28 Models Later: Modeling a Zombie Apocalypse with Real Data

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Introduction

The Zombie Apocalypse has captured public imagination, interacting with real-world scares of global pandemic and introducing the lay person to mathematical population modeling to predict the consequences of outbreaks. However, the most common model, the venerable Susceptible-Infected-Removed (SIR) model [2], poorly describes the course of a zombie apocalypse because zombies act more like hunting animals than passive repositories of disease. In this poster we use techniques of predator-prey modeling and model competition to uncover the mechanisms driving a zombie apocalypse, using data collected from the Humans vs. Zombies (HvZ) game of moderated tag played at USU in Fall 2011 [3] to judge competing models and determine most likely mechanisms. Models were fit to data using minimum sum squared error (SSE) and judged in relation to observations using Bayesian (Schwartz) Information Criterion.

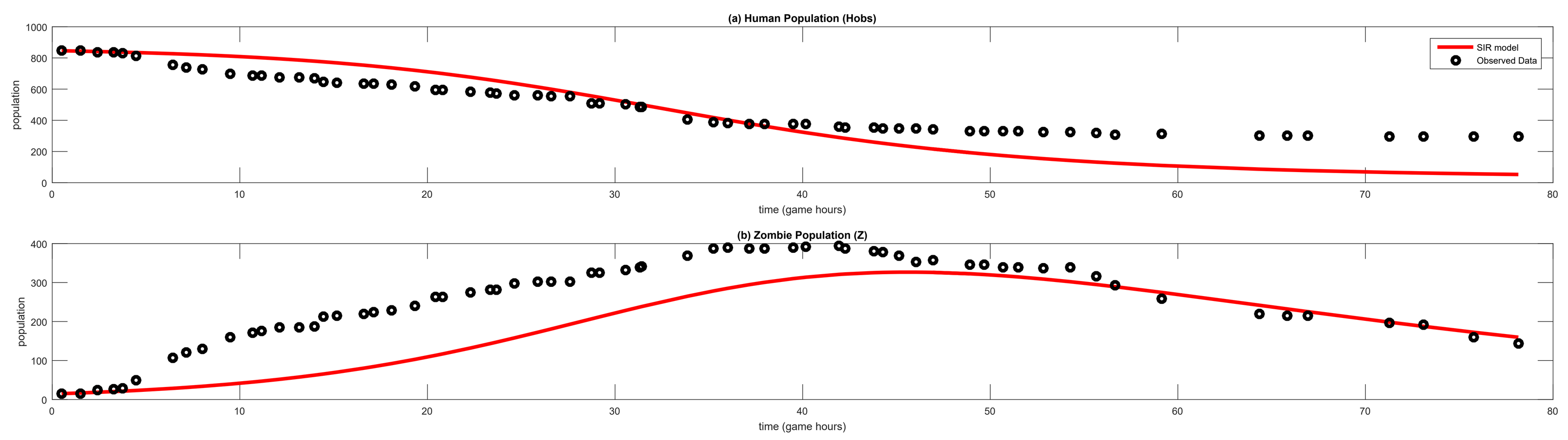
HvZ: Rules governing the Campus Apocalypse (2011)

- Players divided: 846 humans, 15 zombies
  - → No new humans
- Zombies attack humans
  - → One new zombie per attack
  - → Zombies never recover
- Hunting allowed from 7am to 9pm
- Academic buildings off limits
- Zombies starve without daily attacks
- One zombie gets credit per attack
- Humans can quit since human population is calculated, not observed
- Humans fight back with Nerf guns

Failure of SIR Model to Describe HvZ

- Transmission modeled using Law of Mass Action
- Starvation equally likely for each zombie
- Effective model for measles, mumps, rubella

$$\dot{H} = -aHZ$$
$$\dot{Z} = aHZ - mZ$$
$$\dot{R} = mZ$$



Best fit of SIR model to 2011 USU HvZ data. Transmission via random encounter is inaccurate. The model would predict virtually no survivors when in reality 30% of the population survives the apocalypse.

Modeling HvZ as a Predator-Prey System

$$\dot{H}_{\text{obs}} = -\underbrace{A(H,Z)}_{\text{Zombie attacks}}, \quad \dot{H} = -\underbrace{A(H,Z)}_{\text{Zombie attacks}} - \underbrace{Q(H,Z)}_{\text{Human quitting}}, \quad \dot{Z} = \underbrace{A(H,Z)}_{\text{Zombie attacks}} - \underbrace{S(H,Z)}_{\text{Zombie starvation}}$$

Models are fit to data by minimizing

$$\text{SSE} = \sum_{n=1}^{N}\left(\left(H_{obs}(t_n) - H_n\right)^2\right) + \sum_{n=1}^{N}\left(\left(Z(t_n) - Z_n\right)^2\right)$$

over the space of relevant parameters using fmincon in MATLAB.

Bayesian Information Criterion [1]

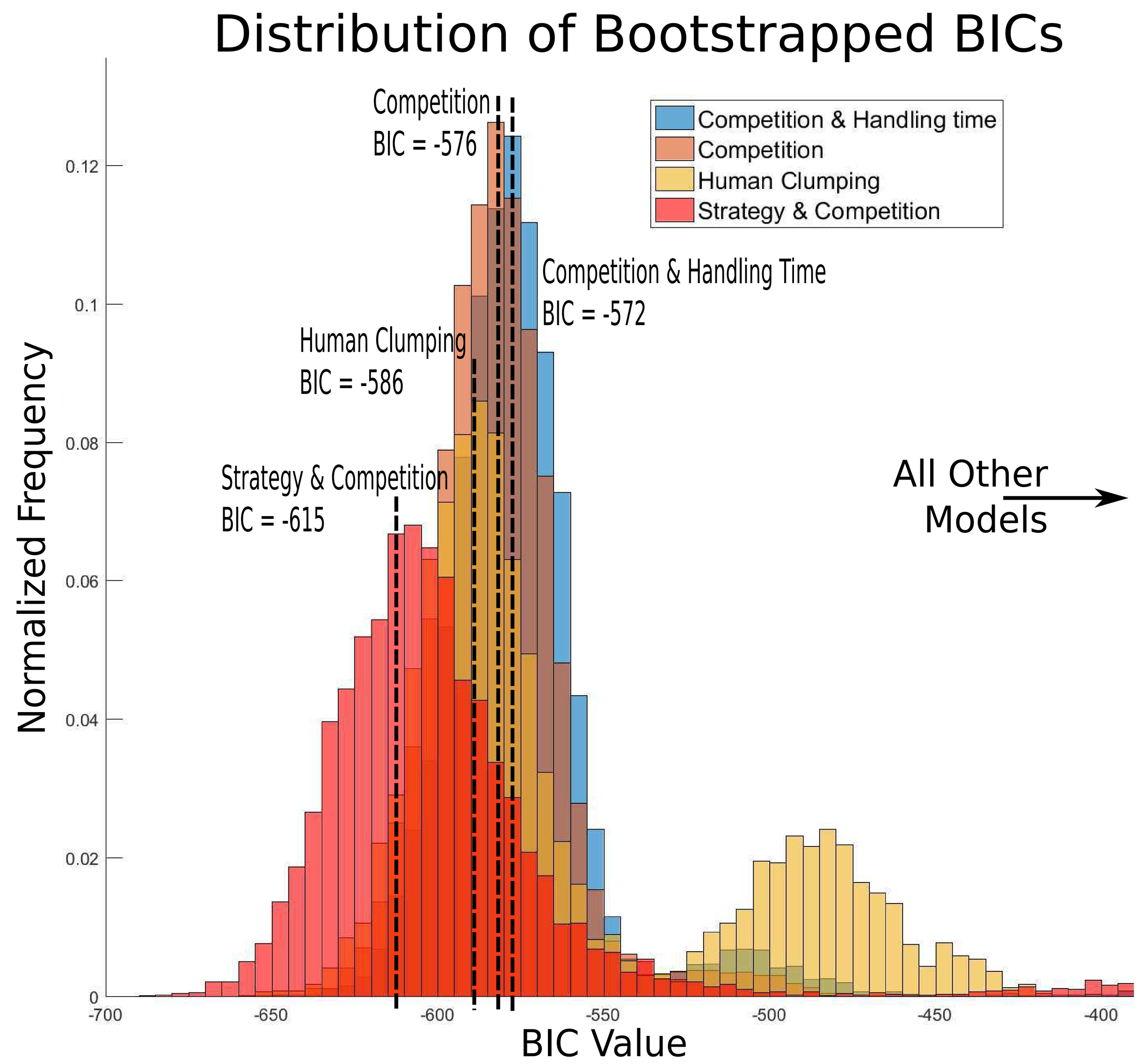
BIC is an info-theoretic metric balancing goodness of fit and model complexity as measured by number of parameters; lower BIC is better. For N data points and K parameters

$$\text{BIC} = \underbrace{2N\ln(2\pi\sigma^2) + \frac{1}{\sigma^2}\sum_{n=1}^{N}\left((\overbrace{Z_n}^{\text{data}} - \overbrace{Z(t_n,\boldsymbol{\theta})}^{\text{prediction}})^2 + (\overbrace{H_n}^{\text{data}} - \overbrace{H_{\text{obs}}(t_n,\boldsymbol{\theta})}^{\text{prediction}})^2\right)}_{\text{goodness of fit}} + \underbrace{(K\ln(N))}_{\text{parameter penalty}}$$

Governing Mechanisms for 28 Models

Attack mechanism	$A(H,Z)$	
Random Encounter	aHZ	Mass Action (SIR)
Saturation due to Prey 'Handling'	$\frac{aHZ}{1+at_hH}$	t_h = handling time
Zombie Competition for Prey	$\frac{aHZ}{1+at_cZ}$	t_c = competition time
Handling & Competition	$\frac{aHZ}{1+at_hH+at_cZ}$	
Effective Hunting Strategy	$a\left(\frac{H}{H_0}\right)^{\theta}Z$	$\theta < 1$ [5]
Strategy & Competition	$\frac{a\left(\frac{H}{H_0}\right)^{\theta}Z}{1+at_hH+at_cZ}$	
Human Clumping	$\frac{aHZ}{\left(1+\frac{r_AZ}{k_A}\right)^{k_A}}$	$k_A >> 1 \rightarrow$ humans evenly distributed [4]
Starvation	$S(H,Z)$	
Per-Capita Starvation	mZ	m = starvation parameter
Uneven Attack Distribution	$\frac{mZ}{\left(1+\frac{r_SH}{k_S}\right)^{k_S}}$	$k_S >> 1 \rightarrow$ humans evenly distributed [4]
Quitting	$Q(H,Z)$	
None	0	
Per-Capita Quitting	qH	q = quitting parameter

Model Competition to Describe Data [1]



Results of 10,000 bootstrap samples indicate that the Strategy & Competition model performs better than its closest competitor, the Human Clumping model, and much better than any other considered model. Best-performing models ALL included Per-Capita Quitting and Uneven Attack Distribution among zombies.

References

[1] Hilborn, Ray and Marc Mangel. *The Ecological Detective*. Princeton Universtiy Press. 1997.

[2] Kermack, W.O. and McKendrick, A.G. *Contributions to the mathematical theory of epidemics, part i*. Proc. Roy. Soc. of Edinburgh. Section A. Mathematics. Vol 138, pp. 55-83. 1927.

[3] Lewis, M. and J. Powell. *Modeling zombie outbreaks: A problem-based approach to improving mathematics one brain at a time*. PRIMUS 26(7): 705-726. 2016.

[4] May, Robert. *Host-Parasitoid Systems in Patchy Environments: A Phenomenological Model*. Journal of Animal Ecology, Vol 47, No 3. October 1978.

[5] Sibly et. al. *On the Regulation of Populations of Mammals, Birds, Fish, and Insects*. Science, Vol 309. 2005.

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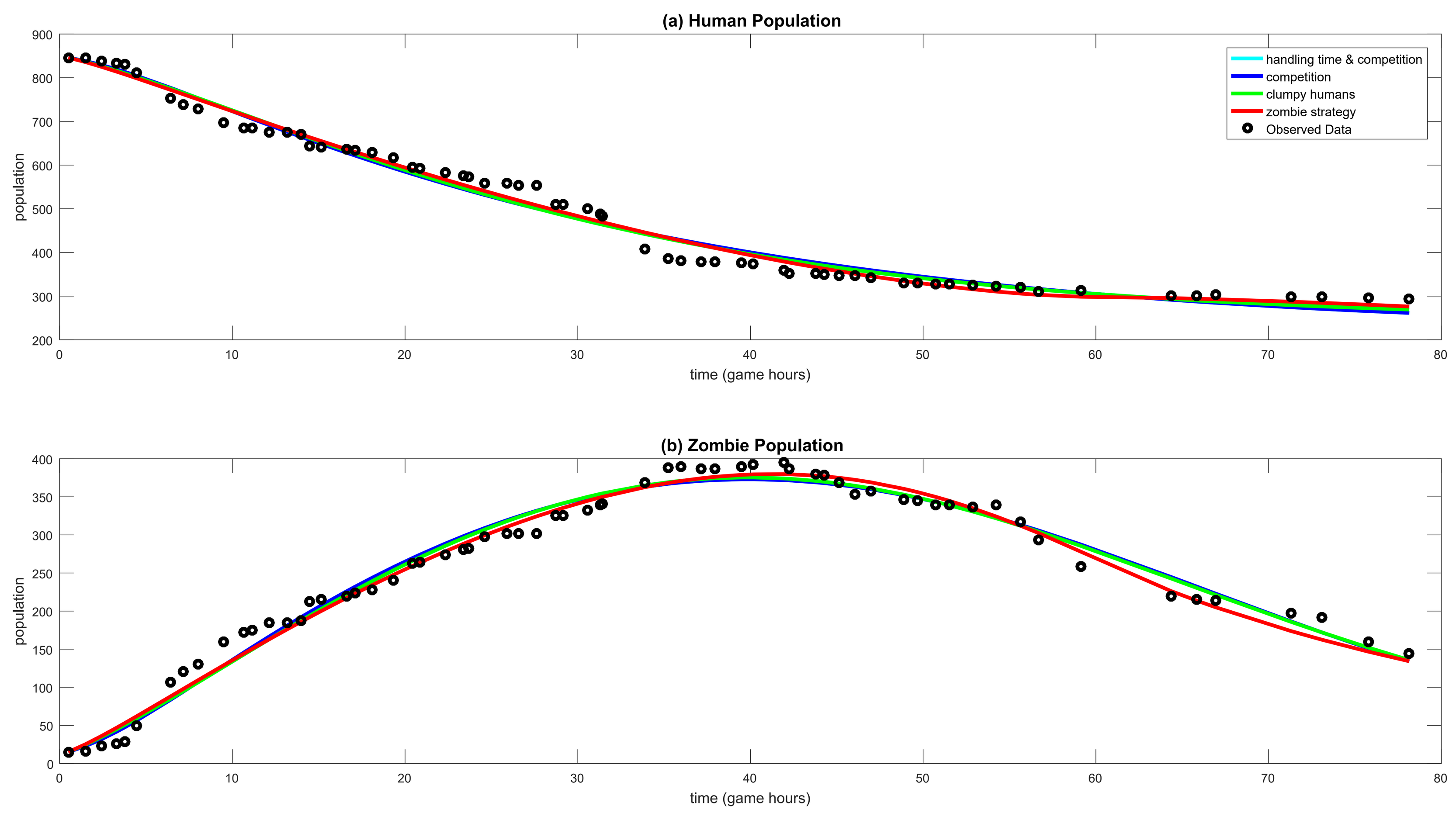
Four Most Successful Models

Attack Mechanisms $A(H,Z)$	Starvation Mechanism $S(H,Z)$: $Q(H,Z)=0$, mZ	$Q(H,Z)=0$, $\frac{mZ}{\left(1+\frac{rH}{k_S}\right)^{k_S}}$	$Q(H,Z)=qH$, mZ	$Q(H,Z)=qH$, $\frac{mZ}{\left(1+\frac{rH}{k_S}\right)^{k_S}}$
aHZ	-163	−179	−414	−223
$\frac{aHZ}{1+at_hH}$	−159	−193	−410	−409
$\frac{aHZ}{1+at_fZ}$	−377	−534	−464	-576
$\frac{aHZ}{1+at_hH+at_fZ}$	−373	−526	−400	-572
$a\left(\frac{H}{H_0}\right)^{\theta_A}Z$	−435	−484	−437	−480
$\frac{a\left(\frac{H}{H_0}\right)^{\theta_A}Z}{1+at_fZ}$	−437	−561	−471	-615
$\frac{aHZ}{\left(1+\frac{rZ}{k_A}\right)^{k_A}}$	−391	−524	−469	-586

Plotted below are the four most successful models compared with data. Each of the four includes per-capita quitting and non-random distribution of attacks among zombies; the best model (red) reflects better-than-random hunting strategy as well as competition among zombies for human prey. BIC ranged from -572 to -615 for these models; by comparison the BIC of the best fitting SIR model was only -163.



Conclusions Regarding HvZ Mechanisms

$$\dot{H}_{\text{obs}} = -\frac{a\left(\frac{H}{H_0}\right)^{\theta}Z}{1+at_cZ}$$

$$\dot{H} = -\frac{a\left(\frac{H}{H_0}\right)^{\theta}Z}{1+at_cZ} - qH$$

$$\dot{Z} = \frac{a\left(\frac{H}{H_0}\right)^{\theta}Z}{1+at_cZ} - \frac{mZ}{\left(1+\frac{r_SH}{k_S}\right)^{k_S}}$$

Fitting of our best model gives parameter values of $a = 1.39$, $t_c = 51.74$, $\theta = .36$, $m = .041$, $r_S = 8.59$, $k_s = 1.48$, and $q = .017$. From this we determine that zombies compete for prey, as $t_c \neq 0$. The game rules dictate that if a group of zombies attack a human only one zombie registers a kill so this makes sense. Additionally, since $\theta < 1$, zombies hunt with a strategy that is more productive than random encounters. Further, since $k_s = 1.48$ attacks are not evenly distributed among zombies. Thus a minority of zombies are responsible for a majority of attacks. Finally, humans are quitting the game at a rate of 1.7% of the population per game hour. Residual variability seems to follow a diurnal cycle, with slightly higher attack rates later in the day. The diurnal variability has not yet been explored but it may explain the small fluctuations of the data.