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SIAM DSWeb 2013 Contest – Teaching Dynamical Systems

Over the last few decades, interest in synchronization of dynamical systems on complex networks has exploded, making it a popular and important area of research in applied mathematics [1]. An important development that has allowed researchers to study and uncover generic mechanisms behind synchronization is the *Kuramoto model* [2,3], which describes the evolution of a population of N globally-coupled phase oscillators, which evolve according to their respective ODEs

$$\dot{\theta}_n = \omega_n + \frac{K}{N}\sum_{m=1}^{N}\sin(\theta_m - \theta_n),$$

where θ_n is the phase of oscillator n, ω_n is the natural frequency of oscillator n [typically drawn from a distribution denoted by $g(\omega)$], and K is the global coupling strength. In addition to this globally coupled formulation, researchers have explored the dynamics of Kuramoto oscillators in other scenarios, including (but not limited to) network structure [4], modular networks [5], and time-delays [6].

My contribution is a piece of software called *Synched* that allows any user, ranging from a first-time dynamical systems student exploring synchronization for the first time to a senior researcher presenting at a conference, to simulate and visualize in real time the phenomenon of synchronization using Kuramoto oscillators. Upon opening the application, a simulation of the standard all-to-all Kuramoto model begins, depicting oscillators as small circles moving along the outer circle according to their evolution ODE and an order parameter, describing the degree of synchronization, depicted by a larger circle in the middle. The natural frequencies are drawn from the Lorentzian distribution $g(\omega) = \pi^{-1}/[1+(\omega-\omega_0)^2]$. This software allows the user to tune both the global coupling strength K and the mean frequency ω_0 to explore the dynamics of the system and observe the phenomenon of synchronization in real time. For instance, when K becomes large enough, a subset of the population, i.e. oscillator that turn yellow, becomes phase-locked, representing a state of partial synchronization. The user can also increase the number of oscillators by choosing between system sizes of N = 49, 100, 400, or 900. *Synched* is for MacOSX and free to download and use for academic purposes at amath.colorado.edu/student/skardal/**Synched**.html.

In addition to the standard all-to-all Kuramoto model, the user can explore the effect of community structure [5], time-delays [6], and clustering [7] by changing the model type. These models introduce more complexity into the system, and have more tunable parameters that the user can change in real time. The specifics of each model, as well as the relevant parameters and their interpretations, are described in detail on the website.

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