

John David Crawford died on 23 August, 1998 from Burkitt's Lymphoma at the age of 44. John David graduated with honors from Princeton University in 1977 and with a doctorate in Physics from the University of California at Berkeley in 1983. His thesis on *Hopf Bifurcation and Plasma Instabilities* was written under the direction of Henry Abarbanel. During his career, John David spent six years at the University of California at San Diego, at first in the Physics Department working on non-neutral plasmas and subsequently at the Institute for Nonlinear Science pursuing his interests in bifurcation theory. In 1989 he held visiting positions at the Mathematics Institute at the University of Warwick and at the Institute for Fusion Studies at the University of Texas at Austin. He joined the faculty of the Department of Physics and Astronomy at the University of Pittsburgh in 1990.

John David's interests ranged from the physics of collisionless plasmas to the mathematics of pattern formation. However, there was a common thread: understanding the development and equilibration of instabilities in diverse systems, be they Hamiltonian or dissipative. This workshop focused on pattern formation in continuous systems, a subject to which John David contributed greatly. He worked on developing group-theoretic methods for use in pattern formation studies of dissipative systems and new techniques for studying bifurcation phenomena in Hamiltonian systems associated with the emergence of an eigenvalue from a continuous spectrum. The former area of research was motivated primarily by his interest in parametrically driven water waves (the Faraday system) and the latter by the beam-plasma instability in the Vlasov-Poisson system.

The Faraday instability is a subharmonic instability and is therefore associated with a Floquet multiplier at  $-1$ . John David's early work with Edgar Knobloch and Hermann Riecke [10]–[12] discussed mode interaction in the Faraday experiment in a circular container, focusing on the dynamics of discrete time- $T$  maps with  $-1$  Floquet multipliers of double multiplicity as appropriate for modes that break the  $O(2)$  symmetry of the container. Of particular interest was the classification of the conditions under which the mixed patterns resulting from such interaction drift azimuthally. Such rotating patterns were observed in experiments by Sergio Ciliberto and Jerry Gollub. His subsequent and classic work on the Faraday system in a square container [18] was also motivated by Gollub's experiments. In this work John David focused on understanding the hidden symmetries, both translations and rotations, introduced into the Faraday system by Neumann boundary conditions [18, 21]. These depend on the modes excited and on their degeneracy. John David's observation that as a result there is a significant difference between the Faraday system in a square container and one with  $D_4$  symmetry but in a nonsquare container [17] was confirmed in subsequent experiments by Gollub and David Lane [20]. Related work on parametrically modulated Hopf bifurcation in systems with  $O(2)$  symmetry [4, 13] predicted that such modulation would stabilize standing waves even in cases in which traveling waves were preferred in the absence of modulation. This prediction was also confirmed in elegant experiments by Victor Steinberg and David Anderock and their colleagues.

At the same time John David continued his studies of bifurcations in collisionless plasmas. Using the technique of spectral deformation developed in landmark papers with Peter Hisslop [8, 9] he was able to understand in detail the appearance of a neutral eigenvalue (or mode) embedded in a continuous spectrum at threshold for instability. In this problem, as in the closely related shear flow problems for ideal fluids, the instability appears when the electron distribution function or shear flow profile are gradually changed, for example, by injecting a beam of fast electrons to create a bump on the tail of the electron distribution or changing the pressure distribution driving the

flow. However, because of the presence of the continuum center manifold theory cannot be used to study the resulting bifurcation. John David's understanding of the structure of the linear problem led him to consider the equilibration of the resulting instability using the instability growth rate  $\gamma$  as the bifurcation parameter. In a remarkable paper [23] he showed that in the limit of fixed (i.e. heavy) ions the instability saturates at  $O(\gamma^2)$  amplitude, in contrast to the  $O(\gamma^{\frac{1}{2}})$  amplitude familiar from dissipative systems. This result is *nonperturbative*, and terms of all orders contribute to the equilibration as  $\gamma \rightarrow 0$  [24]. Thus not only do these instabilities saturate at a much smaller amplitude but they do not have to approach the equilibrium monotonically. The predicted  $\gamma^2$  "trapping" scaling agrees with numerical and experimental observations. Subsequent work by John David's student Anand Jayaraman [27, 30] generalized these conclusions to mobile ions showing that the scaling changes to  $\gamma^{\frac{5}{2}}$ .

While engaged in this work John David realized that similar mathematics applies to the Kuramoto model of phase-coupled oscillators. This model consists of many globally coupled oscillators with frequencies drawn from a prescribed frequency distribution and exhibits a remarkable "phase transition" as the strength  $K$  of the interaction increases in which the oscillators begin to phase-lock. As in the Vlasov-Poisson system the stability problem for the incoherent state has a continuous spectrum and this state loses stability at  $K = K_c$  when an unstable eigenvalue pops out of the neutral continuum. As a result of a calculation to all orders similar to the plasma one, John David showed [22] that the saturated amplitude (the fraction of synchronized oscillators) scales like  $(K - K_c)^{\frac{1}{2}}$  for the Kuramoto model but scales like  $K - K_c$  for more general couplings than assumed by Kuramoto [25, 29]. These results resolve analytically several long-standing issues in both theoretical and numerical studies of this important model.

John David wrote two influential review articles, one on basic bifurcation theory [19] and one with Edgar Knobloch on the use of equivariant bifurcation theory for studies of pattern formation in fluid dynamics [16]. A bibliography of John David's contributions to pattern formation and bifurcation theory is included below.

John David was a consummate scholar, devoted to deep understanding of important and challenging problems. His solutions to these problems were always innovative offering a fresh perspective. At home both in physics and mathematics John David was an invaluable colleague, generous with his time and ideas, and a rare knack for explaining scientific principles to friends, colleagues and students. His lectures were a model of clarity and he was a much sought-after speaker. At the workshop his delight in being back in the milieu he so loved was almost palpable. He will be greatly missed by all of us.

Edgar Knobloch  
October, 1998

Click "Next" (below) for bibliography.

# References

- [1] J.D. Crawford and S. Omohundro. On the global structure of period doubling flows, *Physica D* **13** (1984) 161–180.
- [2] J.D. Crawford and E. Knobloch. Symmetry breaking bifurcations in  $O(2)$  Maps, *Phys. Lett. A* **128** (1988) 327–331.
- [3] J.D. Crawford and E. Knobloch. Classification of  $O(2)$  symmetric Hopf bifurcation: no distinguished parameter, *Physica D* **31** (1988) 1–48.
- [4] H. Riecke, J.D. Crawford and E. Knobloch. Time-modulated oscillatory convection, *Phys. Rev. Lett.* **61** (1988) 1942–1945.
- [5] J.D. Crawford and E. Knobloch. On degenerate Hopf bifurcation with broken  $O(2)$  symmetry, *Nonlinearity* **1** (1988) 617–652.
- [6] J.D. Crawford, M. Golubitsky, and W. Langford. Modulated rotating waves in  $O(2)$  mode interaction, *Dynamics and Stability of Systems* **3** (1988) 159–175.
- [7] J.D. Crawford and P. Hislop. Vlasov equation on a symplectic leaf, *Phys. Lett. A* **134** (1988) 19–24.
- [8] J.D. Crawford and P. Hislop. Application of the method of spectral deformation to the Vlasov-Poisson model, *Ann. Phys.* **189** (1989) 265–317.
- [9] P. Hislop and J.D. Crawford. Application of the method of spectral deformation to the Vlasov-Poisson system II, *J. Math. Phys.* **189** (1989) 2819–2837.
- [10] J.D. Crawford, E. Knobloch, and H. Riecke. Competing parametric instabilities with circular symmetry, *Phys. Lett. A* **135** (1989) 20–24.
- [11] J.D. Crawford, E. Knobloch, and H. Riecke. Mode interactions and symmetry, in *Proc. Intern. Conf. on Singular Behavior and Nonlinear Dynamics*, vol. 1, S. Pnevmatikos *et al.* (eds), World Scientific, 1989, 271–291.
- [12] J.D. Crawford, E. Knobloch, and H. Riecke. Period-doubling mode interactions with circular symmetry, *Physica D* **44** (1990) 340–396.
- [13] H. Riecke, J.D. Crawford and E. Knobloch. Temporal modulation of a subcritical bifurcation to travelling waves, in *New Trends in Nonlinear Dynamics and Pattern-Forming Phenomena: The Geometry of Nonequilibrium*, P. Coullet and P. Huerre (eds), NATO ASI Series B **237**, Plenum Press, 1991, 61–64.
- [14] J.D. Crawford, M. Golubitsky, M.G.M. Gomes, E. Knobloch, and I. Stewart. Boundary conditions as symmetry constraints, in *Singularity Theory and its Applications, Warwick 1989* Part 2, R.M. Roberts and I.N. Stewart (eds), Lecture Notes in Mathematics, Springer-Verlag, 1991, 63–79.

- [15] J.D. Crawford. Amplitude equations on unstable manifolds: singular behavior from neutral modes, in *Modern Mathematical Methods in Transport Theory* (Operator Theory: Advances and Applications, vol. 51), W. Greenberg and J. Polewczak (eds), Birkhauser Verlag, 1991, 97–108.
- [16] J.D. Crawford and E. Knobloch. Symmetry and symmetry-breaking bifurcations in fluid dynamics, *Annu. Rev. Fluid Mech.* **23** (1991) 341–387.
- [17] J.D. Crawford. Surface waves in non-square containers with square symmetry, *Phys. Rev. Lett.* **67** (1991) 441–444.
- [18] J.D. Crawford. Normal forms for driven surface waves: boundary conditions, symmetry, and genericity, *Physica D* **52** (1991) 429–457.
- [19] J.D. Crawford. Introduction to bifurcation theory, *Rev. Mod. Phys.* **63** (1991) 991–1037.
- [20] J.D. Crawford, J.P. Gollub, and David Lane. Hidden symmetries of parametrically forced waves, *Nonlinearity* **6** (1993) 119–164.
- [21] J.D. Crawford.  $D_4 \times T^2$  Mode interactions and hidden rotational symmetry, *Nonlinearity* **7** (1994) 697–739.
- [22] J.D. Crawford. Amplitude expansions for instabilities in populations of globally coupled oscillators, *J. Stat. Phys.* **74** (1994) 1047–1084.
- [23] J.D. Crawford. Universal trapping scaling on the unstable manifold of a collisionless electrostatic mode, *Phys. Rev. Lett.* **73** (1994) 656–659.
- [24] J.D. Crawford. Amplitude equations for electrostatic waves: universal singular behavior in the limit of weak instability, *Phys. Plasmas* **2** (1995) 97–128.
- [25] J.D. Crawford. Scaling and singularities in the entrainment of globally coupled oscillators, *Phys. Rev. Lett.* **74** (1995) 4341–4344.
- [26] J.D. Crawford.  $D_4$ -symmetric maps with hidden Euclidean symmetry, in *Pattern Formation: Symmetry Methods and Applications*, J. Chadam et al. (eds) Amer. Math. Soc., 1995, 93–124.
- [27] J.D. Crawford and A. Jayaraman. Nonlinear saturation of electrostatic waves: mobile ions modify trapping scaling, *Phys. Rev. Lett.* **77** (1996) 3549–3552.
- [28] J.D. Crawford and E. Knobloch. Amplitude equations for coupled electrostatic waves in the limit of weak instability, *J. Plasma Phys.* **60** (1998) 159–180.
- [29] J.D. Crawford and K.T.R. Davies. Synchronization of globally coupled oscillators: singularities and scaling for general couplings, *Physica D* **125** (1999) 1–46.
- [30] J.D. Crawford and A. Jayaraman. Amplitude equations for electrostatic waves: Multiple species, *J. Math. Phys.*, **39** (1998) 4546–4576.
- [31] J.D. Crawford and A. Jayaraman. First principles justification of a “Single Wave Model” for electrostatic instabilities, *Phys. Plasmas*, **6** (1999) 666–673.