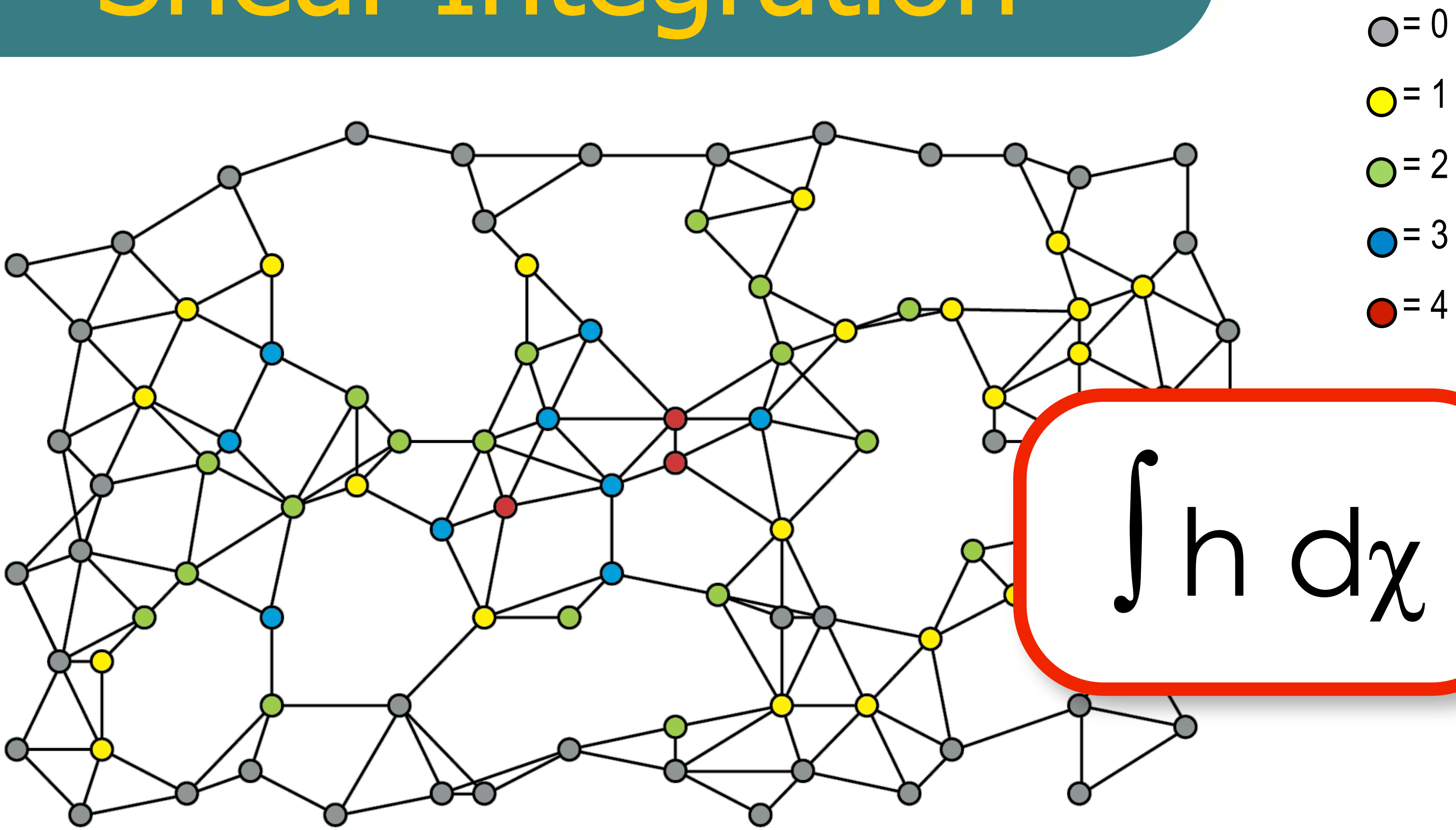
🟢 = 2

🔵 = 3

🔴 = 4

teaser

Sheaf Integration



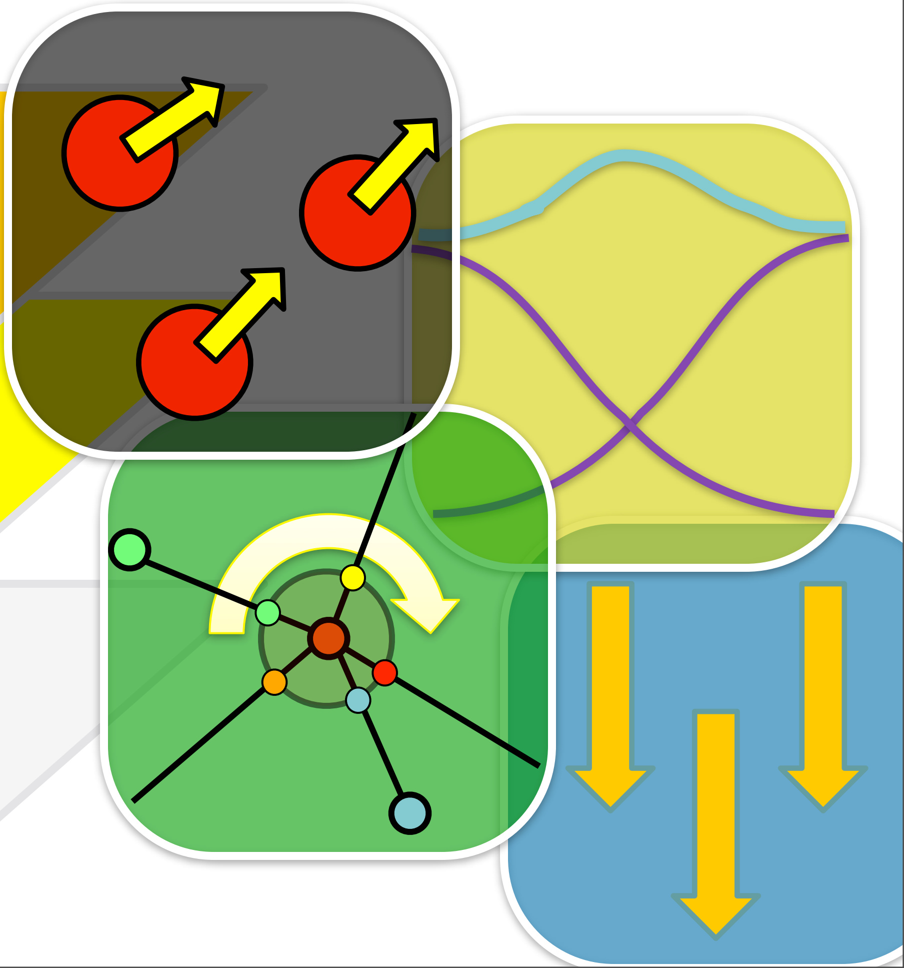
$$\int h \, d\chi$$

Homological Nets

stochastics

dynamics

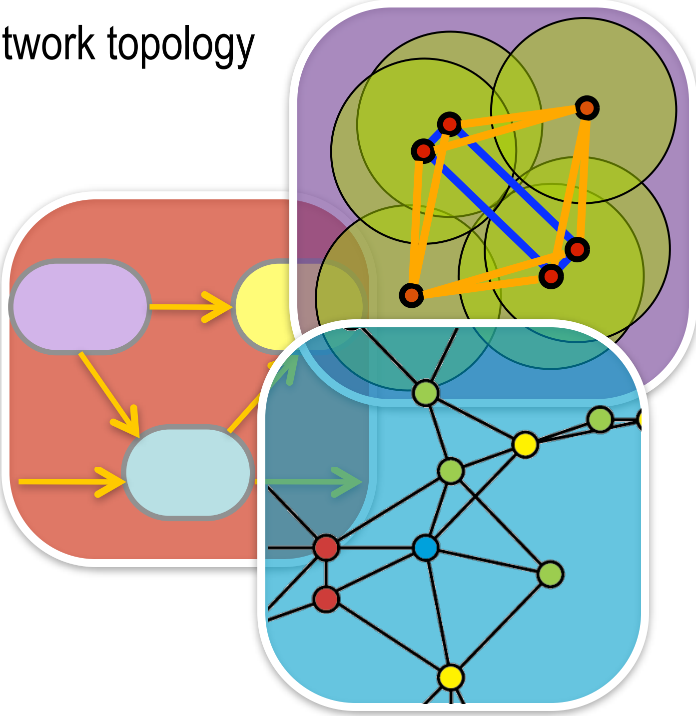
topology



fin

Closing Credits

These examples of algebraic topology in place of network topology brought to you via collaboration with



Homological Coverage

Persistence

V. de Silva

Euler Integration

Y. Baryshnikov

...and via the generous support of DARPA

Technology Flow

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Consider the flow of techniques from mathematics to engineering/science

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Prime example: invariant manifolds in the USA the sermon

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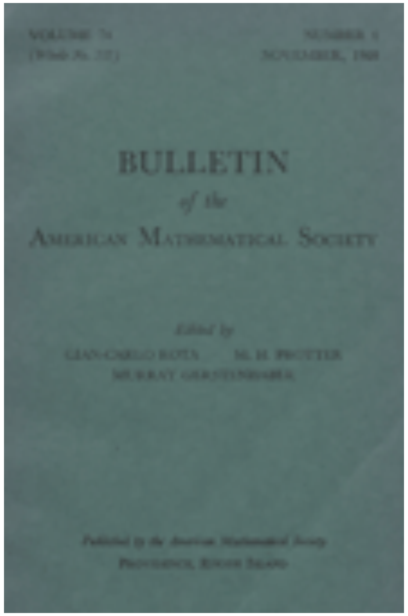

BULLETIN
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AMERICAN MATHEMATICAL SOCIETY

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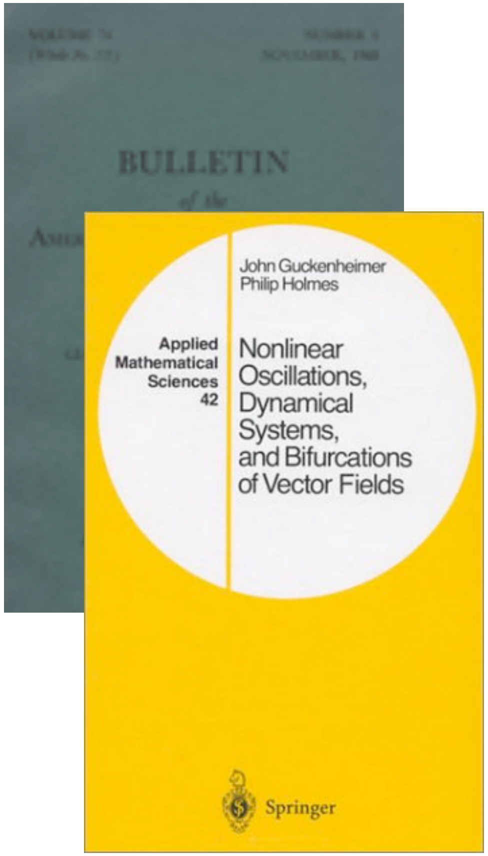

BULLETIN
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John Guckenheimer
Philip Holmes

Applied
Mathematical
Sciences
42

Nonlinear
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Dynamical
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and Bifurcations
of Vector Fields

Springer

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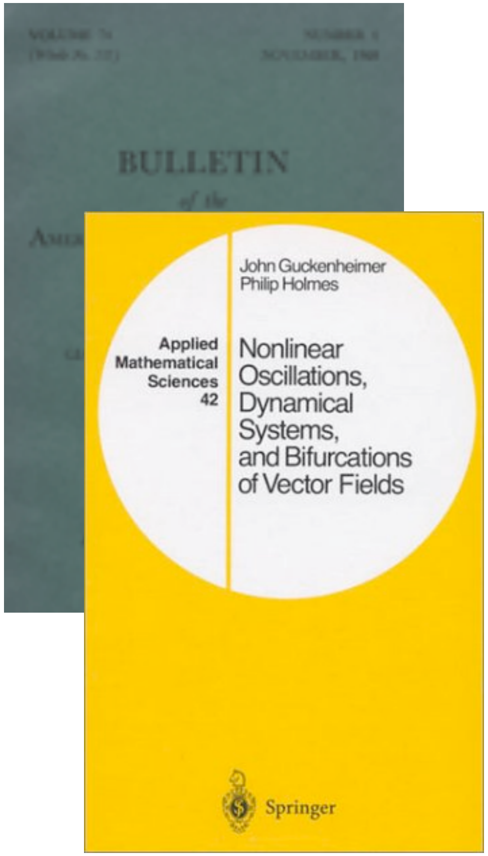

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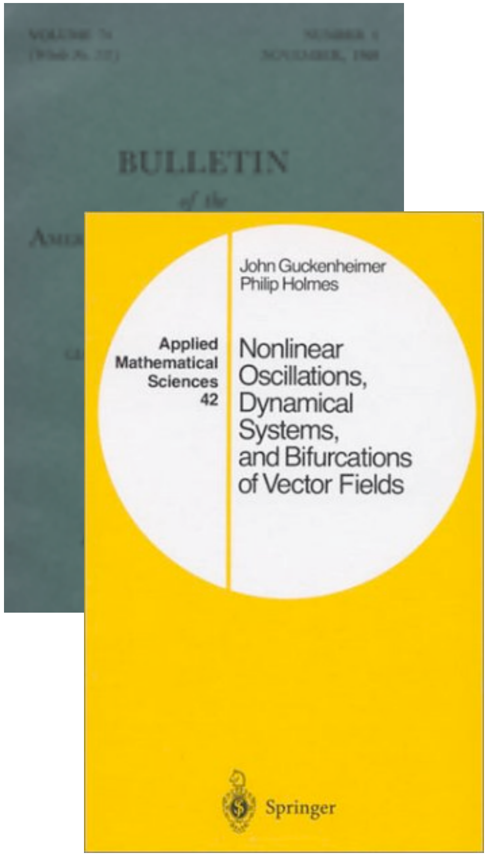
Prediction: homological methods are on the same trajectory



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