Suppression of Complex Collective Dynamics in a Network of Theta Neurons by Synaptic Diversity

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Abstract

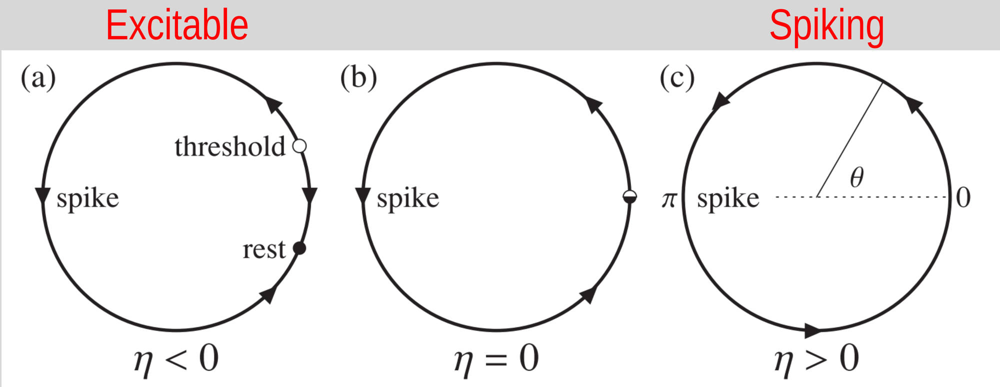
In the wake of a study that examined the collective dynamics of a large network of uniformly coupled Type 1 neurons [1], we explore how macroscopic behavior changes with respect to alterations in different neuronal parameters of a model that includes synaptic diversity. Our analysis demonstrates that heterogeneity in synaptic strength increases the robustness of equilibrium states and increasing synaptic diversity suppresses the emergence of the collective rhythmic state.

Background

Single Theta Neuron

$$\dot{\theta} = (1-\cos\theta) + (1+\cos\theta)\eta$$

θ is the phase variable [0, 2π) characterizing state of the neuron and η is the input current [2].



η represents the excitability of the neuron and controls whether the neuron is inherently resting or spiking, the spiking threshold, and the spiking rate.

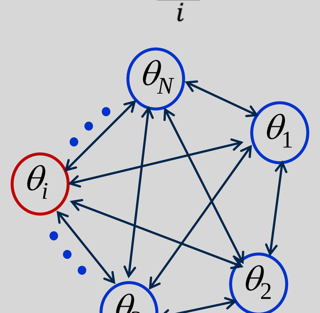
Model Formulation

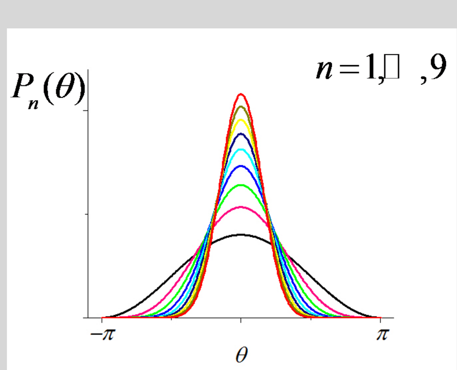
Network of Theta Neurons

$$\dot{\theta}_j = (1-\cos\theta_j) + (1+\cos\theta_j)[\eta_j + I_{syn}]$$

Network Connectivity

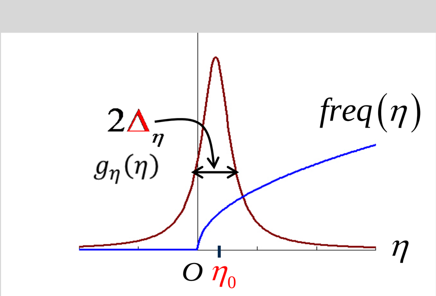
$$I_{syn} = \frac{k}{N}\sum_i^N P_n(\theta_i) \qquad P_n(\theta_i) = a_n(1+\cos\theta_i)^n \quad [3]$$

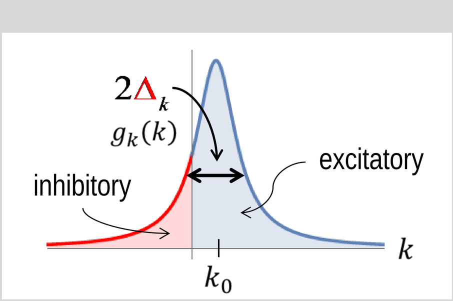




Network Heterogeneity

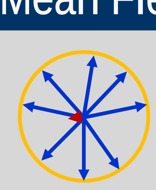
$$g_\eta(\eta) = \frac{1}{\pi}\frac{\Delta_\eta}{(\eta-\eta_0)^2+\Delta_\eta^2} \qquad g_k(k) = \frac{1}{\pi}\frac{\Delta_k}{(k-k_0)^2+\Delta_k^2}$$

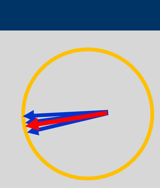




Macroscopic Mean Field

$$z = \rho e^{i\psi} \left(= x+iy\right) \equiv \frac{1}{N}\sum_{i=1}^N e^{i\theta_i}$$


Incoherent


Coherent

Reduction

We follow the mean-field approach by considering the limit $N\to\infty$. The network can be described by a probability density function $F(\theta,\eta,k,t)$ which satisfies the continuity equation,

$$\frac{\delta F}{\delta t} + \frac{\delta}{\delta\theta}(F v_\theta) = 0. \qquad (1)$$

The velocity v_θ of a neuron and the order parameter are continuum versions of their discrete equations:

$$v_\theta = (1-\cos\theta) + (1+\cos\theta)\left[\eta + k a_n \int_0^{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\theta',\eta',k',t)(1-\cos\theta')^n d\eta' dk' d\theta'\right] \qquad (2)$$

$$z(t) \equiv \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} F(\theta,\eta,k,t) e^{i\theta}\, dk \qquad (3)$$

To obtain a reduced dynamical system, we substitute the Ott-Antonsen ansatz [4],

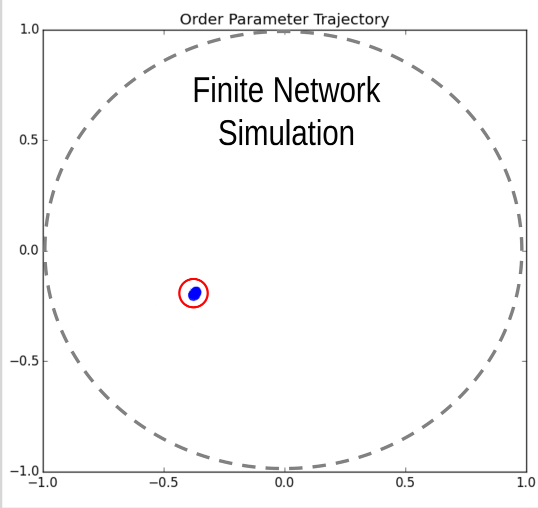
$$F(\theta,\eta,k,t) = \frac{g(\eta,k)}{2\pi}\left\{1 + \sum_{q=1}^{\infty}(\alpha^*(\eta,k,t)^q e^{iq\theta} + \text{c.c.})\right\} \qquad (4)$$

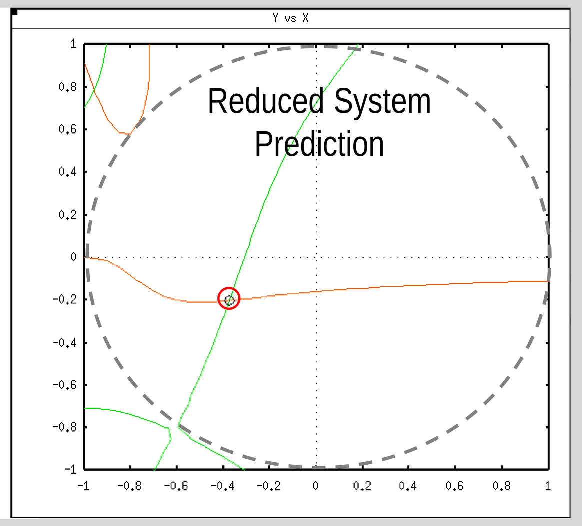
in which $g(\eta,k)=g_\eta(\eta)*g_k(k)$ is the joint probability distribution for η and k, into (1) and (3) and obtain:

$$\dot{z} = -i\frac{(z-1)^2}{2} + \frac{(z+1)^2}{2}\left[-(\Delta_\eta + \Delta_k H(z,n)) + i(\eta_0 + k_0 H(z,n))\right]$$

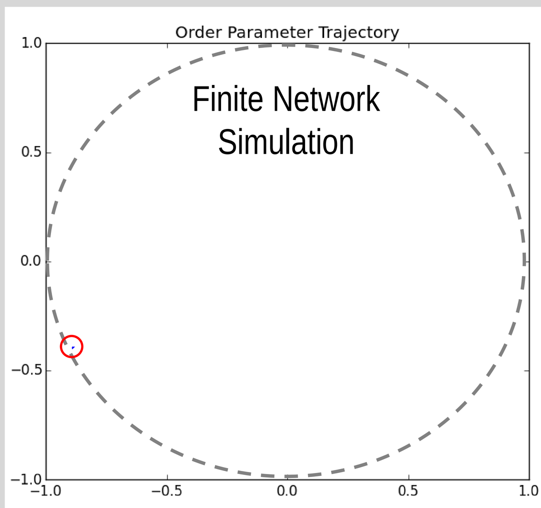
Possible Macro States

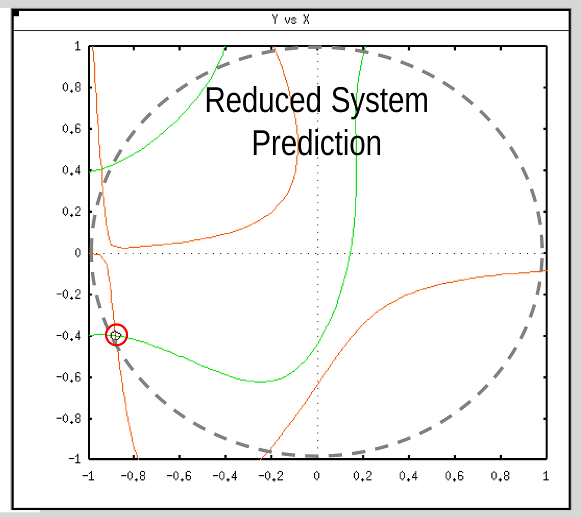
Partially Synchronous Spiking



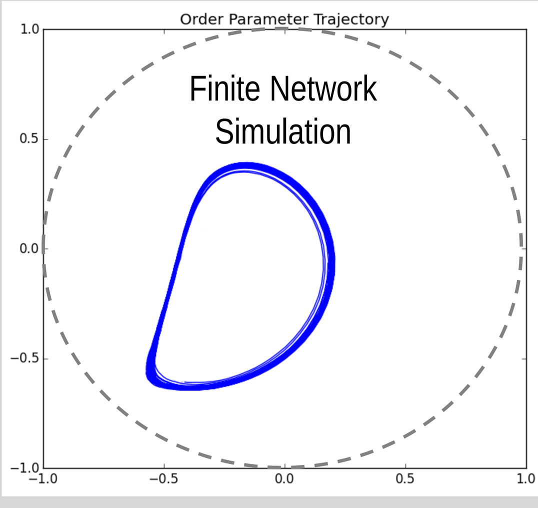


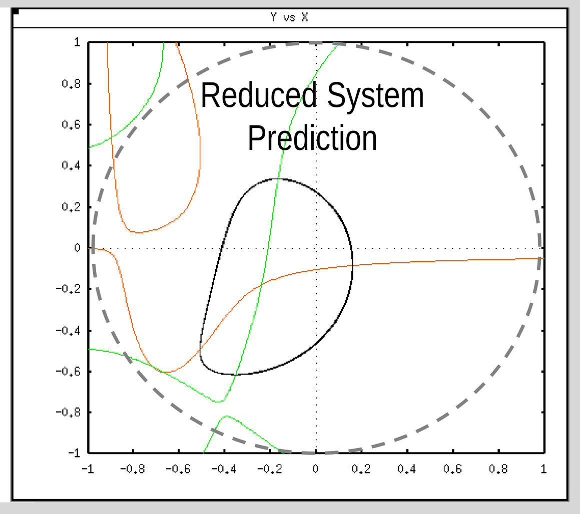
Partially Synchronous Resting





Collective Periodic Wave





Sparsely Coupled Network

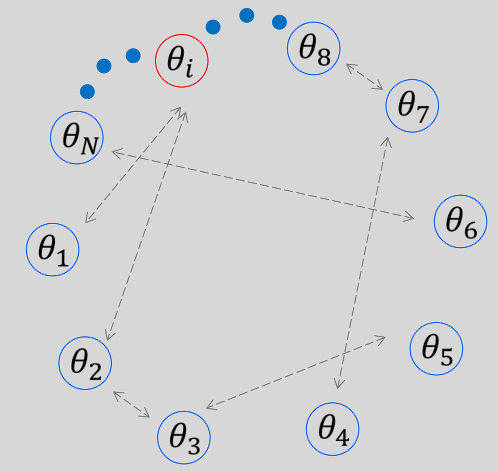
Conditions

$k_0 = 0$
Δ_k small

Most connections are very weak.

Result

Macroscopic mean field settles into a PSS equilibrium whether $\eta_0 < 0$ or $\eta_0 > 0$.



Largely Diverse Network

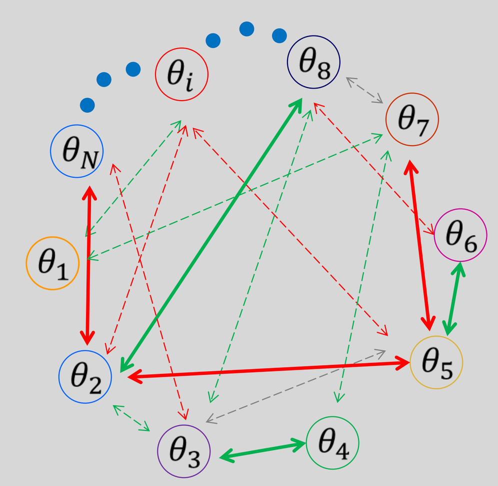
Conditions

Δ_k large
Δ_η large

There are all types of connections.

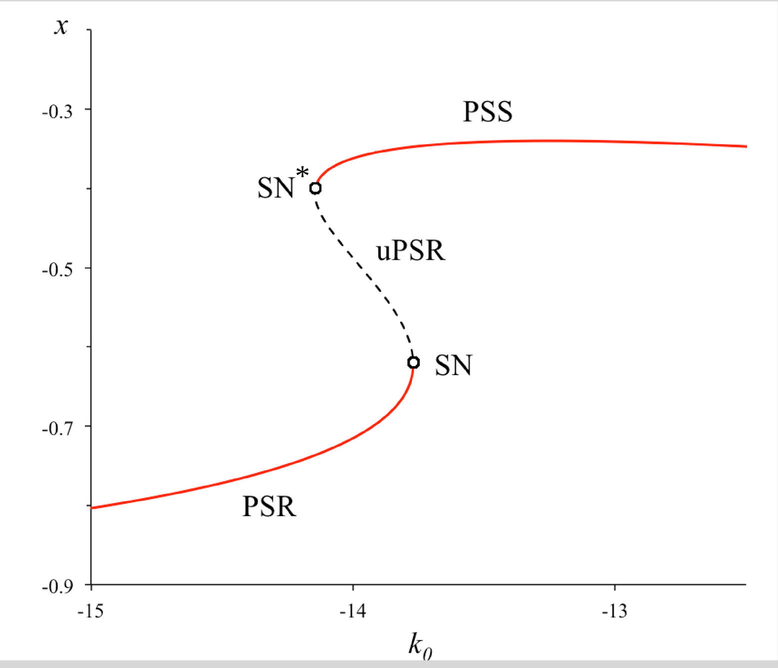
Result

Macroscopic mean field settles into a PSS equilibrium that tends towards $z(t) = -1$ as both Δ_η and Δ_k increase.

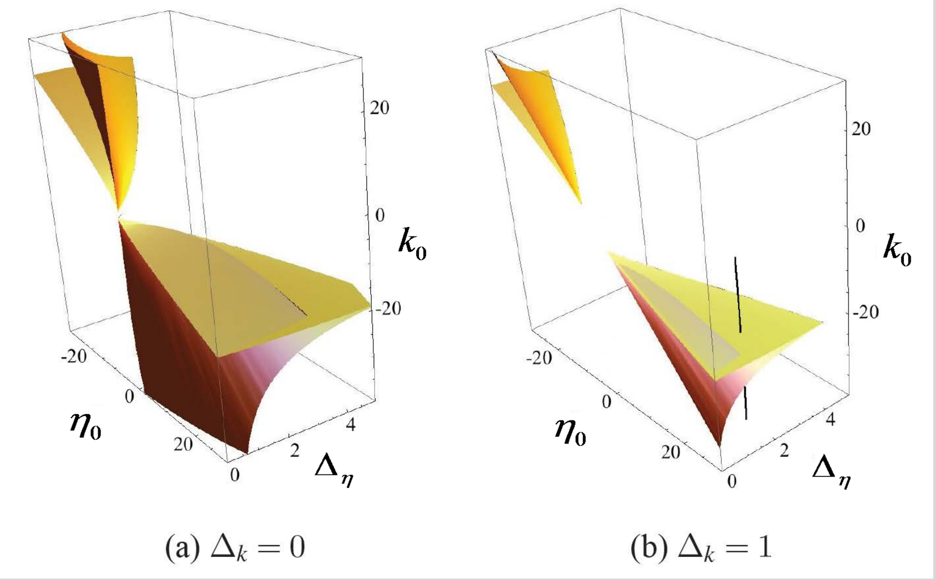


Effects of Synaptic Diversity: Saddle Node Bifurcation

The following one dimensional bifurcation diagram shows the equilibria when k_0 varies while $\eta_0 = 20$, $\Delta_\eta = 2$, and $\Delta_k = 1$.



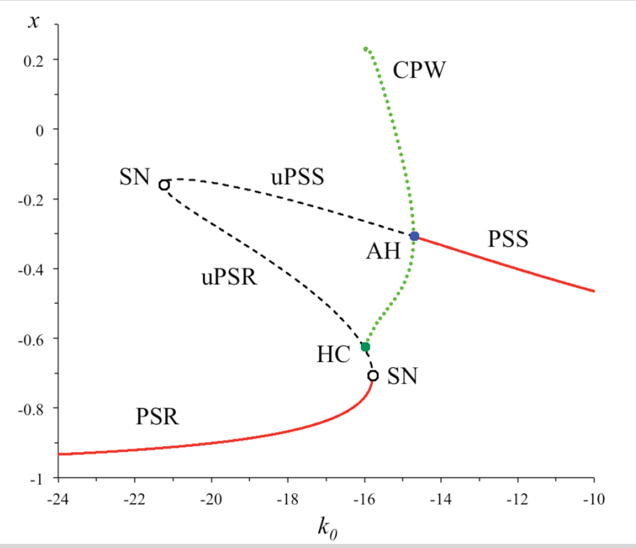
Saddle Node Bifurcation Evolution



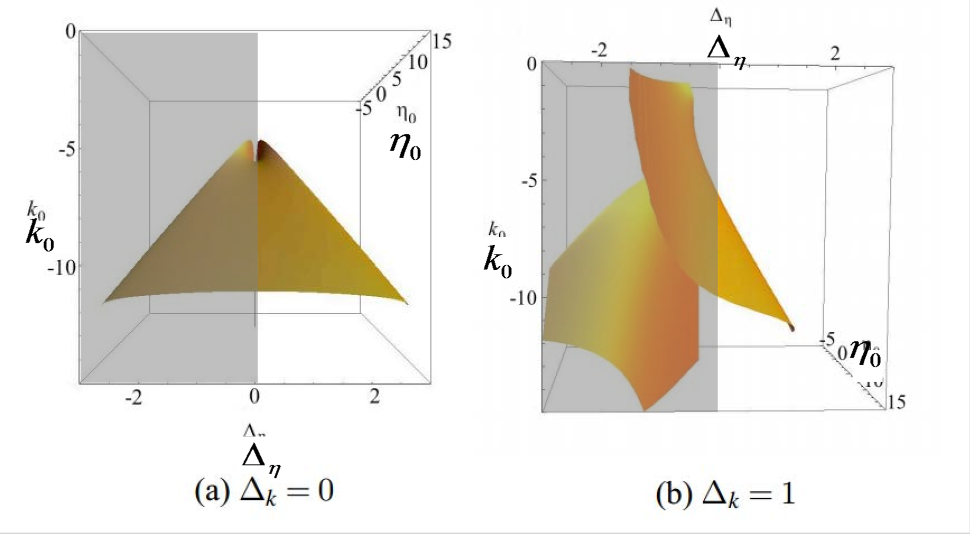
An increase in Δ_k decreases the complexity of macroscopic behaviors. The bifurcation surfaces shift into $\Delta_k < 0$, which is not physical

Effects of Synaptic Diversity: Hopf BIfucation

The following one dimensional bifurcation diagram shows the equilibria when k_0 varies while $\eta_0 = 24$, $\Delta_\eta = 0.5$, and $\Delta_k = 1$.

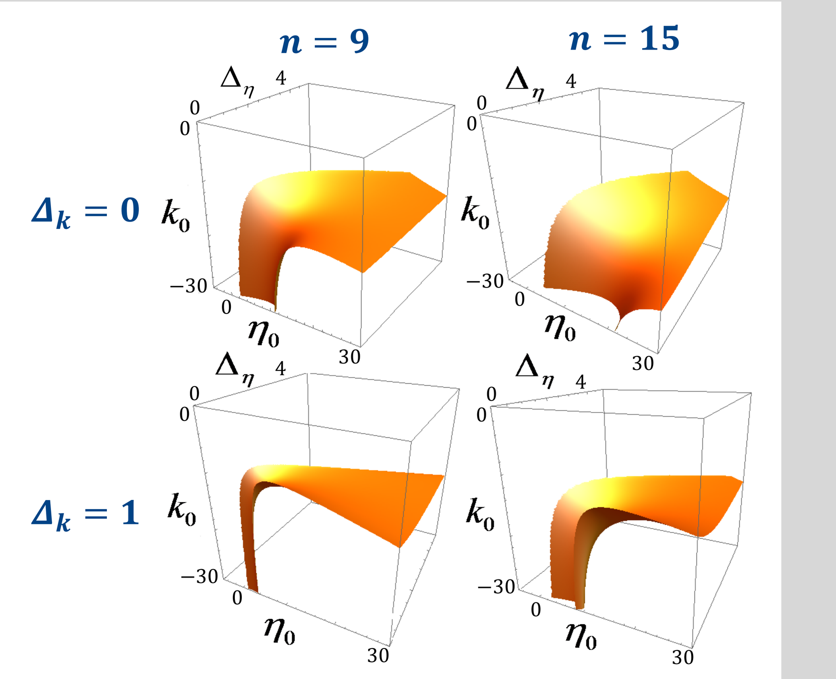


Andronov-Hopf Bifurcation Evolution



The Hopf bifurcation surface also shifts in the negative Δ_η direction as Δ_k increases.

Impact of Sharpness Parameter n



Heterogeneity in k allowed sharpness parameter n to impact macroscopic complexity. As n increased, robustness decreased.

Conclusions

- Obtained a low dimensional ODE for the asymptotic macroscopic mean field of a heterogeneous network of theta neurons with synaptic diversity
- Demonstrated that greater heterogeneity in coupling leads to decreased macroscopic complexity
- Demonstrated that heterogeneity in synaptic strength increases the impact of pulse sharpness on bifurcation complexity
- Our work suggests that maintaining both robustness and flexibility in a network requires moderate diversity in synaptic strength

References

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