STABILITY AND BIFURCATION OF SWITCHED DYNAMICAL SYSTEM (SBSDS)

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This is an automated tool for the steady-state time -domain analysis as well as the stability analysis of Switched Dynamical Systems (SDS). It is based on the fundamental solution matrix and can be applied to any switching dynamical system.

STEPS TO FOLLOW FOR USING THE TOOL:

MATLAB provides a mechanism to generate Graphical User Interface (GUI), by using GUIDE (the standard tool within MATLAB). Two .m files have been provided: SBSDS_GUI.fig and SBSDS_GUI.m.

- Double-click on the file SBSDS_GUI.m file in the folder. Once
- The MATLAB Command prompt opens up, ‘Run’ the .m file
- The graphical user interface named SBSDS_GUI opens up.
- You can load some existing systems by clicking on ‘**Load System**’ (Top left corner of the GUI). It directs to folder ‘Systems’ inside the ‘SBSDS’ directory. Inside ‘Systems’, different files for different systems are already stored with the .mat extension.
- A new system can be saved after giving appropriate inputs to 6 blocks namely: (i) **System Description, (ii) Parameters, (iii) System Info, (iv) Bifurcation Parameters, (v) Initial Conditions, (vi) ODE Solver Parameters.** Then save the given inputs to the six blocks by clicking ‘**Save System**’. Give suitable name with .mat extension and put it inside ‘Systems’ folder.

The following will show how to give inputs to the six different blocks:

(i) System Description:

 Write the system description in brief. This is optional i.e., the input to this section will not be included in the main program. Example: Current Mode Controlled Buck Converter.

(ii) System Info:

 System type: (a) Piecewise-Linear ODE, (b) Piecewise-Nonlinear ODE, (c) Piecewise-Linear DAE etc.

 The ‘**Dimensions**’ of the system and ‘**Number of Subsystems**’ need to be entered. The ‘**Type**’ of periodic orbit (i.e. Non-Autonomous Time Periodic/Autonomous) is to be selected. An input dialog box is used to receive the value of the fixed/ variable time period. For Non-Autonomous Time period System the Time period (T) appears in the list of possible bifurcation parameters.

NOTE: The user must select the system type and type of the periodic orbit as no default value is given.

(iii) Parameters:

 System parameters can be added by pressing "**Add**" in the parameters panel. They are entered in accordance with 'Name=Value' following the MATLAB syntax e.g. L=1e-3, C=4.5e-6.

 The" **Modify**" and "**Delete**" buttons are used to modify/ delete the selected entry in the parameters.

NOTE: The bifurcation parameter is to be provided in the parameter list also.

(iv) Subsystems:

The information about the various subsystems is to be entered in this panel.

 M: denotes the i^{th} number of subsystem.

 f(*x*,t): The vector fields of the particular subsystem are entered in this box.

 J: The Jacobian Matrix for the particular subsystem are entered in this box.

 h1@cond1@cond2#h2@cond1@cond2: The various switching conditions for the particular subsystem are to be added in this list box. The different switching conditions are separated by "**#**". Additional conditions within a particular switching surface are added with "**@**".**NOTE**: The entire entry must be enclosed within square brackets ([])

 Transitions: The transitions from the particular subsystem are entered in this list box. These transitions must be order with the switching conditions entered in the [1@2#2] cond list box. **NOTE:** The entry must be enclosed within square brackets ([])

 Grad(h): The gradient of the switching conditions is to be entered in this list box. These must be same order as the entries in the switching conditions list box.

 del(h)/del(t): The time derivative of the switching conditions is to be entered in the list box. These must be same order as the entries in the switching conditions list box.

(iv) Bifurcation Parameters:

 All parameters entered in the **Parameters** list appear in the list of bifurcation parameters (**Bif. Par**). From these the required bifurcation parameter is to be selected and its **Range** [Initail:Stepsize:Final] is to be provided.

 The list of switching surfaces appears in the list of **Poincare section**. Here too, the required Poincare section is to be selected.

(v) Initial Conditions:

 The initial conditions, i.e., the **Initial State, Xo** and the **Initial Subsystem** is to be entered in this list.

(vi) ODE Solver Parameters:

 Type of ODE solver can be selected from the available list depending upon the requirements of the system (stiff or nonstiff).

 Settings: The ODE solver settings are also mandatorily required to be entered by the user for speed and accuracy. Only three are included namely Relative Tolerance ('**RelTol**'), Absolute Tolerance ('**AbsTol**') and Maximum step-size ('**MaxStep**')

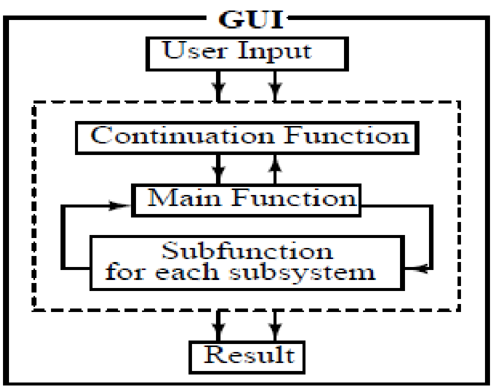
Results:

Once the inputs have been provided to the data blocked, the user can obtain the time-domain simulation or the Stability Analysis of the given systems.

1. Steady-State Analysis: The "**Run**" button starts the time-domain simulation. On pressing this pushbutton, the user requires to input **the number of cycles** and the **Pre-iterates**. The time-domain simulation and the bifurcation diagram is then run for the specified number of

cycles and plotted up to the number of pre-iterates. In the **Graphical** section user can see the W**aveforms** of the selected state variables and **Brute-Force Bifurcation Diagram** for the selected range of bifurcation parameter.
NOTE: The appropriate number of cycles should be entered so that the transients die out.
2. **Stability Analysis** -- The "**Compute**" button runs the stability analysis by the Newton-Raphson method. For this method we have to put three parameters namely, **Convergence Tolerance, Maximum iterations,** and **Periodicity**. Periodicity is needed because there may have many periodic orbits.
User can see the fixed point, Eigenvalues, subsystem sequences, stability status from the **Numerical** section in **Display** section. From the **Graphical** section we get Time Response, Phase Space, Path-Following Bifurcation Diagram, Locus of Max. Eigenvalue and Locus of all Eigenvalues.

Structure of the program inside GUI

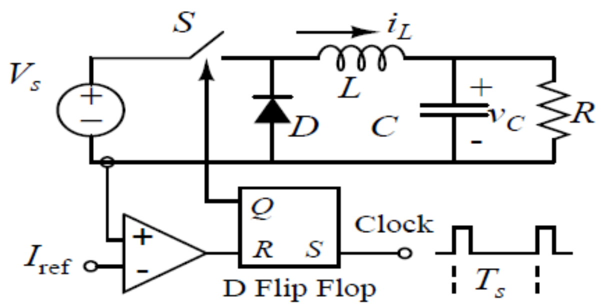


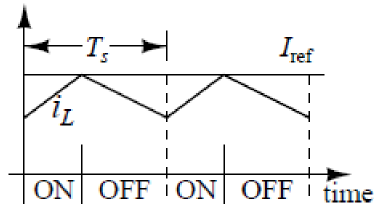
By giving input to the edit fields in the GUI, the existing system for analysis are kept in the folder "System" (xx.mat format). After loading the existing system in the GUI and Clicking the "**Run**" and "**Compute**" pushbutton, .m files are generated. All the .m files are kept in the "temp_m_files_SS" and "temp_m_files_SA" folder. There are 7 .m files for a system with 2 subsystems.
The subfunctions (temp_parameters.m, temp_subsystem1.m, temp_fields.m, temp_subsystem2.m, temp_sw_surface.m) are called by the main function temp_main_NR.m. The subfunctions also depend on others according to the requirement. Finally to show the result, temp_continuation.m calls temp_main_NR.m. Results are stored in .mat format for future use.

The steps to follow for loading the system in GUI:

(a) Determine the type of system (Piecewise-linear or Piecewise nonlinear etc) along with the type of periodic orbit (autonomous or nonautonomous).
(b) Write down a set of **first-order** differential equations for each subsystem (M_i).
(c) Find out the Jacobian for each subsystem.
(d) Find out all the switching surfaces (h_1,h_2..) for transition from one subsystem to another. (transition diagram)
(e) Find out the gradient and rate of change of the switching conditions.
(f) List of all parameters values.

Example: Buck Converter with Current-Mode Control





Current Waveform

Parameters
$V_s = 20$ V, $R_L = 10\ \Omega$, $f_s = 30$ kHz,
$L = 0.62$ mH, $C = 1$ mF, $I_{\text{ref}} = 1$ A.

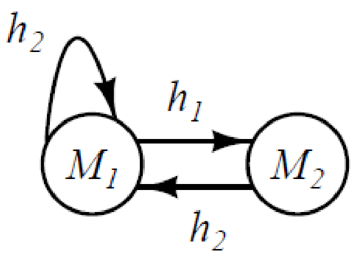
From "hybrid system" point of view the system can be modeled as

$$\frac{dx}{dt} = \begin{cases} M_1 : \boldsymbol{A}_1 x + \boldsymbol{B}_1 u & \text{S is ON} \\ M_2 : \boldsymbol{A}_2 x + \boldsymbol{B}_2 u & \text{S is OFF} \end{cases}$$

where, $x = [i_L \ v_C]^T = [x_1 \ x_2]^T$, $u = V_s$ and the coefficient matrices are

$$\boldsymbol{A}_1 = \boldsymbol{A}_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The switching surfaces are given by $h_1 : x_1 - I_{\text{ref}} = 0$, and $h_2 : t \ mod \ T_s = 0$.



Transition diagram

EXAMPLE 1: Data input of the above system [buck_current.mat]
System Description: Current-mode controlled Buck Converter
System Info:

Piecewise Linear ODE
Dimensions: 2
Number of Subsystems:2
Type: Non Autonaumous Time Periodic
Time Period=1/(30e3)

Subsystems:

Subsystem 1:

f1=[-x(2)/L+Vs/L;x(1)/C-x(2)/(C*R)]
J1=[0,-1/L;1/C,-1/(C*R)]
h1=[x(1)-Iref#t-count*T]
Transitions:[2#1]
grad (h)=[1;0]#[0]
del(h)/del(t)=[0#inf]

Subsystem 2:

f2=[-x(2)/L;x(1)/C-x(2)/(C*R)]
J2=[0,-1/L;1/C,-1/(C*R)]
h2=[t-count*T]
Transitions:[1]
grad (h)=[0]
del(h)/del(t)=[inf]

Bifurcation Parameter:

Bif. Par: Vs
Range: 20:1:20
Poincare Section=[t-count*T]

Initial Conditions:

Initial State, Xo=[0.79;8.8]
Initial Subsystem=1

ODE Solver Parameter:

Type: ode45
Settings: Relative Tolerance (‘**RelTol**’)=1e-4,
Absolute Tolerance (‘**AbsTol**’)=1e-6 and
Maximum step-size (‘**MaxStep**’)=1e-7

Steady-State Analysis:
Number of cycles=20
Pre-iterates=10

Stability Analysis: Convergence Tolerance=1e-3
Max.Iteration Number=100
Periodicity=1